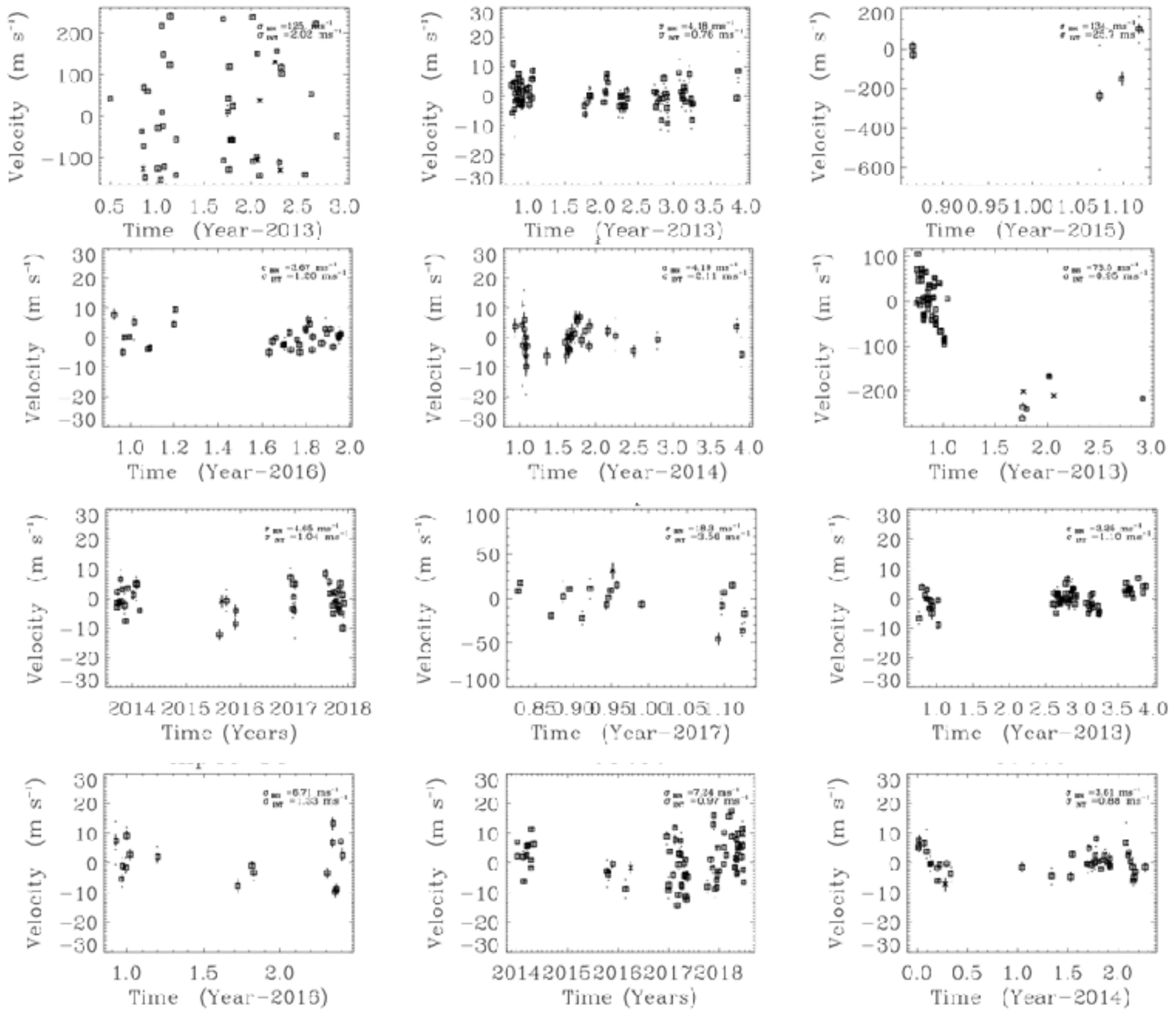
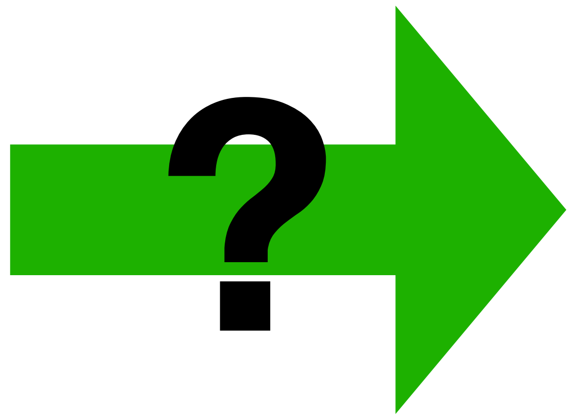
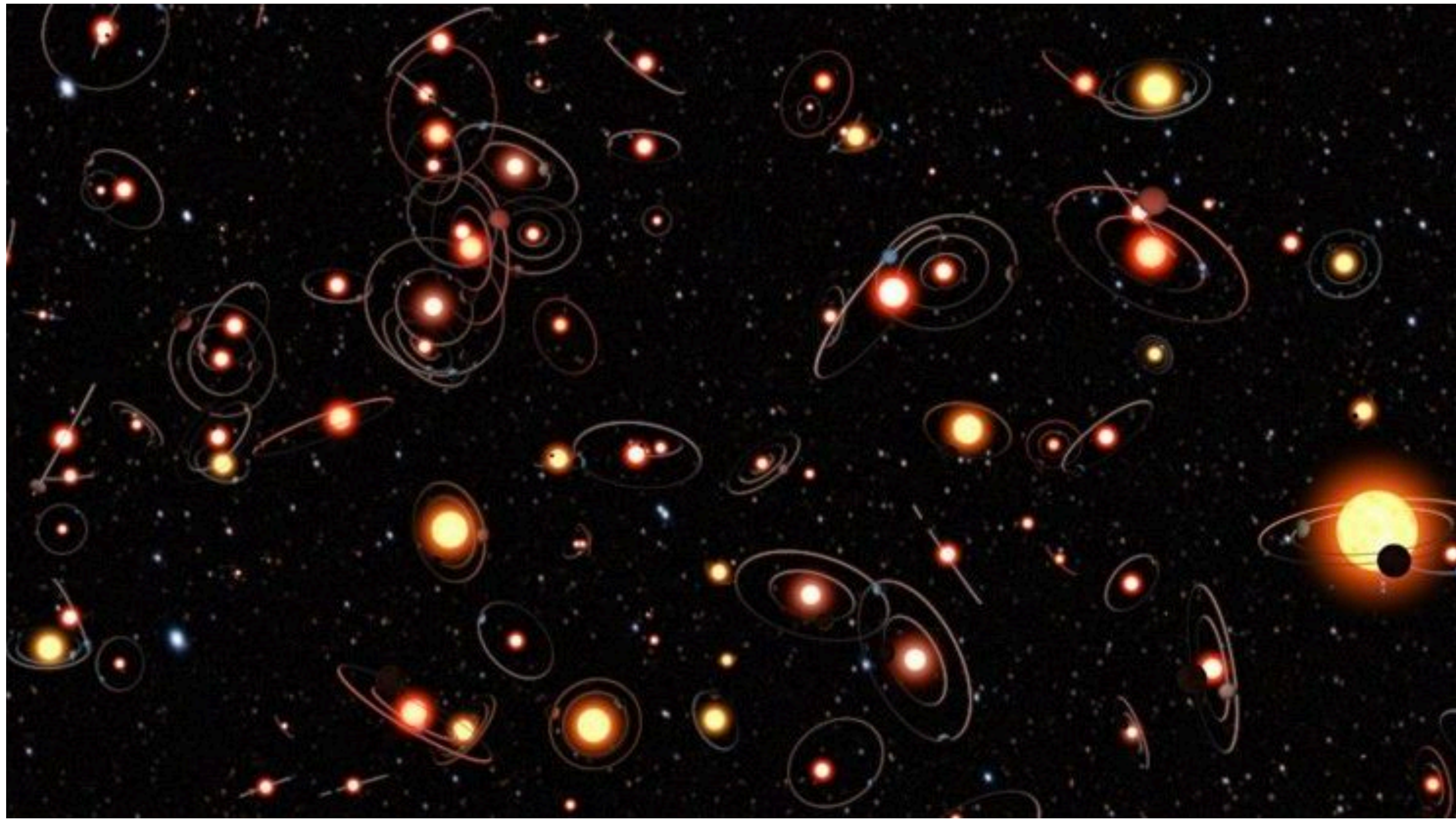
Orbit Determination and Degeneracy of Models



