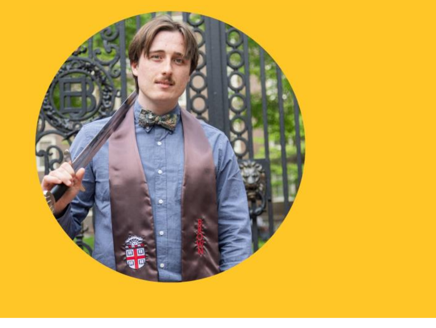
Higher-Order Mean-Motion Resonances Can Form in Type-I Disk Migration

School of Earth and Space Exploration
Arizona State University

Finnegan Keller [1, 2, 3] Fei Dai [3, 4, 5] Wenrui Xu [6]

[1]School of Earth and Space Exploration, Arizona State University [2]Department of Physics, Brown University [3]Institute for Astronomy, University of Hawai'i
[4]Division of Geological and Planetary Sciences, California Institute of Technology [5]Department of Astronomy, California Institute of Technology
[6]Center for Computational Astrophysics, Flatiron Institute



I. Background

- There is mounting evidence that *Kepler*-like planets [~ 0.1AU, $< 4R_\oplus$, $N_p \geq 3$, 1, 2] could have formed initially in chains of mean-motion resonance (MMR) through Type-I (non-gap-opening) convergent disk migration [e.g. 3].
- The order of an MMR is defined by the difference of two integers involved ($|p-q|$, e.g. 3:2 is first-order, 5:3 is second-order, 8:5 is third-order) and the strength of the MMR scales as orbital eccentricity raised to the power of the order [($\propto e^{|p-q|}$ 4, 5].
- For most *Kepler*-like planetary systems, the orbital eccentricity is low [$\lesssim 0.05$ e.g. 6, 7], so the weakness of any higher-order resonances could aid the disruption of initially resonant *Kepler*-like systems [8].
- Several multi-planet systems contain planet pairs near higher-order resonances (Fig. 4), including Kepler-29 bc: 9:7 [9], TOI-178 bc: 5:3 [10], TOI-1136 ef: 7:5 [8], Kepler-138 cde: 5:3-5:3 [11], and TRAPPIST-1 bcd: 8:5-5:3 [12].

II. Methods

- We focus on Type-I migration of low-mass planets that do not carve a gap in protoplanetary disks [13] after they have grown to their final masses.
- Disk migration prescriptions were implemented using the symplectic `WHFAST` integrator [14, 15] with the `type_I_migration` [16] scheme in `REBOUNDx` [17] and `REBOUND` [18].
- By drawing select initial conditions from the NASA Exoplanet Archive, simulated planetary systems retain the stellar-mass-planet-size correlation [e.g. 19] and the 'peas-in-a-pod' pattern [e.g. 20].
- The hallmark of true resonance is the libration (oscillation with a bounded amplitude) of a resonant angle in the presence of a separatrix, a generalized coordinate for the resonant Hamiltonian [4]. These resonant angles are linear combinations of the relevant planets' mean longitudes and the (mixed) longitude of pericenters [e.g. 21, 22]. We identify two-body and three-body MMRs with libration amplitudes under 90°.
- We examined all 36 first-, second-, and third-order MMRs ranging from period ratio of 1.1 (11:10) to 4 (4:1). This set encompasses the smallest observed pairwise period ratios [Kepler-36bc 7:6, see 23] and the widest third-order resonance, 4:1.

