Resonance Capture and Stability Analysis for Planet Pairs under Type I Disk Migration

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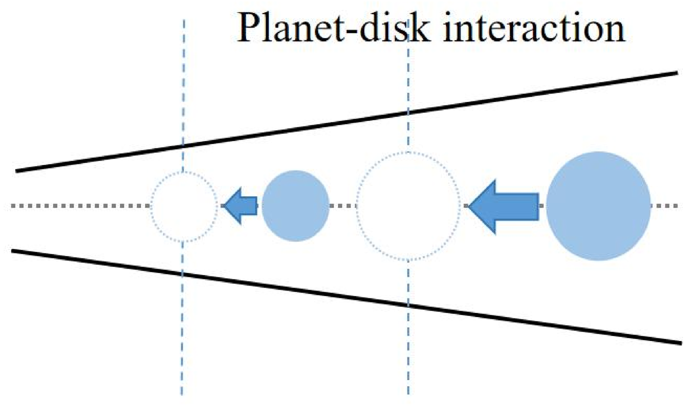
Introduction

Low-mass planets embedded in protoplanetary disks interact with surrounding disk gas and undergo type I orbital migration. Resonance capture can be a natural outcome of the planet pairs featured by convergent migration. The key question is to understand under which condition this capture can occur and its following dynamical evolution.

Type-I migration



Mean motion resonance (MMR)

- $\frac{P_o}{P_i} = j : j-k$ (e.g, 2:1)
- Resonant angle φ at a fixed value
  $\varphi = j\lambda_o - (j-k)\lambda_i - k\varpi_{o,i}$

Convergent Migration can lead to MMR trapping

Method

For low eccentricities, the dissipative effect of planet-disk interaction can be parameterized by:

$$\frac{1}{L}\frac{dL}{dt} = -\frac{1}{\tau_m},$$
$$\frac{1}{a}\frac{da}{dt} = -\frac{1}{\tau_a} = -\frac{2}{\tau_m} + \frac{2e^2}{1-e^2}\frac{1}{\tau_e},$$
$$\frac{1}{e}\frac{de}{dt} = -\frac{1}{\tau_e},$$

We assume that $\tau_m^{-1} = \tau_{m,o}^{-1} - \tau_{m,i}^{-1}$ is the relative angular momentum damping rate for convergent migration, and $\tau_e = \tau_{e,i} = \tau_{e,o}$ represents the eccentricity damping timescale.

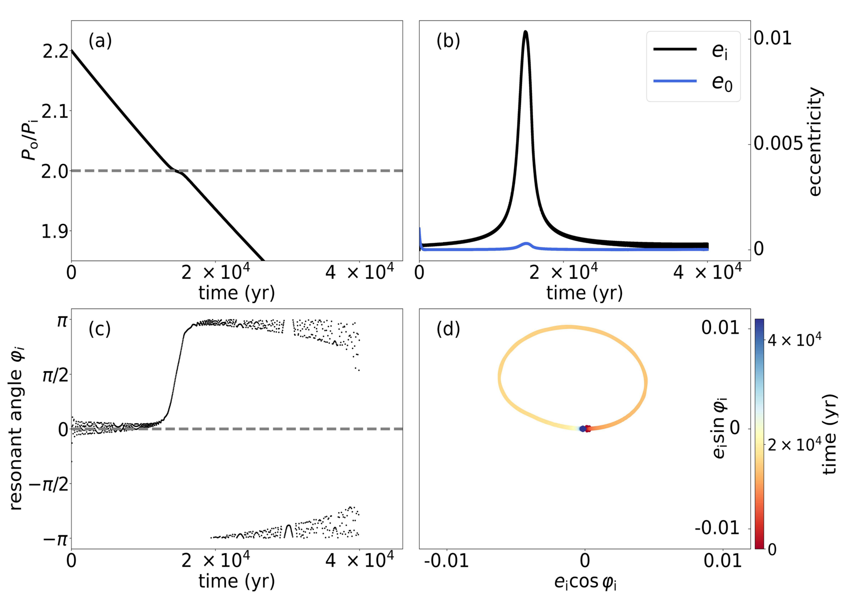
We treat τ_m and τ_e as free but constant parameters and focus on the following two questions:

I. In which condition can planets be trapped in MMR ?

II. Are traps stable or unstable?

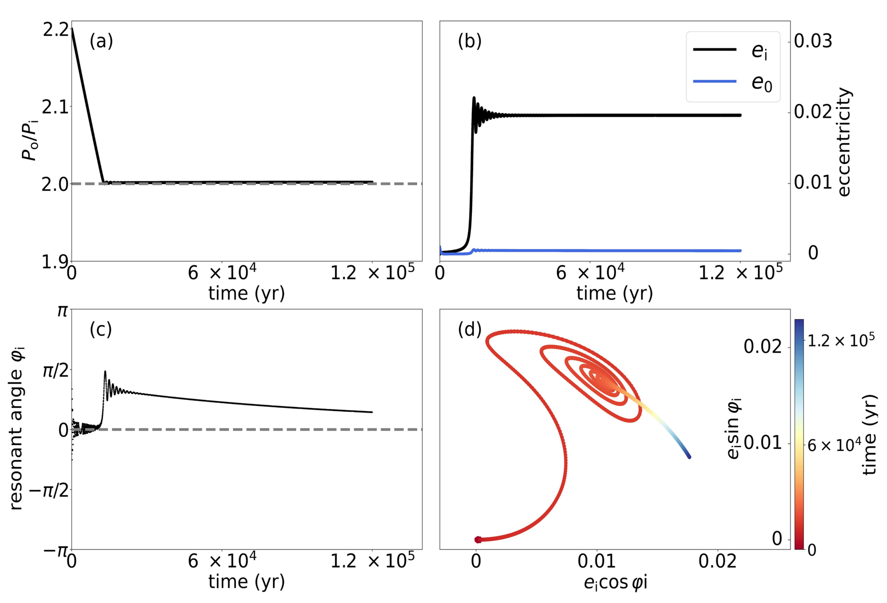
Analytical Framework

I. resonance trapping requires both relatively weak eccentricity damping and slow migration.



No Trap

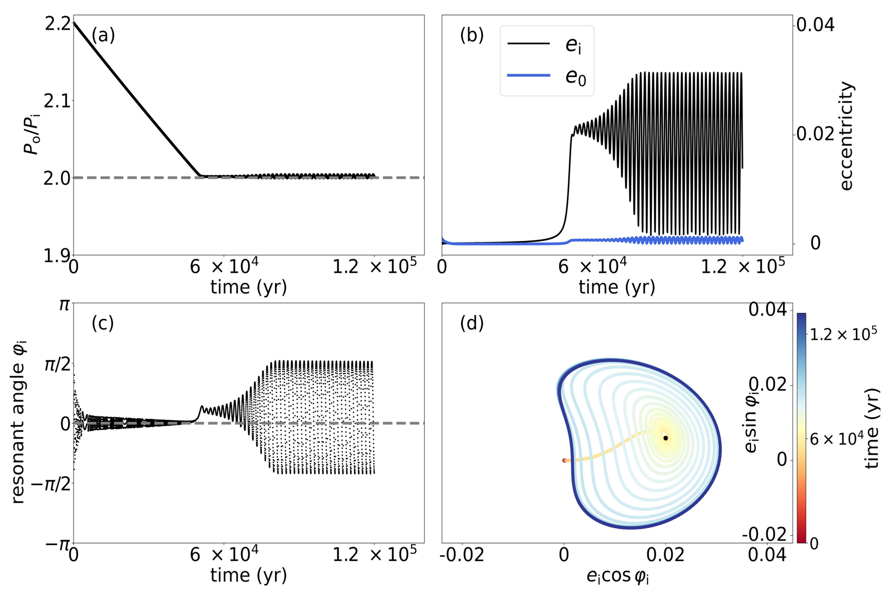
The planets directly bypasses the resonance.