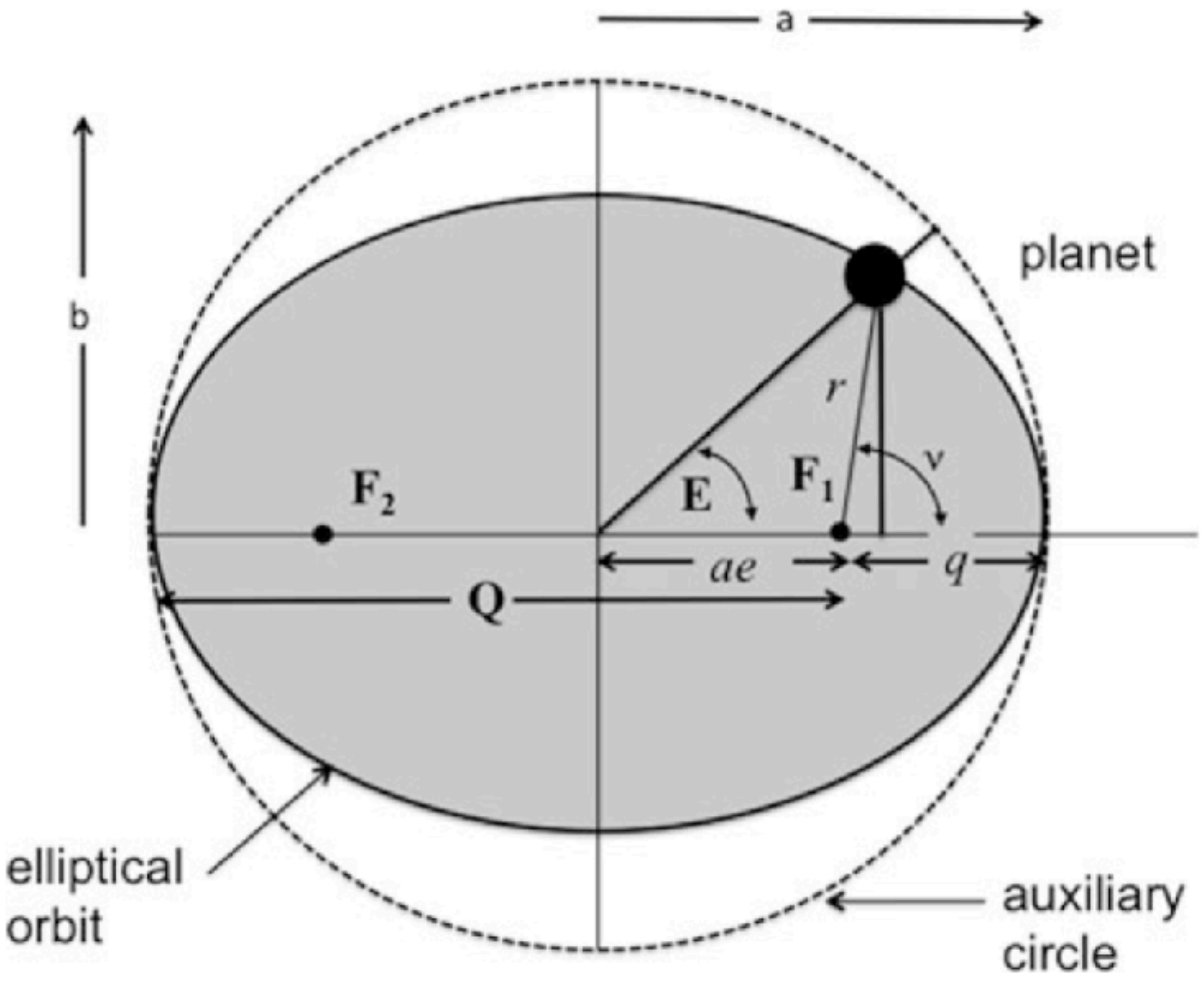


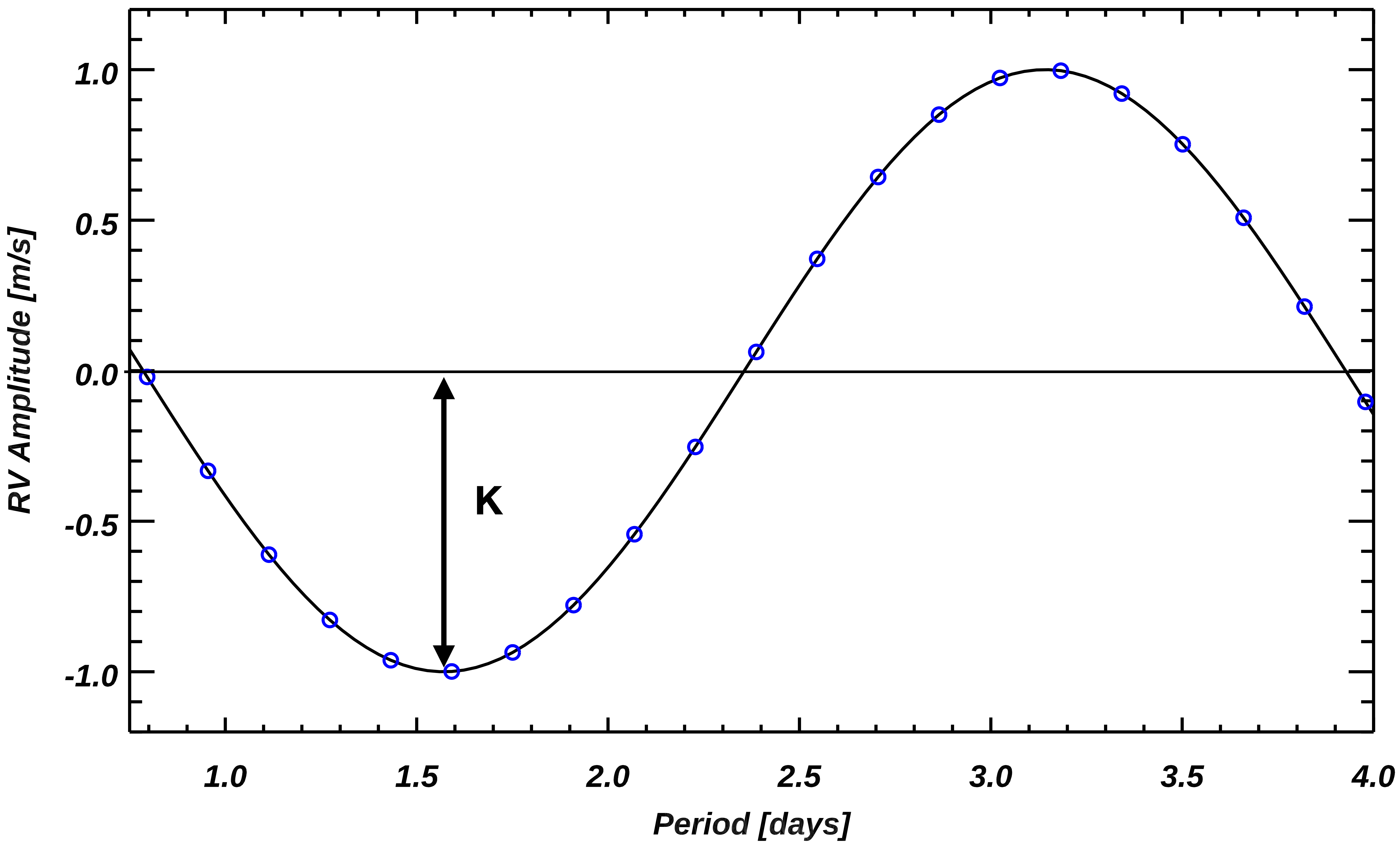


Jennifer Burt
Torres Fellow
MIT Kavli Institute

Kosiarek 2018
c)
$P_b = 2.225174 \pm 7.2e\text{-}05$ days
$K_b = 3.24 \pm 0.57$ m s^{-1}
$e_b = 0.00$
RV [m s^{-1}]
Phase
Teske 2018
Radial Velocity (m s^{-1})
Time since midtransit (days)
planet b
planet c
planet d
Fulton 2015
c)
e)
g)
RV [m s^{-1}]
Phase
Vogt 2015
RV, Planet b [m/s]
RV, Planet f [m/s]
RV, Planet c [m/s]
RV, Planet g [m/s]
RV, Planet d [m/s]
Residuals [m/s]
Time [JD]

Fitting RV data





$$V_{\mathrm{mod}}(t) = K\left[\cos(f+\omega) + e\cos(\omega)\right]$$

$$K = \left(\frac{2\pi G}{P}\right)^{\frac{1}{3}} \frac{M_{pl} * \sin(i)}{\left(M_{star} + M_{pl}\right)^{\frac{2}{3}}}$$

Orbital parameters

Period [days] : Time it takes the planet to complete one orbit around its host star

Semi-amplitude [m/s] : Velocity of the reflex motion the planet imparts on its host star