III. A Case Study Higher-Order Resonance

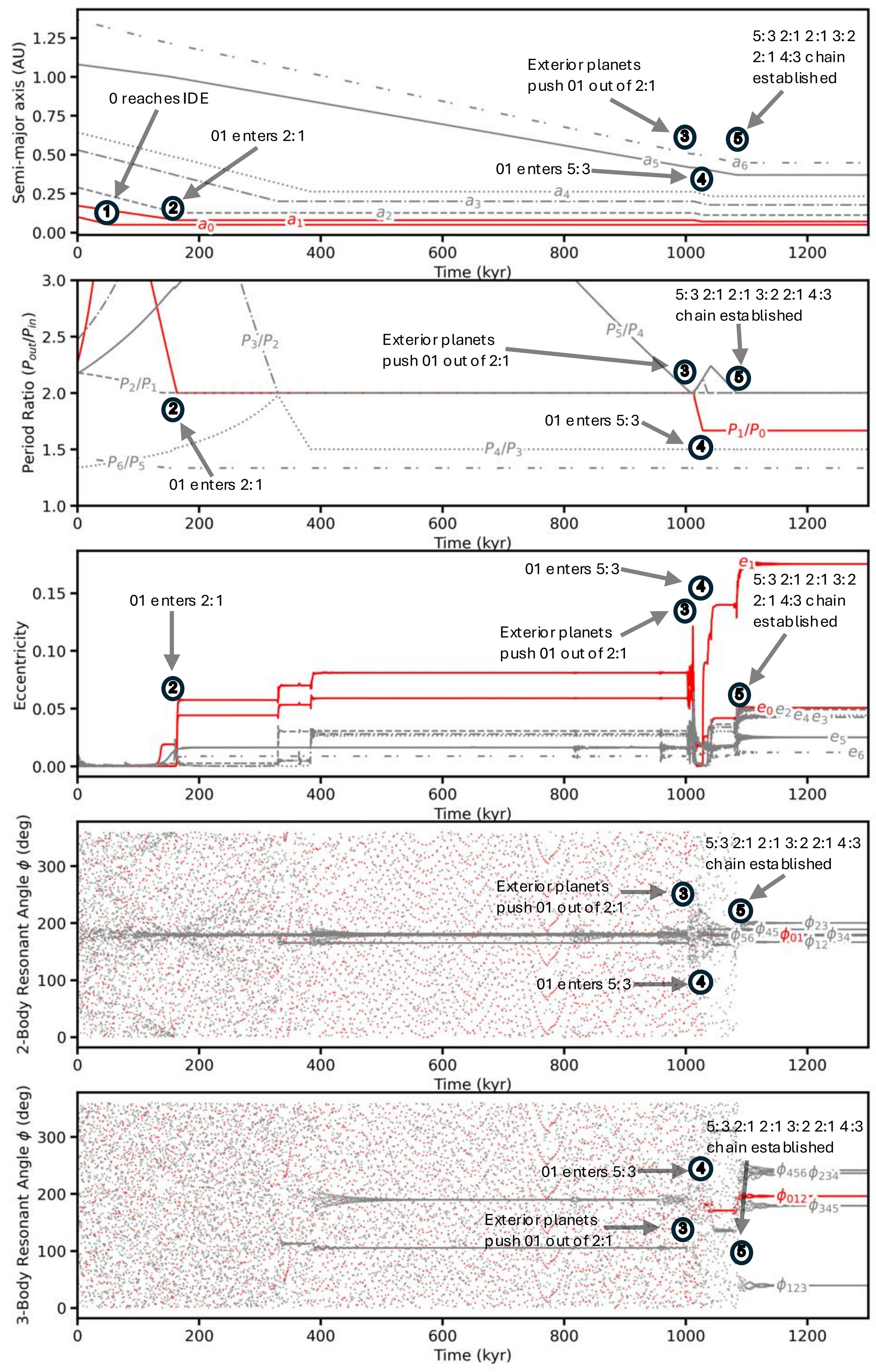


Figure 1. The migration history of a resonant chain with a second-order resonance (innermost two planets 0 and 1, shown in red). Planet 0 and 1 were initially captured into a 2:1 resonance. However, as longer-period planets joined the resonant chain, the 2:1 resonance broke and the planets captured into the nearby 5:3 resonance. A few key milestones of the evolution have been labeled.

IV. Population-Level Results

- We ran ~ 6000 disk migration simulations. We found that $720/5494$ or $13.03 \pm 0.45\%$ contain at least one second-order resonance and $98/5494$ or $1.77 \pm 0.18\%$ contain at least one third-order resonance.
- These fractions of higher-order resonance critically depend on the prior range of disk surface density we assumed ($10 - 10,000$ g cm^{-2}). More robustly, the relative proportion of individual resonances (e.g. the fraction of planets in 5:3 v.s. 7:5 MMR) in our simulations are in good agreement with observation (see Fig. 2).
- Notably, within each order of MMR, the resonances with smaller period ratios (defined P_{out}/P_{in}) are increasingly rare, both in simulations and observations. This is because the planets have to avoid being captured into all preceding MMRs before reaching the deeper resonances [16].
- The prevalence of the 2:1 and 3:1 MMRs are overestimated in our simulations likely because: (a) middle planets may be missed in transit observations since planets in the same system have small but finite mutual inclinations [e.g. 24] and (b) middle planets can be removed through long-term dynamical instability after disk dispersal [25].