Stable Trap

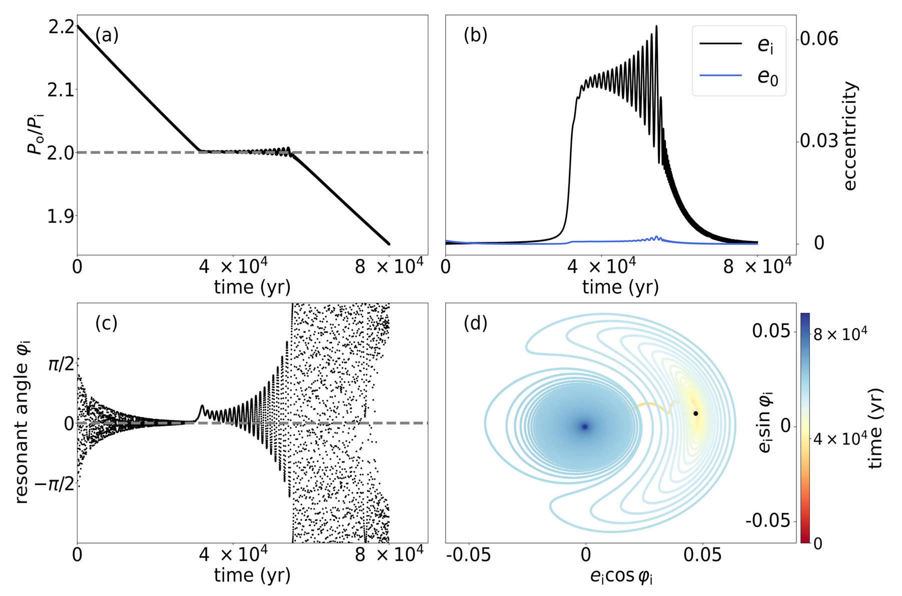
The planets get locked in resonance, and the system's orbital parameters converge towards an equilibrium value.

II. After trapping, the stability of the system weakens as τ_m/τ_e decreases



Overstable Trap

The planets are locked in resonance with librating resonant angles, and the system's orbital parameters librate around equilibrium values with finite amplitudes