Eccentricity : Ellipticity of the planet's orbit

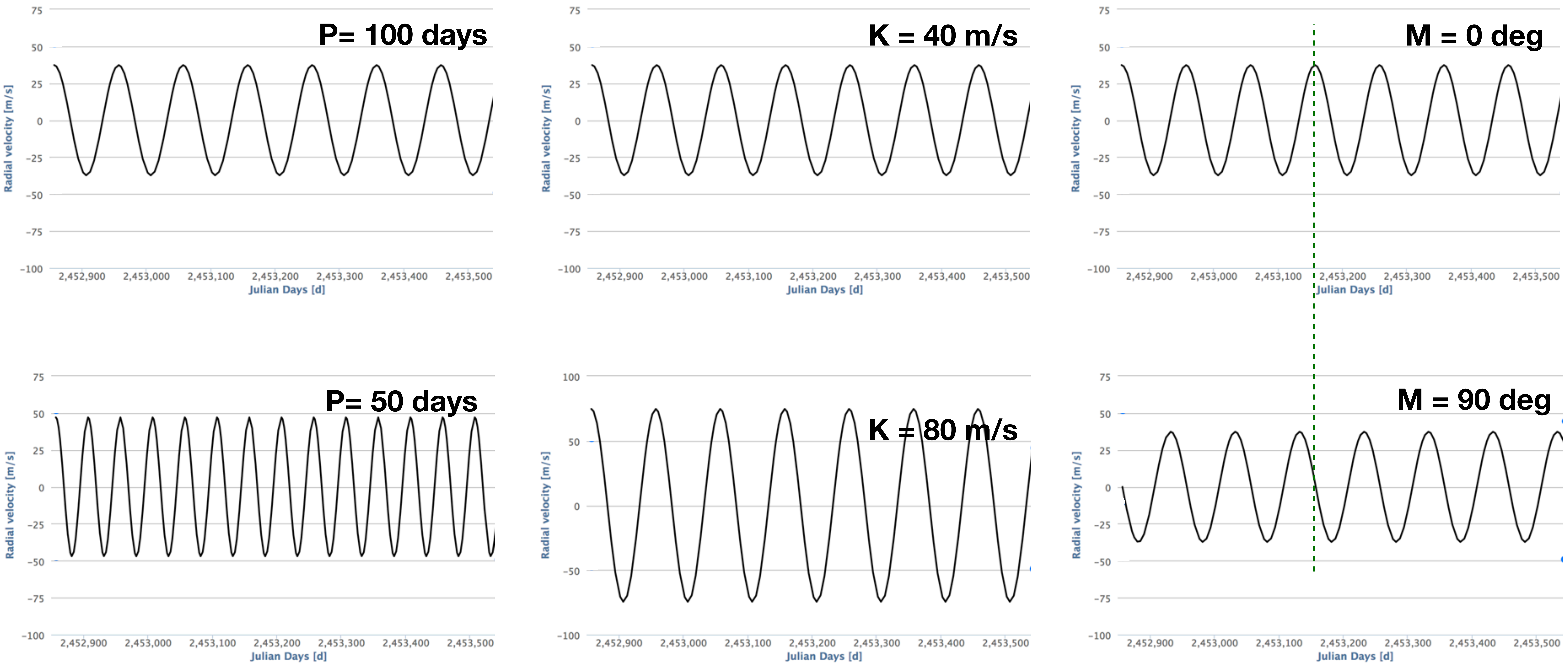
Longitude of periastron [deg] : Orbital angle at which the planet goes through periastron

Time of periastron [JD] : Date when the planet passes through its periastron point

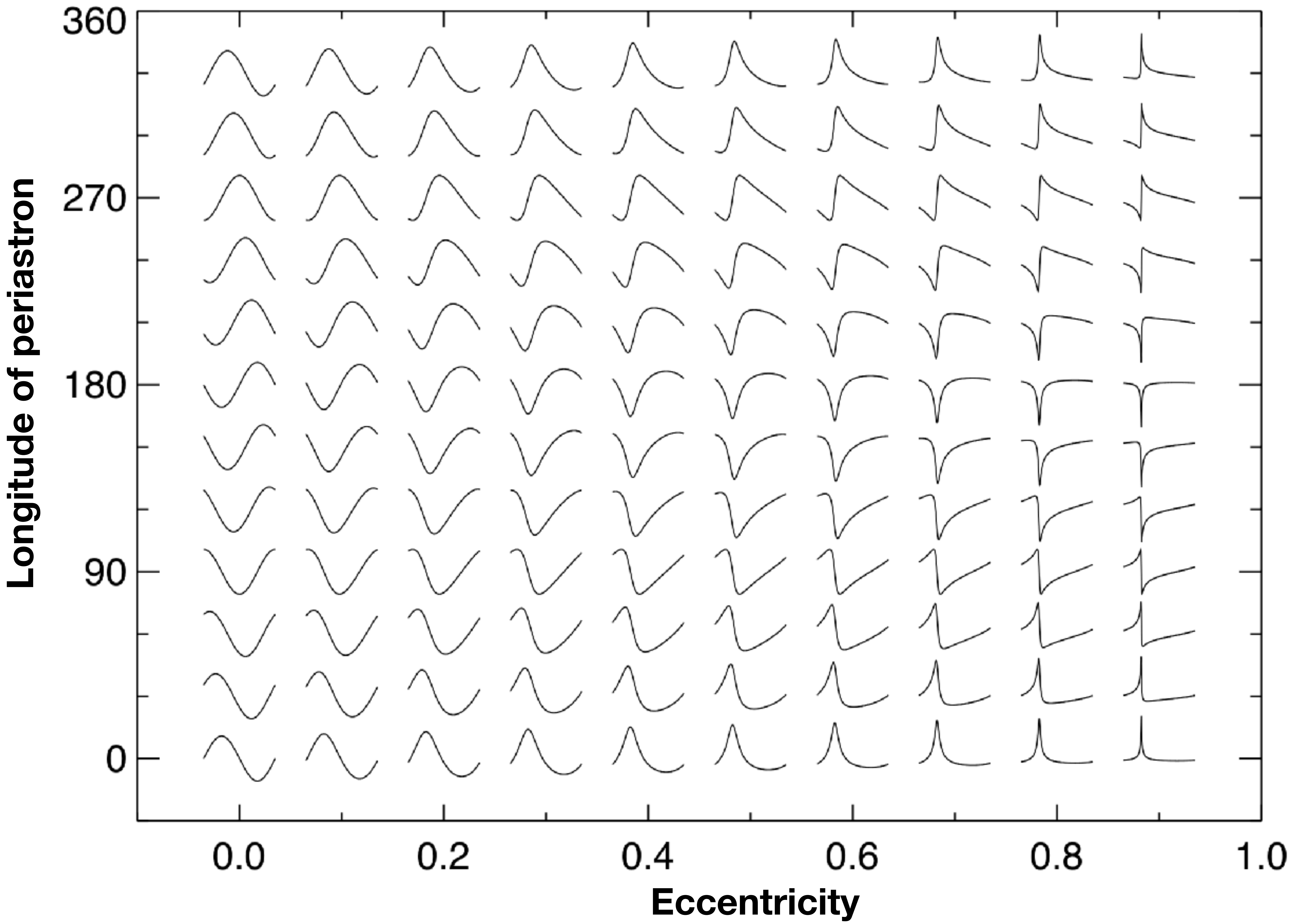
Mean anomaly [deg] : Angular distance from pericenter the planet would have if ecc = 0

RV offset [m/s] : Offset between each data point's RV and the RV zero point

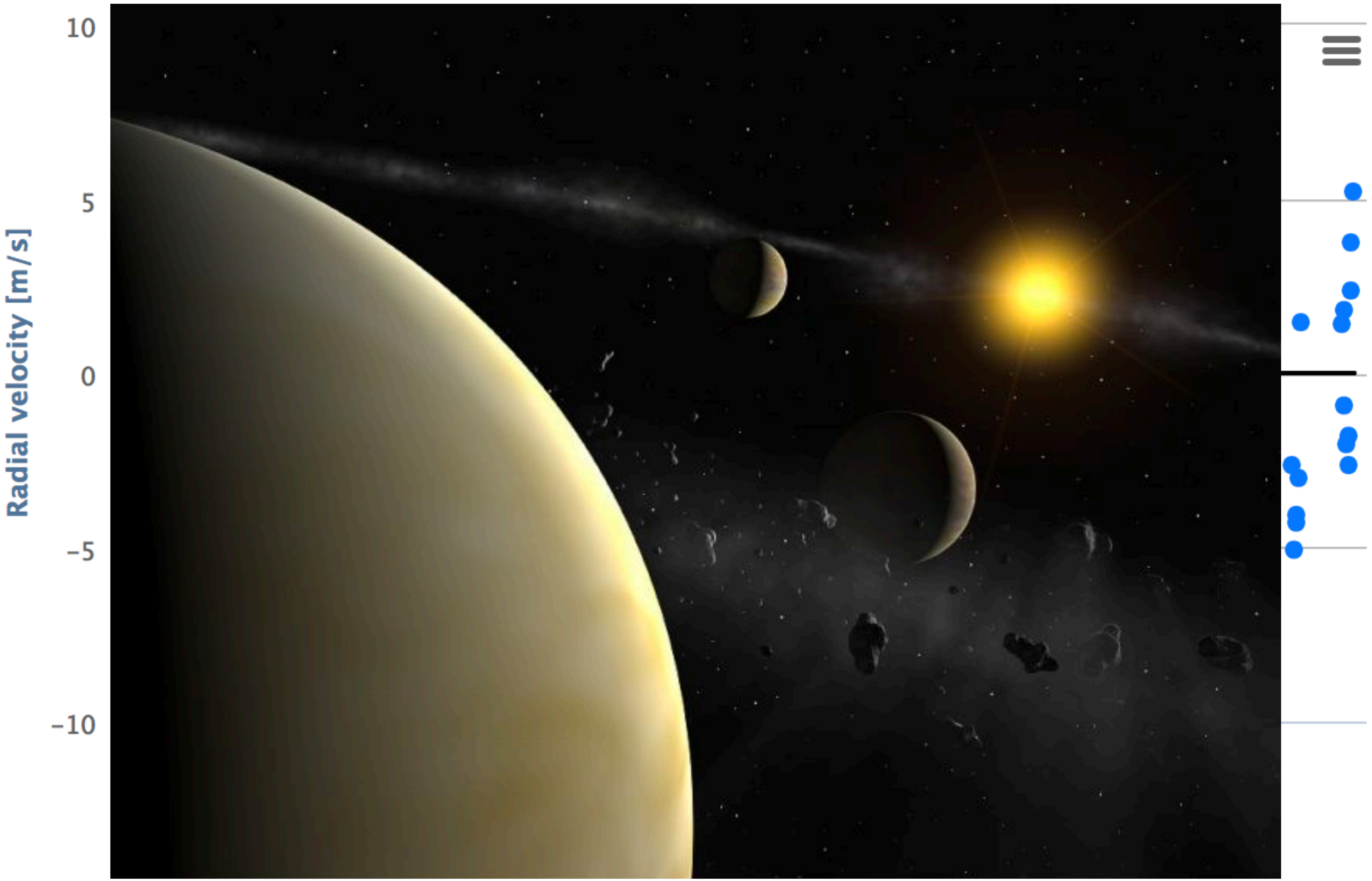
What do these do to a Keplerian signal?