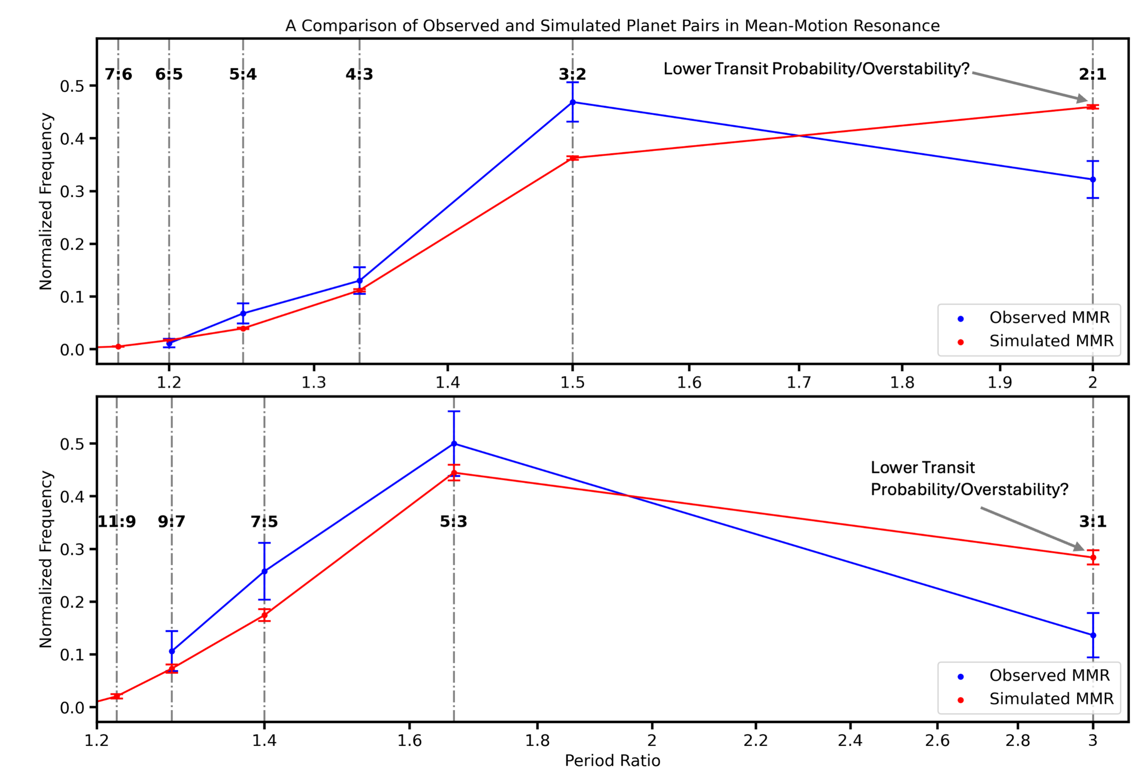


Figure 2. The relative frequencies of individual first (top panel) and second-order (bottom panel) MMR in our simulations and in the confirmed all-ages planet sample from [3]. Note in the simulated samples, all planet pairs have librating resonant angles. In the observed sample, there is not enough information to determine the dynamical state of the planets. Simulations produced more 2:1 and 3:1 resonance than observed. Smaller transit probabilities for inclined planets may underestimate planet counts (and thus overestimate planet separations). Alternatively, overstability may preferentially remove 2:1 and 3:1 MMRs directly [26, 27, 28].

- Higher-order MMRs do not require initial period ratios that are commensurate with the final resonance. Only 43 out of the second-order 1124 pairs started with a period ratio within 2% of the final resonance. Similarly, only 6 out of the 151 third-order pairs started with near-commensurate period ratios.
- In Fig. 3, we show the cumulative distributions of the initial and final period ratios for all planet pairs that end up in second-order MMRs. There are no discernible peaks in the initial period distribution near the final resonances. This result holds across order.
- Furthermore, we found that the initial periods of different orders had statistically indistinguishable initial period ratios. The p-values from a KS test [29] between the first and second-order was 0.2, and between first- and third-order were 0.4.
- The explanation is simple: most higher-order MMR have to undergo substantial migration.