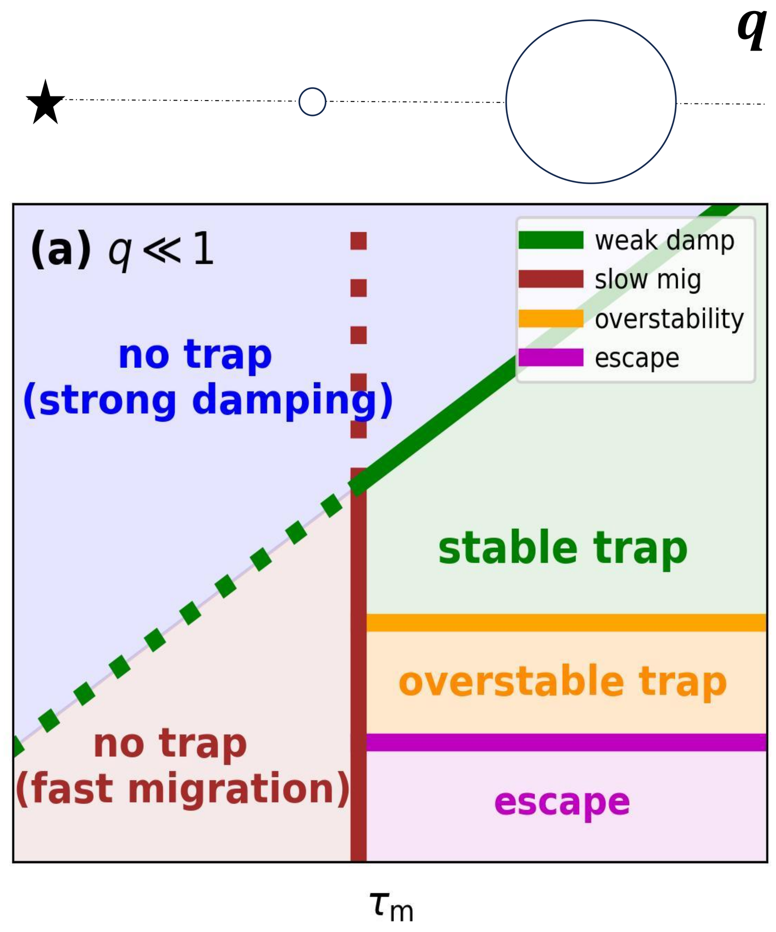


Escape

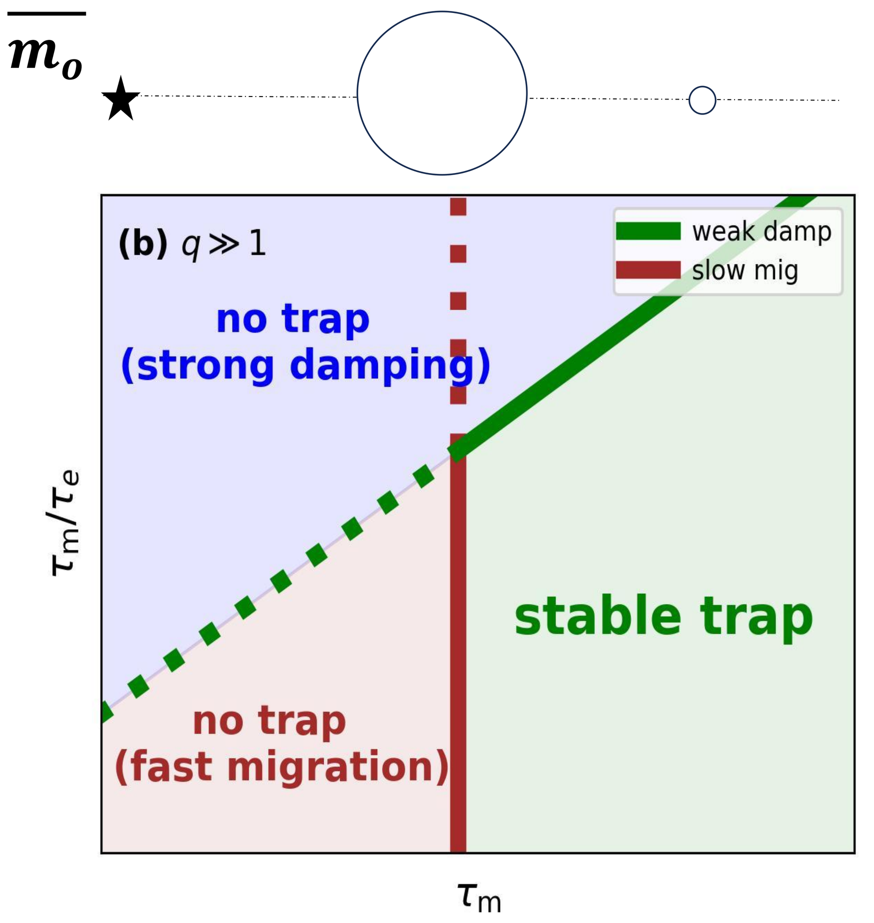
The planets are temporarily trapped in resonance. The system's orbital parameters initially librate around equilibrium values with growing amplitudes, and eventually leading to escape.

III. After trapping, the stability of the system weakens as mass ratio q increases

Outer more massive : q<<1 $\quad q = \frac{m_i}{m_o} \quad$ Inner more massive : q>>1



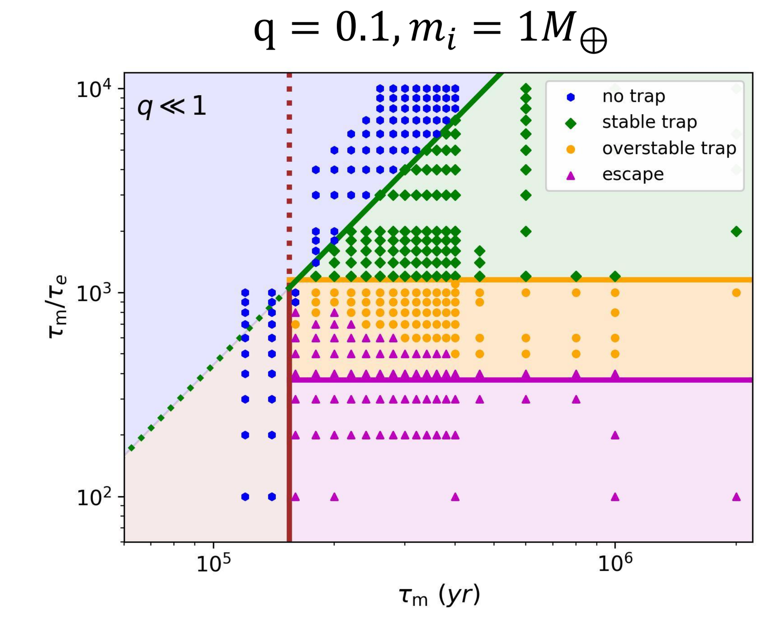
Stable trap, to overstable trap, to escape