What do these do to a Keplerian signal?



Fitting RV data

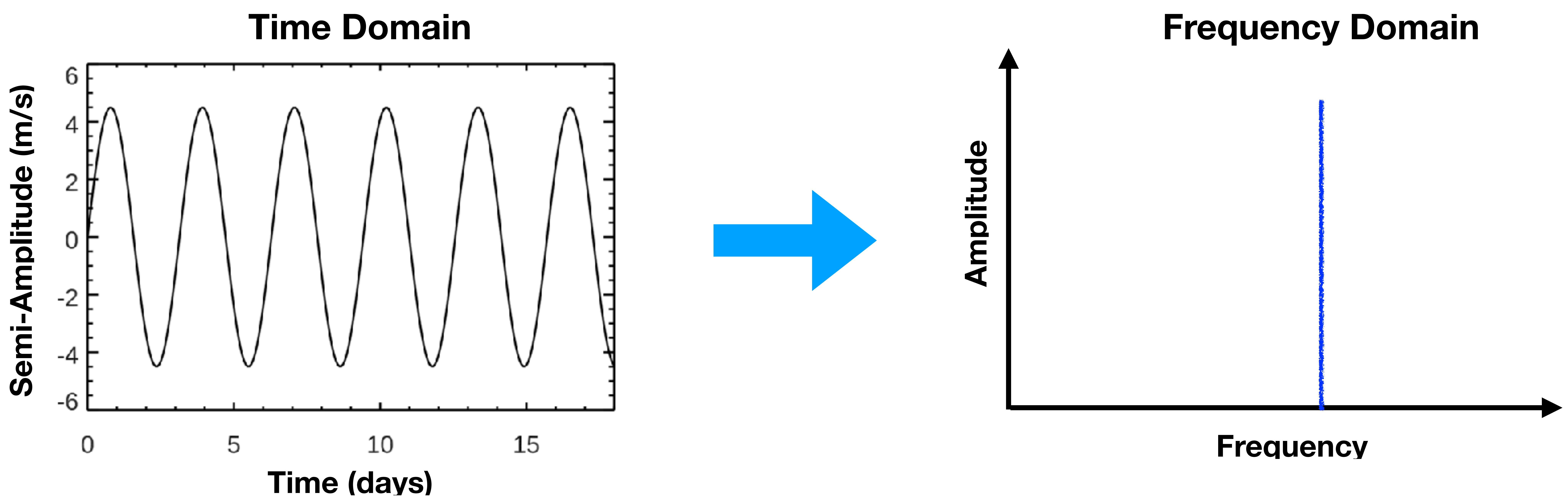


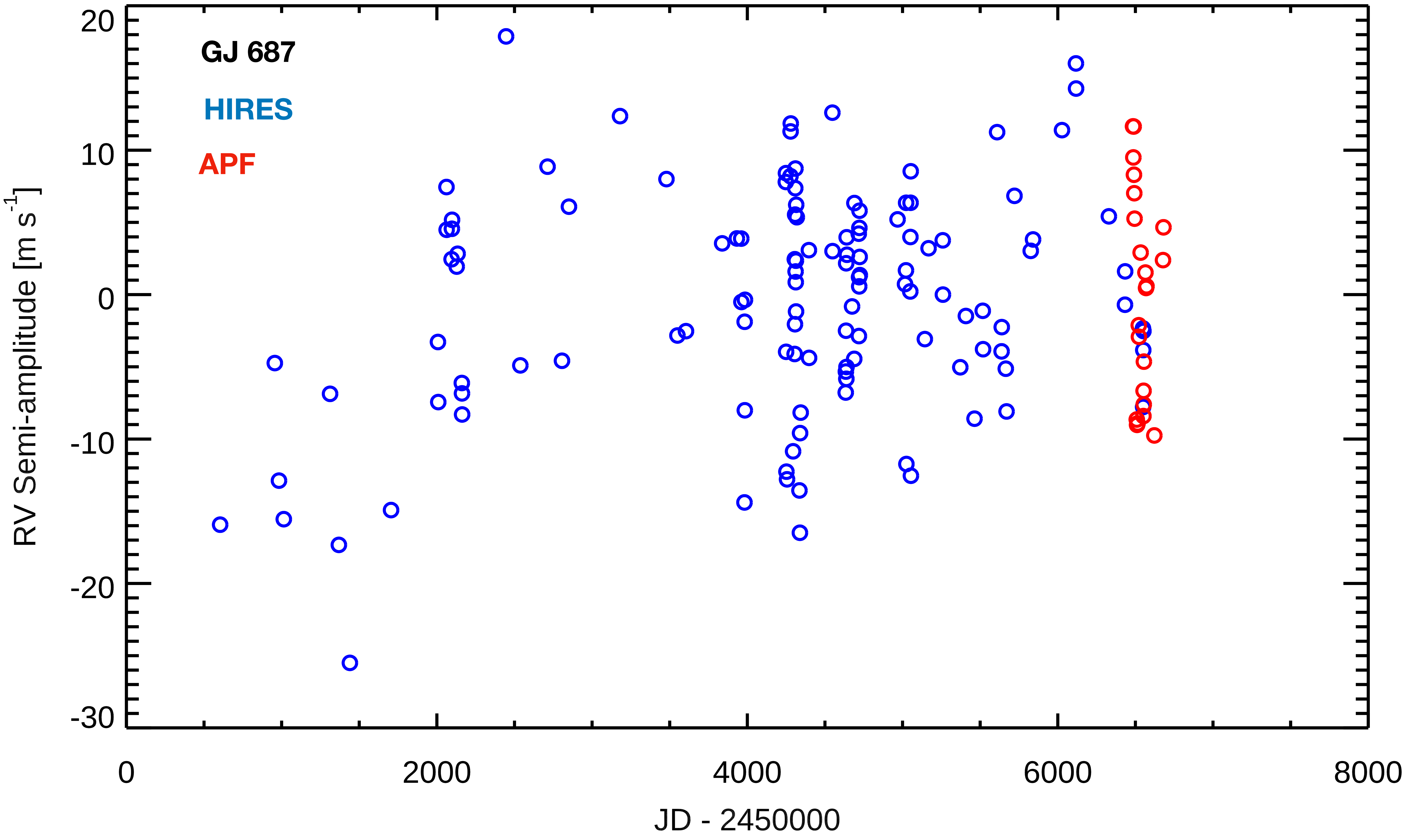
Lovis 2006

Periodograms: idealized case

Goal: identify periodic signals [planetary orbits] in time series data [RV semi-amplitudes]

Method: Take discrete Fourier transform (DFT) of time series data: $\mathrm{DFT}_X(\omega) = \sum_{i=1}^{N} X(t_j) e^{-i\omega t_j}$