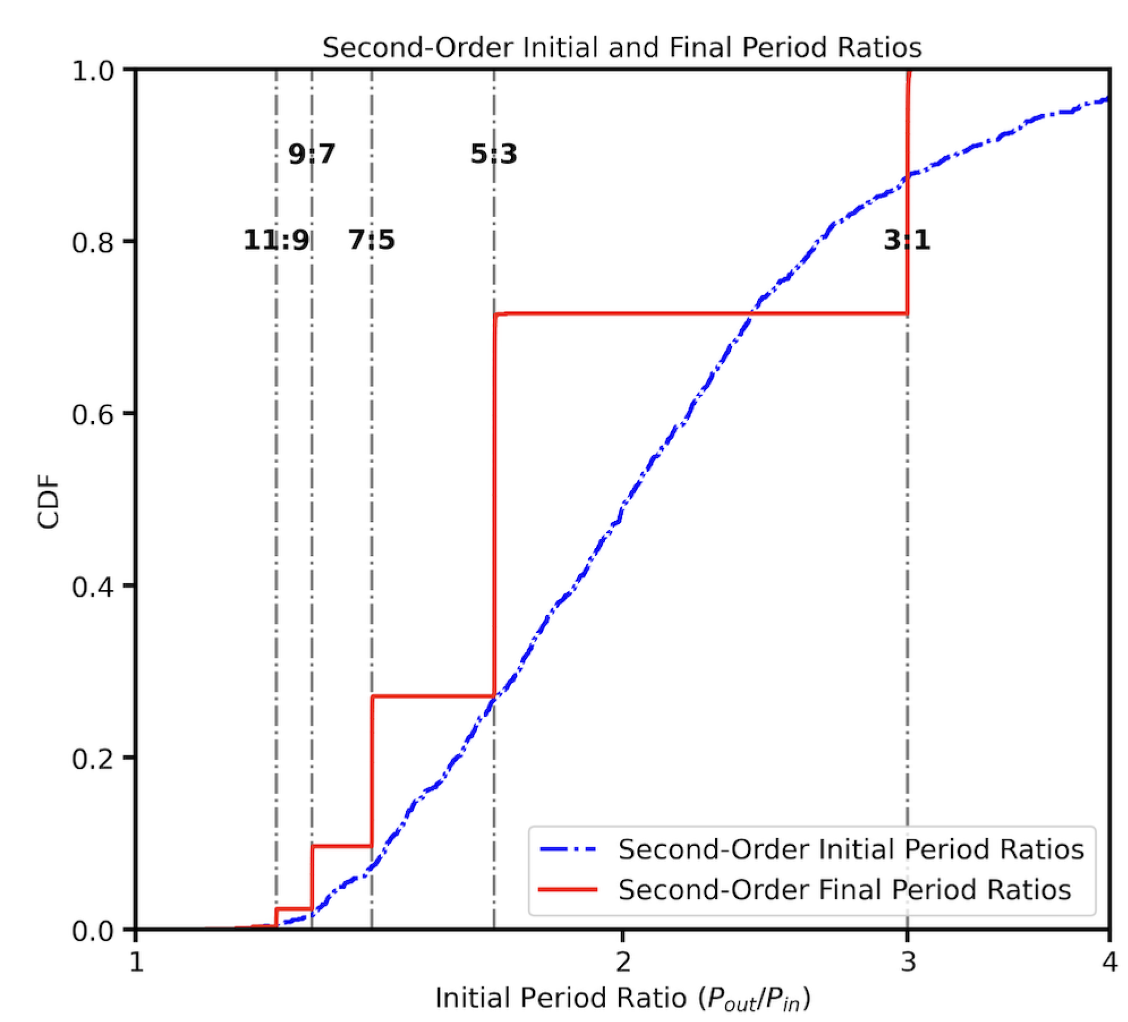


Figure 3. The initial period ratios and final period ratios for planet pairs that end up in second-order MMRs. Notice that these planet pairs, which end up engaged in higher-order mean-motion resonance, need not begin with commensurate period ratios.