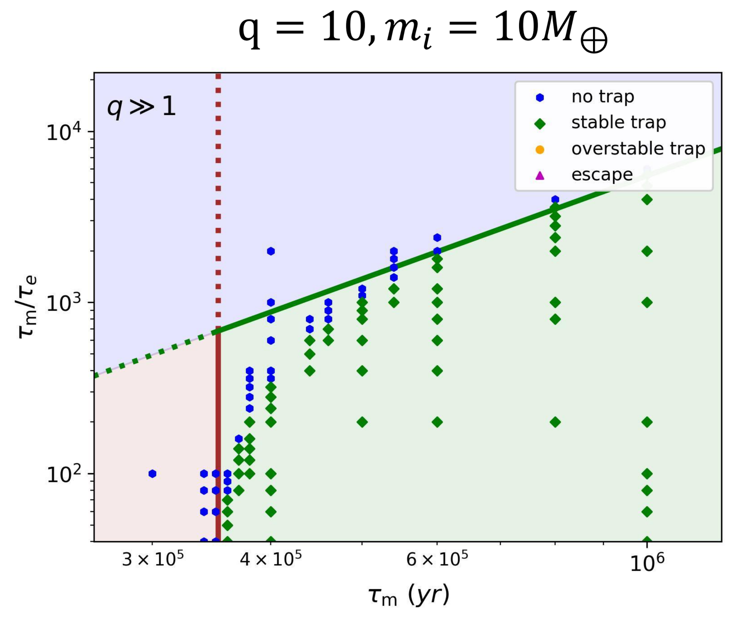
All stable traps

Results

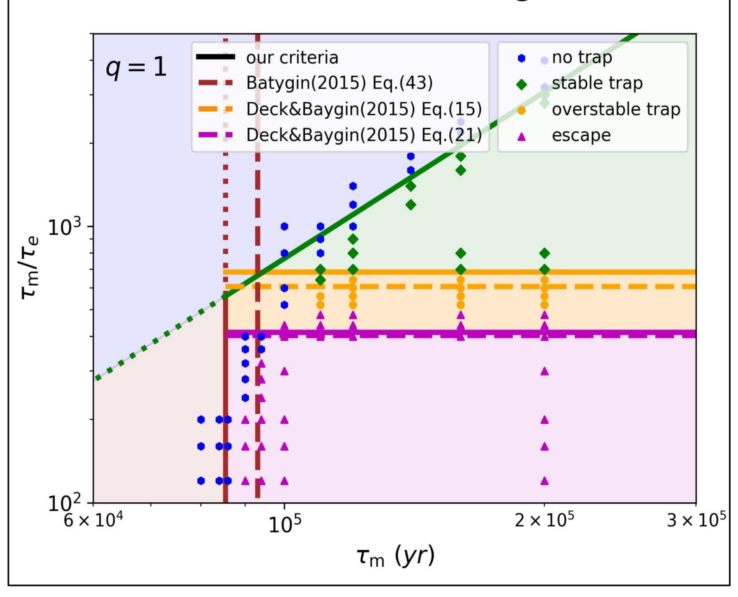
$q = 0.1, m_i = 1M_\oplus$



$q = 10, m_i = 10M_\oplus$



$q = 1, m_i = 10M_\oplus$



Article Link: **https://arxiv.org/abs/2501.12650**
Acknowledgements: This work is supported by REBOUND (Rein & Liu 2012) and REBOUNDX (Tamayo et al. 2020)

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