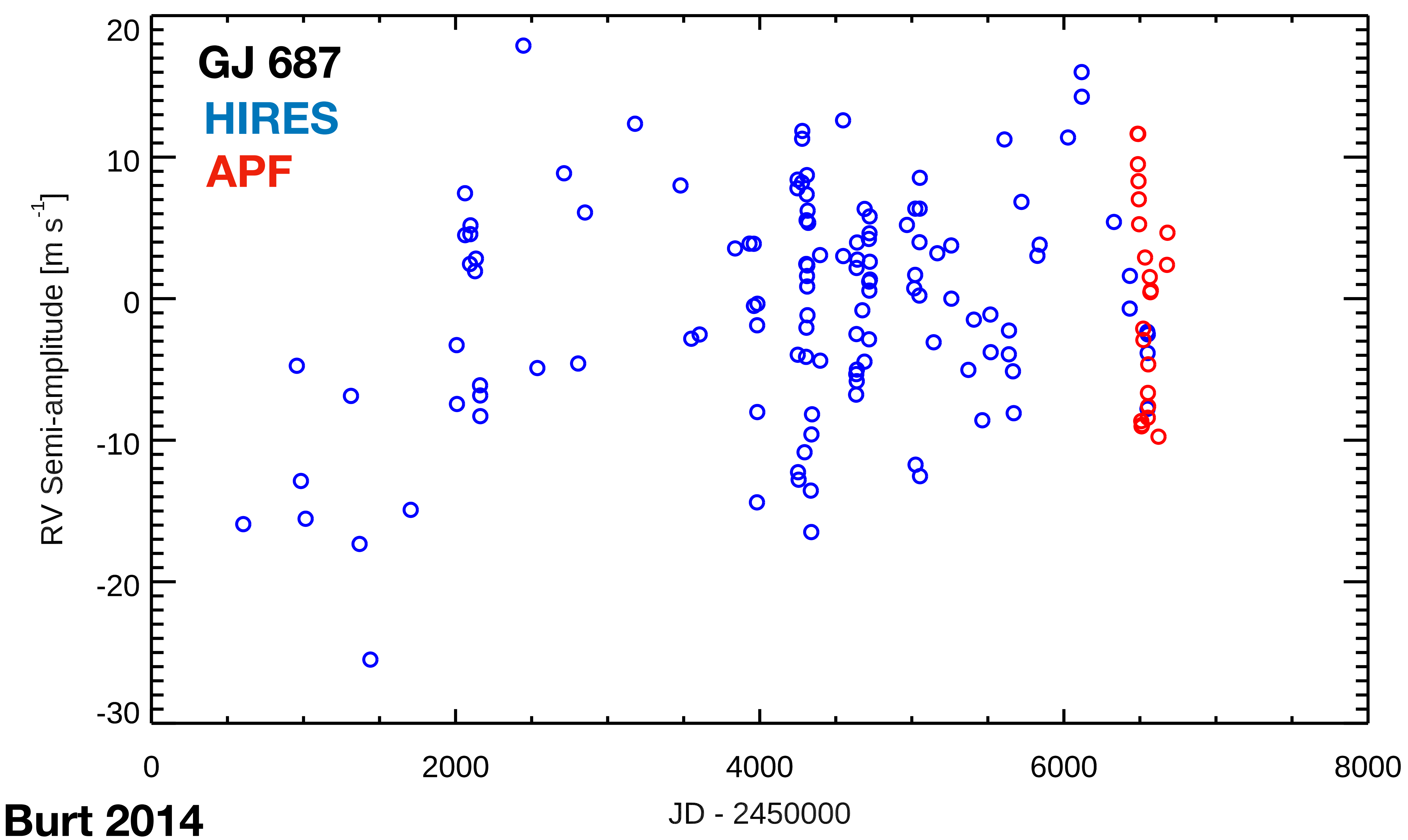

GJ 687
HIRES
APF
RV Semi-amplitude [m s^{-1}]
20
10
0
-10
-20
-30
0
2000
4000
6000
8000
JD - 2450000

Burt 2014

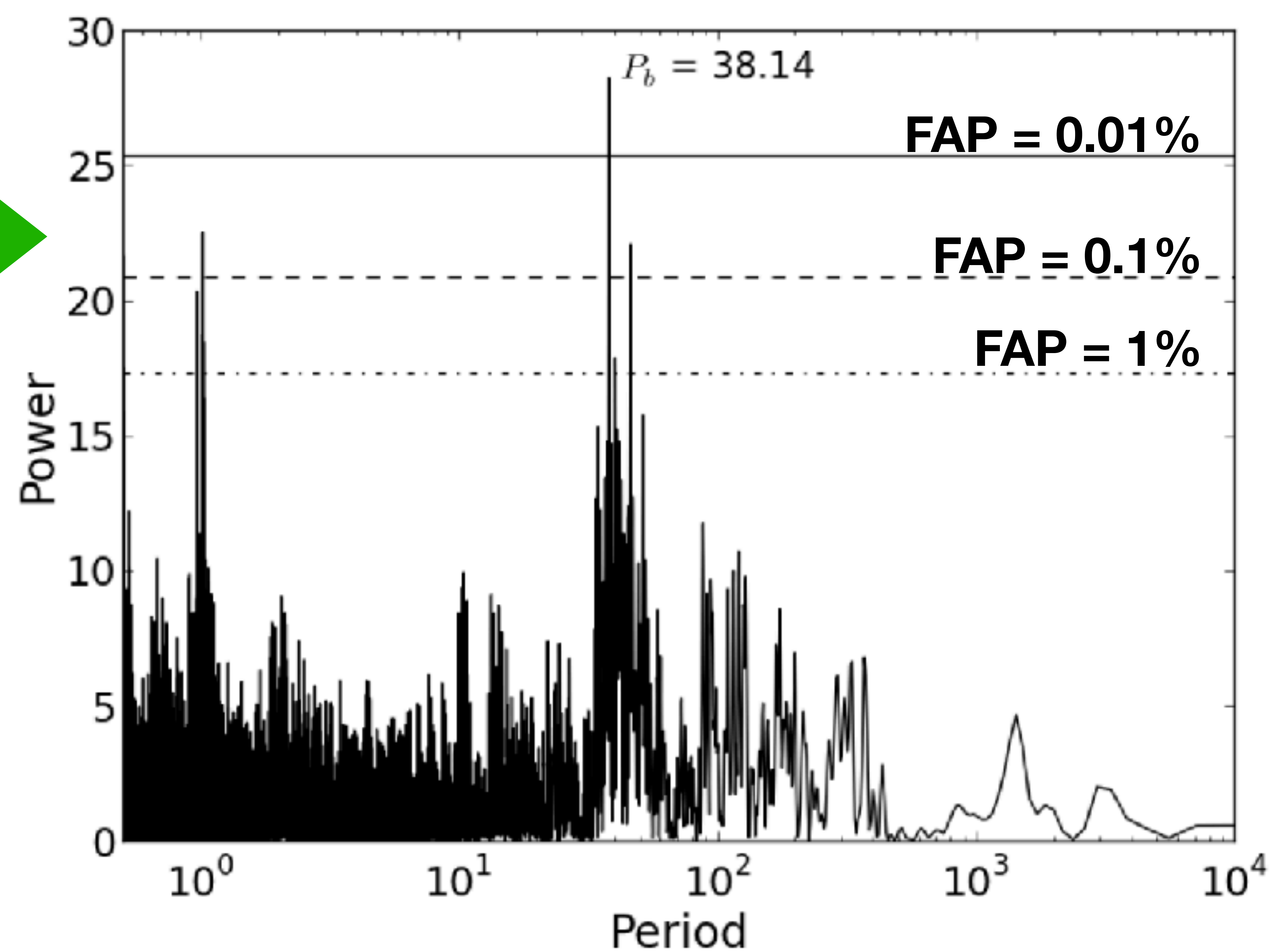
Periodograms: what we actually do

Lomb-Scargle (LS) Periodogram: Able to handle data that is unevenly sampled (phew!)