V. Conclusion

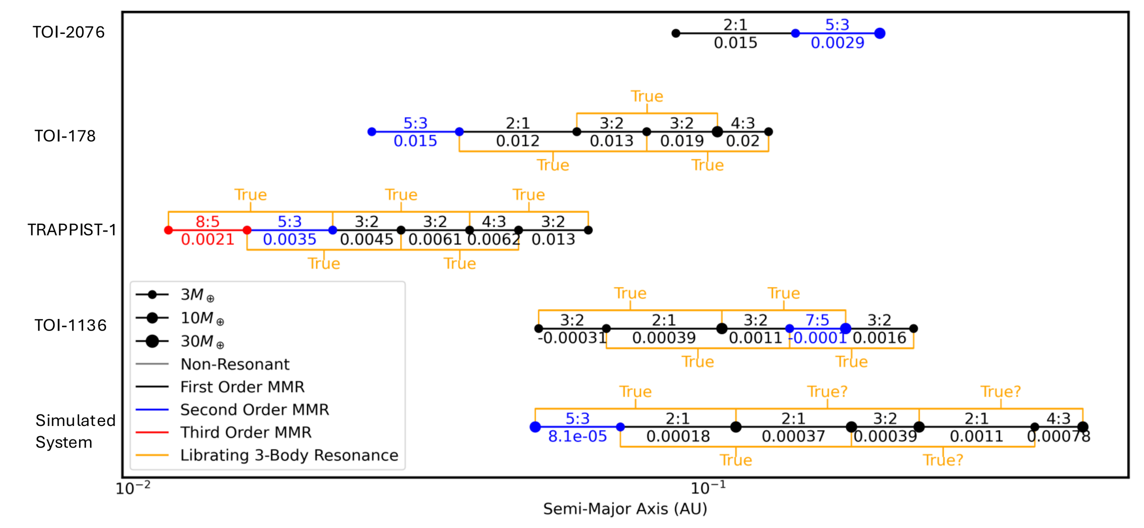


Figure 4. A subset of observed resonant chains and the example system. We mark the most proximal MMRs and observed librating triplets.

- We performed $\sim 6,000$ Type-I simulations of multi-planet systems with initial conditions that mimic the observed *Kepler* sample.
- We found that Type-I migration coupled with a disk inner edge can produce second- and third-order resonances in a manner that is consistent with observations (Figs 2 and 4).
- Planets that end engaged in a higher-order resonance need not begin near the resonance (Figs 1 and 3).
- For further motivation, methods, and findings, see the preprint [30].

PREPRINT

VI. References

[1] Joshua N. Winn and Daniel C. Fabrycky. The Occurrence and Architecture of Exoplanetary Systems. , 53:409–447, August 2015.

[2] Wei Zhu and Subo Dong. Exoplanet Statistics and Theoretical Implications. , 59:291–336, September 2021.

[3] Fei Dai, Max Goldberg, Konstantin Batygin, Jennifer van Saders, Eugene Chiang, Nick Choksi, Rixin Li, Erik A. Petigura, Gregory J. Gilbert, Sarah C. Millholland, Yuan-Zhe Dai, Luke Bouma, Lauren M. Weiss, and Joshua N. Winn. The prevalence of resonance among young, close-in planets. *The Astronomical Journal*, 168(6):239, nov 2024.

[4] C. D. Murray and S. F. Dermott. *Solar system dynamics*. Cambridge university press, 1999.

[5] Sam Hadden. An Integrable Model for the Dynamics of Planetary Mean-motion Resonances. , 158(6):238, December 2019.

[6] Sam Hadden and Yoram Lithwick. Kepler Planet Masses and Eccentricities from TTV Analysis. , 154(1):5, July 2017.

[7] Ji-Wei Xie, Subo Dong, Zhaohuan Zhu, Daniel Huber, Zheng Zheng, Peter De Cat, Jianning Fu, Hui-Gen Liu, Ali Luo, Yue Wu, Haotong Zhang, Hui Zhang, Ji-Lin Zhou, Zihuang Cao, Yonghui Hou, Yuefei Wang, and Yong Zhang. Exoplanet orbital eccentricities derived from LAMOST-Kepler analysis. *Proceedings of the National Academy of Science*, 113(41):11431–11435, Oct 2016.

[8] Fei Dai, Kento Masuda, Corey Beard, Paul Robertson, Max Goldberg, Konstantin Batygin, Luke Bouma, Jack J Lissauer, Emil Knudstrup, Simon Albrecht, et al. Toi-1136 is a young, coplanar, aligned planetary system in a pristine resonant chain. *The Astronomical Journal*, 165(2):33, 2023.

[9] Cezary Migaszewski, Krzysztof Goździewski, and Federico Panichi. The origin and 9:7 MMR dynamics of the Kepler-29 system. , 465(2):2366–2380, February 2017.

[10] Adrien Leleu, Yann Alibert, NC Hara, Matthew John Hooton, TG Wilson, P Robutel, J-B Delisle, J Laskar, S Hoyer, C Lovis, et al. Six transiting planets and a chain of laplace resonances in toi-178. *Astronomy & Astrophysics*, 649:A26, 2021.