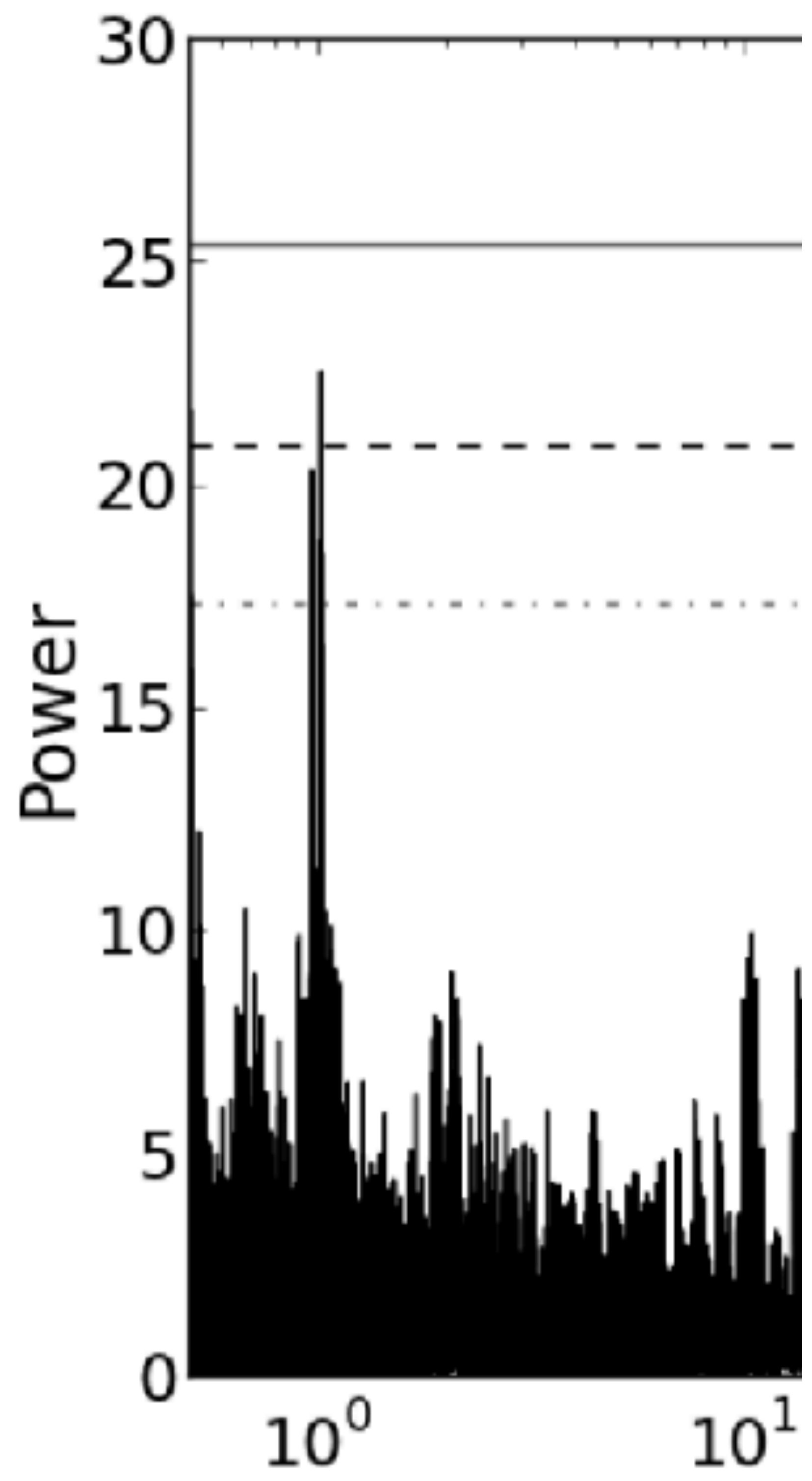
My rule of thumb: False Alarm Probability below 1% suggests periodic signal in the data, and anything below 0.1% merits real investigation”