[11] Daniel Jontof-Hutter, Jason F. Rowe, Jack J. Lissauer, Daniel C. Fabrycky, and Eric B. Ford. The mass of the Mars-sized exoplanet Kepler-138 b from transit timing. , 522(7556):321–323, June 2015.

[12] Shuo Huang and Chris W. Ormel. The dynamics of the TRAPPIST-1 system in the context of its formation. , 511(3):3814–3831, April 2022.

[13] W. Kley and R. P. Nelson. Planet-Disk Interaction and Orbital Evolution. , 50:211–249, September 2012.

[14] Jack Wisdom and Matthew Holman. Symplectic maps for the N-body problem. , 102:1528–1538, October 1991.

[15] Hanno Rein and Daniel Tamayo. whfast: a fast and unbiased implementation of a symplectic wisdom–holman integrator for long-term gravitational simulations. *Monthly Notices of the Royal Astronomical Society*, 452(1):376–388, July 2015.

[16] Kaltrina Kajtazi, Antoine C Petit, and Anders Johansen. Mean motion resonance capture in the context of type i migration. *Astronomy & Astrophysics*, 669:A44, 2023.

[17] Daniel Tamayo, Hanno Rein, Pengshuai Shi, and David M. Hernandez. REBOUNDx: a library for adding conservative and dissipative forces to otherwise symplectic N-body integrations. , 491(2):2885–2901, January 2020.

[18] H. Rein and S. F. Liu. REBOUND: an open-source multi-purpose N-body code for collisional dynamics. , 537:A128, January 2012.

[19] Yanqin Wu. Mass and Mass Scalings of Super-Earths. , 874(1):91, March 2019.

[20] L. M. Weiss, H. T. Isaacson, G. W. Marcy, A. W. Howard, E. A. Petigura, B. J. Fulton, J. N. Winn, L. Hirsch, E. Sinukoff, and J. F. Rowe. The California-Kepler Survey. VI: Kepler Multis and Singles Have Similar Planet and Stellar Properties Indicating a Common Origin. *ArXiv e-prints*, August 2018.

[21] W. Sessin and S. Ferraz-Mello. Motion of two planets with periods commensurable in the ratio 2:1 solutions of the hori auxiliary system. *Celestial Mechanics*, 32(4):307–332, April 1984.

[22] Sam Hadden. An Integrable Model for the Dynamics of Planetary Mean-motion Resonances. , 158(6):238, December 2019.

[23] Joshua A Carter, Eric Agol, William J Chaplin, Sarbani Basu, Timothy R Bedding, Lars A Buchhave, Jørgen Christensen-Dalsgaard, Katherine M Deck, Yvonne Elsworth, Daniel C Fabrycky, et al. Kepler-36: a pair of planets with neighboring orbits and dissimilar densities. *Science*, 337(6094):556–559, 2012.

[24] W. Zhu, C. Petrovich, Y. Wu, S. Dong, and J. Xie. About 30% of Sun-like Stars Have Kepler-like Planetary Systems: A Study of Their Intrinsic Architecture. , 860:101, June 2018.

[25] Andre Izidoro, Masahiro Ogihara, Sean N. Raymond, Alessandro Morbidelli, Arnaud Pierens, Bertram Bitsch, Christophe Cossou, and Franck Hersant. Breaking the chains: hot super-Earth systems from migration and disruption of compact resonant chains. , 470(2):1750–1770, September 2017.

[26] Peter Goldreich and Hilke E. Schlichting. Overstable Librations can Account for the Paucity of Mean Motion Resonances among Exoplanet Pairs. , 147(2):32, February 2014.

[27] Katherine M. Deck and Konstantin Batygin. Migration of Two Massive Planets into (and out of) First Order Mean Motion Resonances. , 810(2):119, September 2015.

[28] Wenrui Xu and Dong Lai. Migration of planets into and out of mean motion resonances in protoplanetary discs: analytical theory of second-order resonances. , 468(3):3223–3238, July 2017.

[29] Vance W Berger and YanYan Zhou. Kolmogorov–smirnov test: Overview. *Wiley statsref: Statistics reference online*, 2014.

[30] Finnegan Keller, Fei Dai, and Wenrui Xu. Higher-order mean-motion resonances can form in type-i disk migration. *arXiv preprint arXiv:2504.12596*, 2025.

fmkeller@asu.edu, Sagan Summer Workshop 2025, Pasadena California