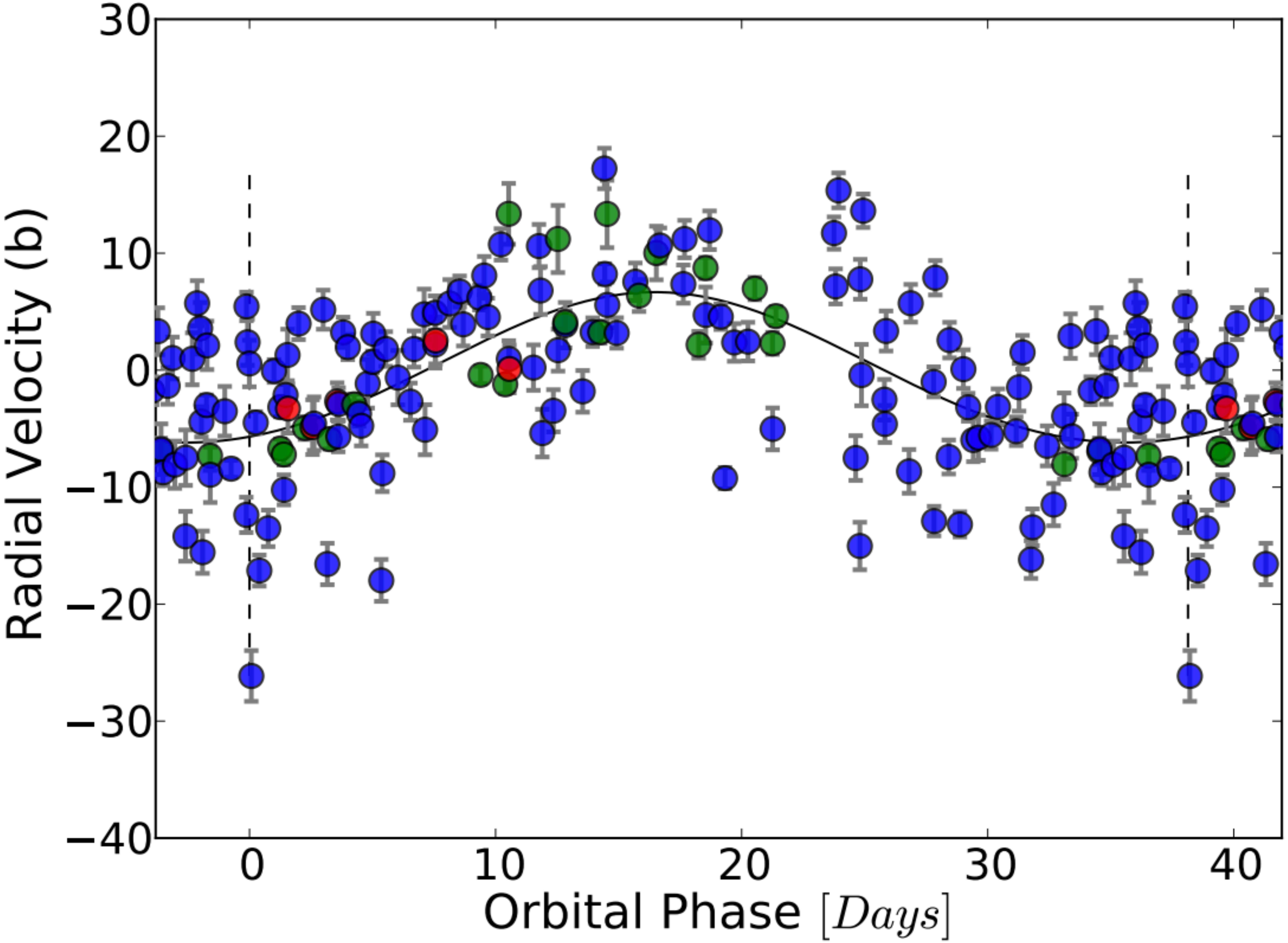


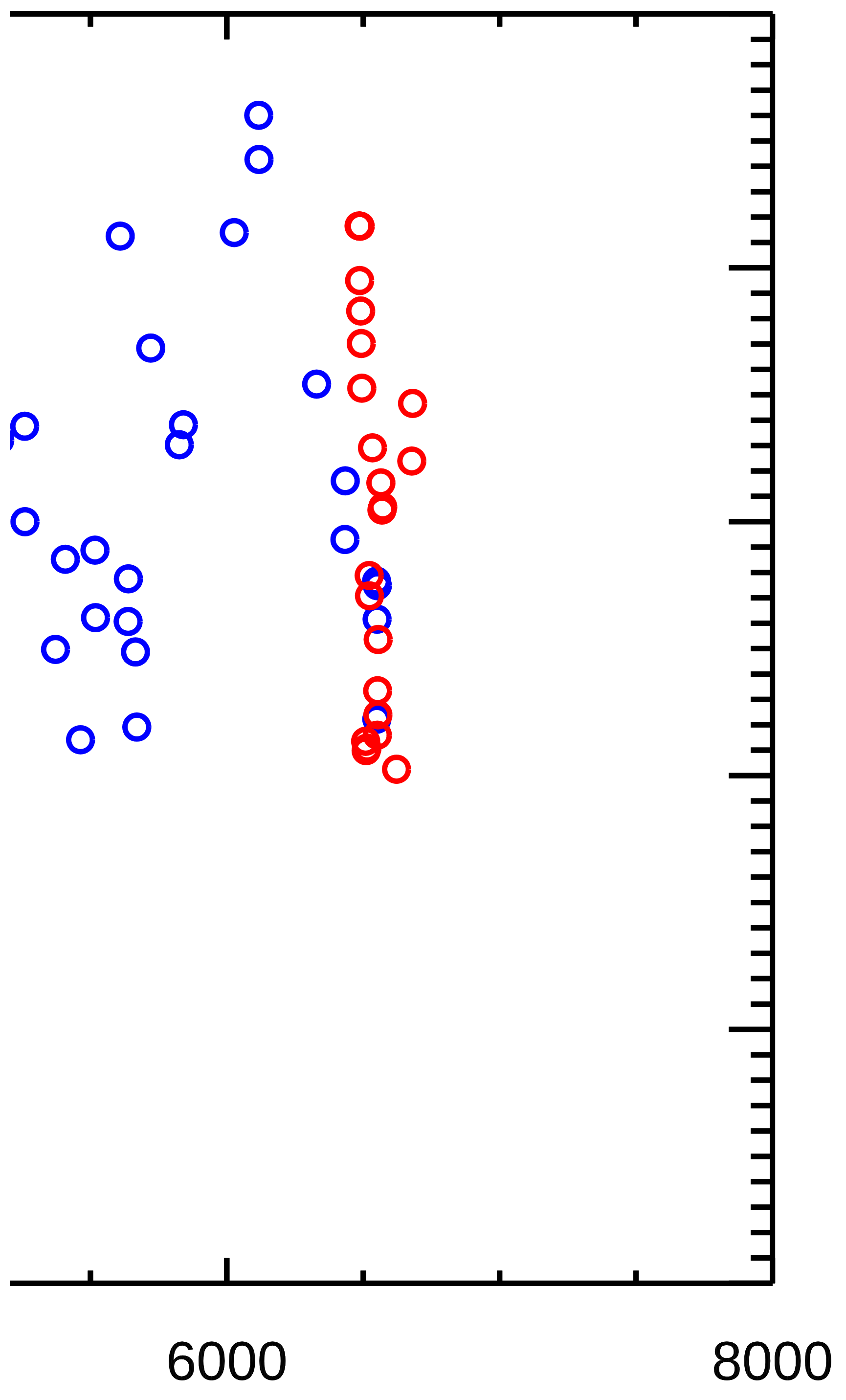
Burt 2014



Periodograms: what we actually do

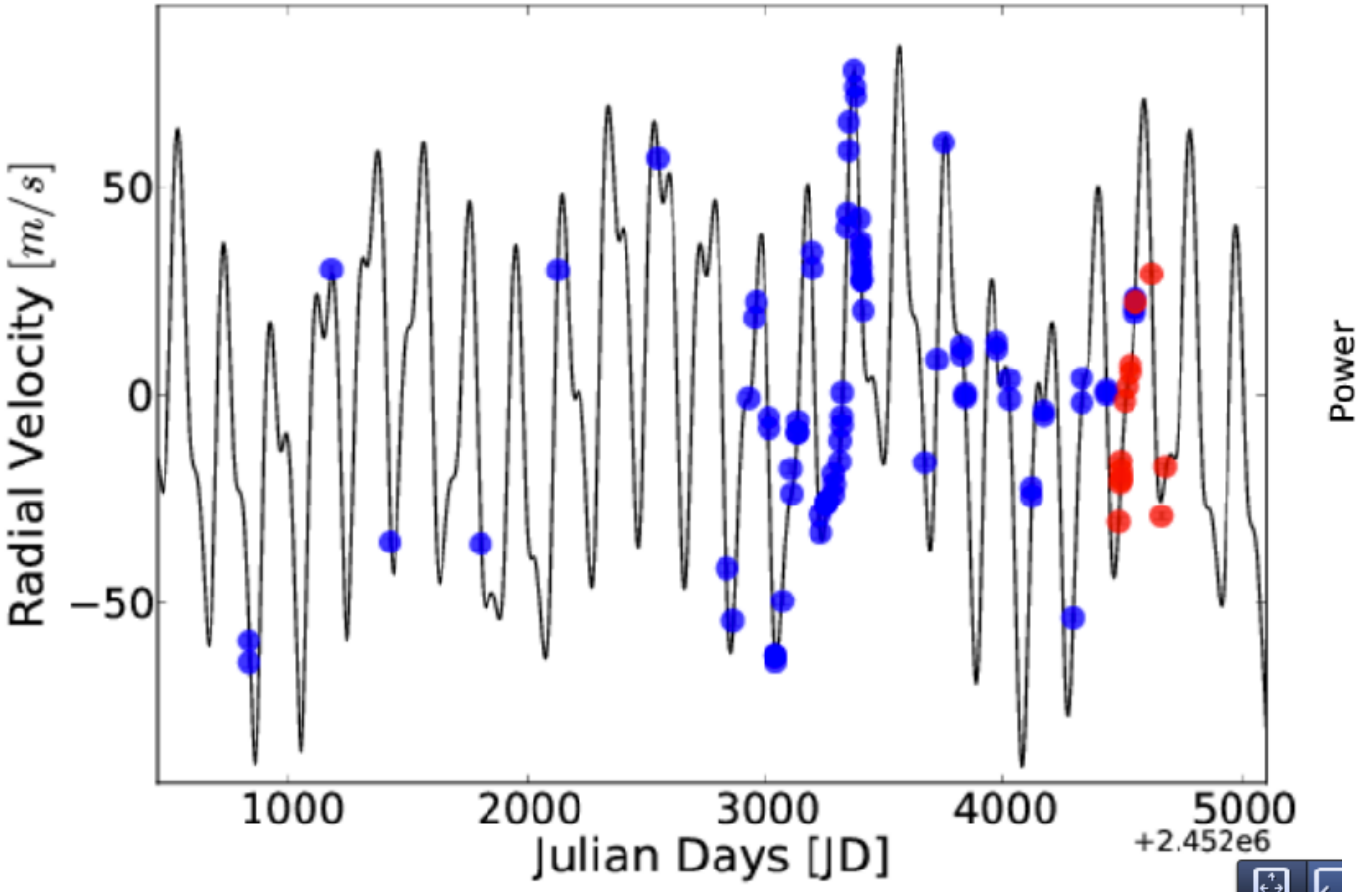


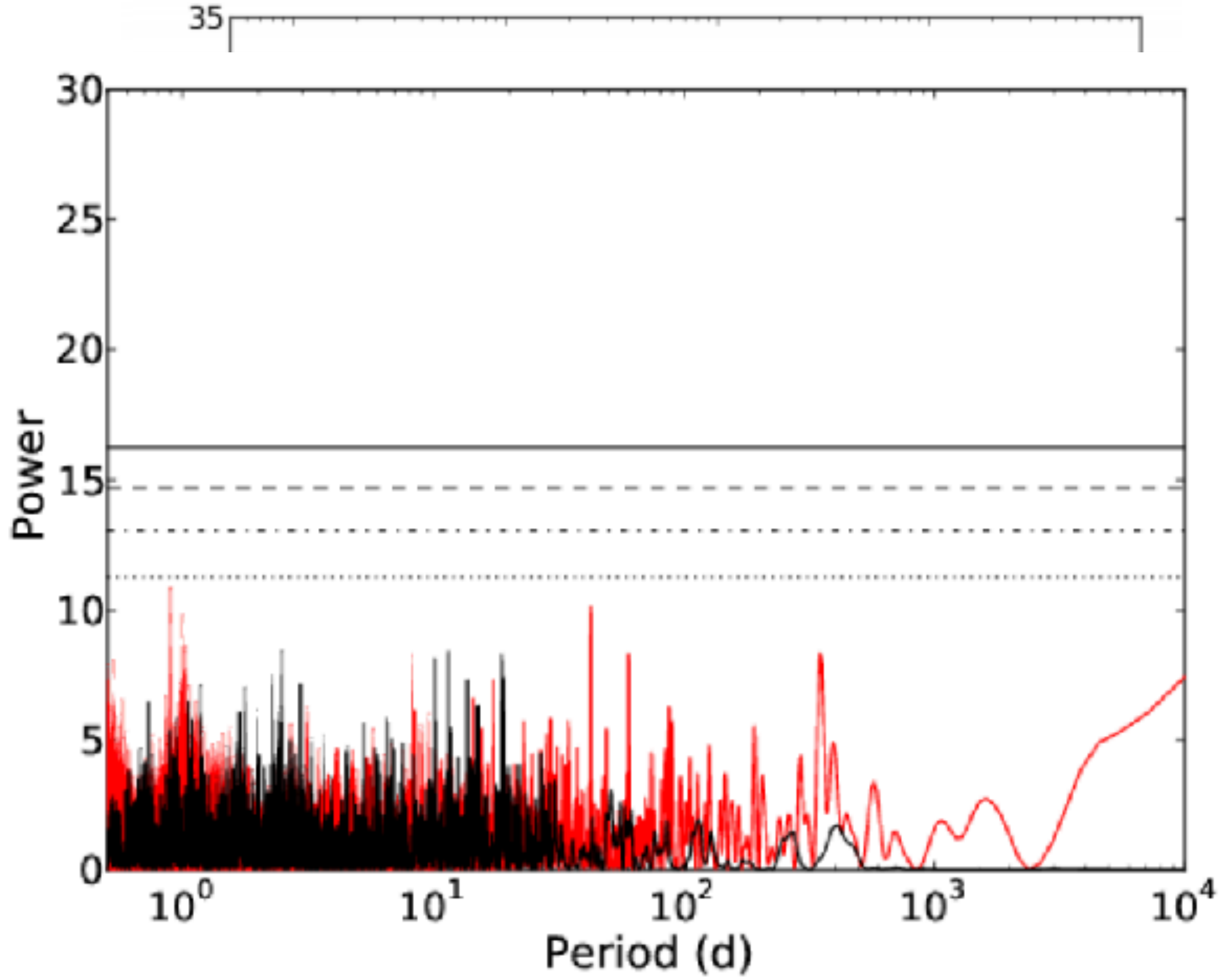


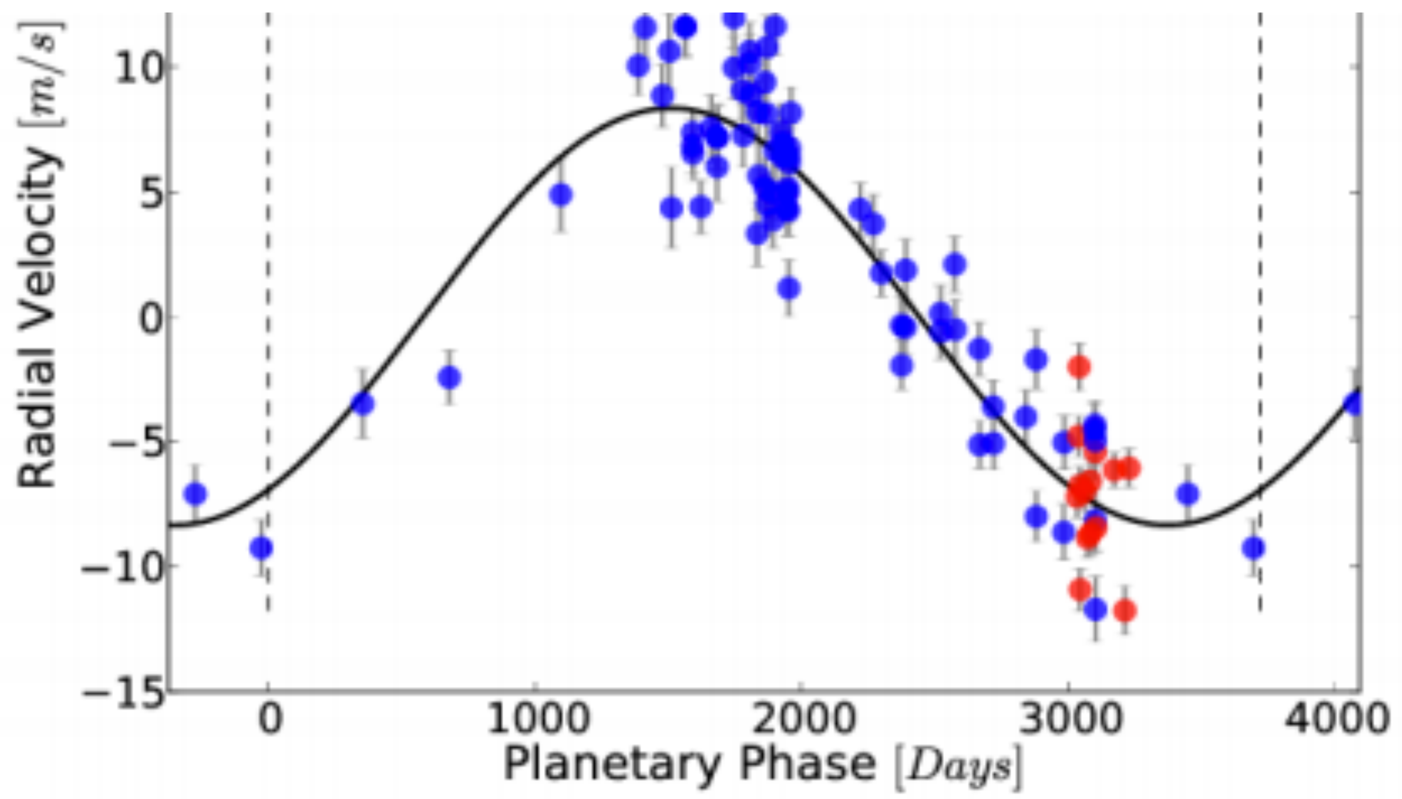


Burt 2014

Fitting orbital parameters : HD 141399





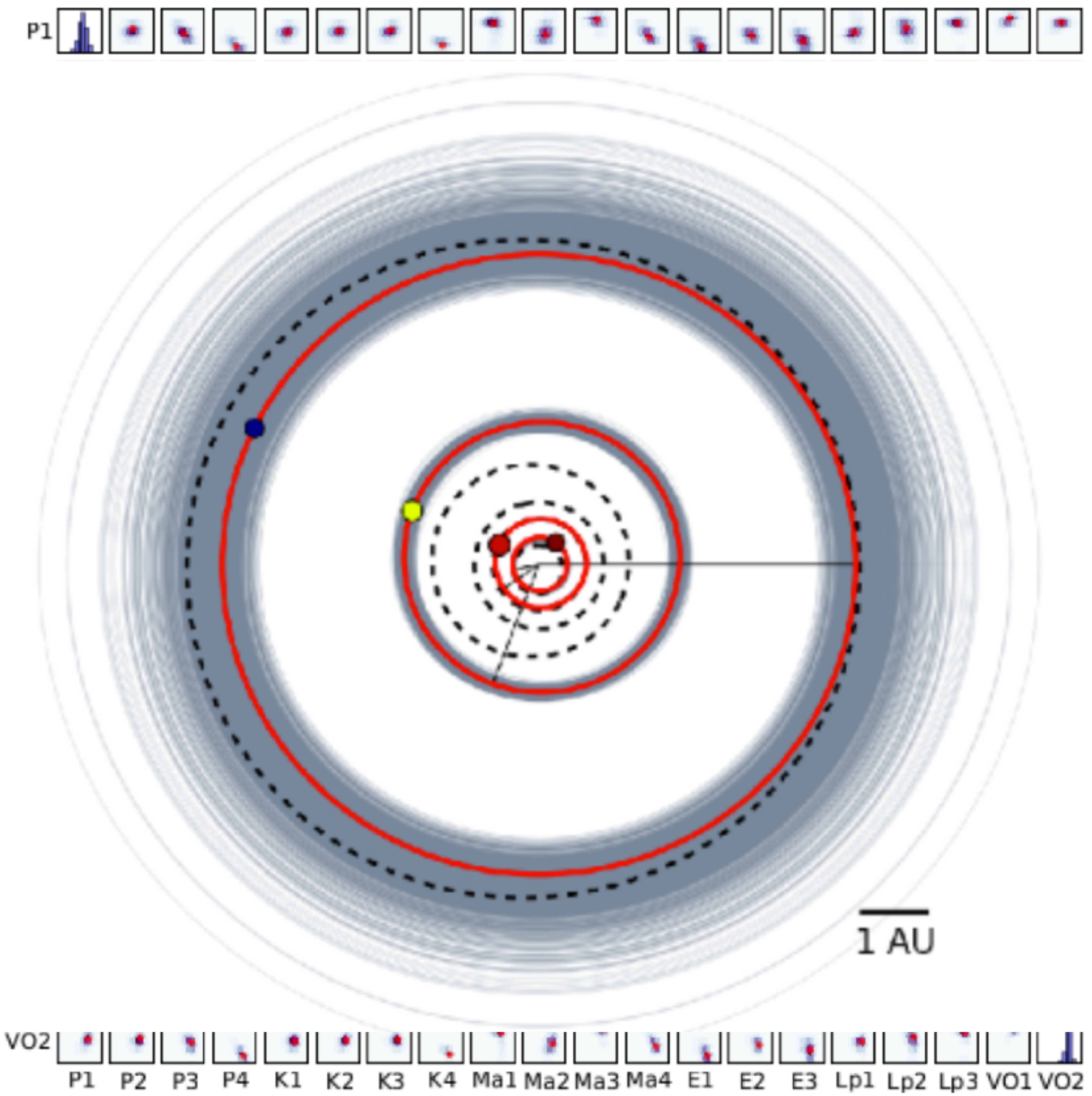


Vogt 2014

Fitting orbital parameters : HD 141399 MCMC

Self-consistent 4-planet model for the HD 141399 System

		Best fit	Errors
Period (d)	b	94.35	(0.059)
	c	202.08	(0.099)
	d	1070.35	(8.178)
	e	3717.35	(555.081)
RV Half-Amplitude ($m\,s^{-1}$)	b	18.8	(0.551)
	c	43.51	(0.591)
	d	22.28	(0.63)
	e	8.34	(1.239)
Mean Anomaly (deg)	b	224.63	(54.09)
	c	303.75	(15.165)
	d	273.89	(39.812)
	e	153.93	(23.889)
Eccentricity	b	0.04	(0.03)
	c	0.05	(0.013)
	d	0.06	(0.029)
	e	0.0	(Fixed)
Longitude of Periastron (deg)	b	191.37	(55.088)
	c	214.74	(14.457)
	d	249.16	(38.966)
	e	0.0	(Fixed)
Time of Periastron (JD)	b	2452774.98	(15.371)
	c	2452663.34	(8.537)
	d	2452019.53	(119.538)
	e	2451244.36	(555.624)
Semi-Major Axis (AU)	b	0.4225	(0.00018)
	c	0.7023	(0.00023)
	d	2.1348	(0.01086)
	e	4.8968	(0.46122)
Mass ($M_{\rm Jup}$)	b	0.46	(0.025)
	c	1.36	(0.067)
	d	1.22	(0.067)
	e	0.69	(0.164)
First Observation Epoch (JD)		2452833.85	
Velocity Offset (KECK)		0.61 $m\,s^{-1}$	(1.7)
Velocity Offset (APF)		1.48 $m\,s^{-1}$	(1.89)
χ^2		5.81	
RMS		2.36 $m\,s^{-1}$	
Jitter (KECK)		2.35 $m\,s^{-1}$	(0.281)
Jitter (APF)		2.59 $m\,s^{-1}$	(0.729)

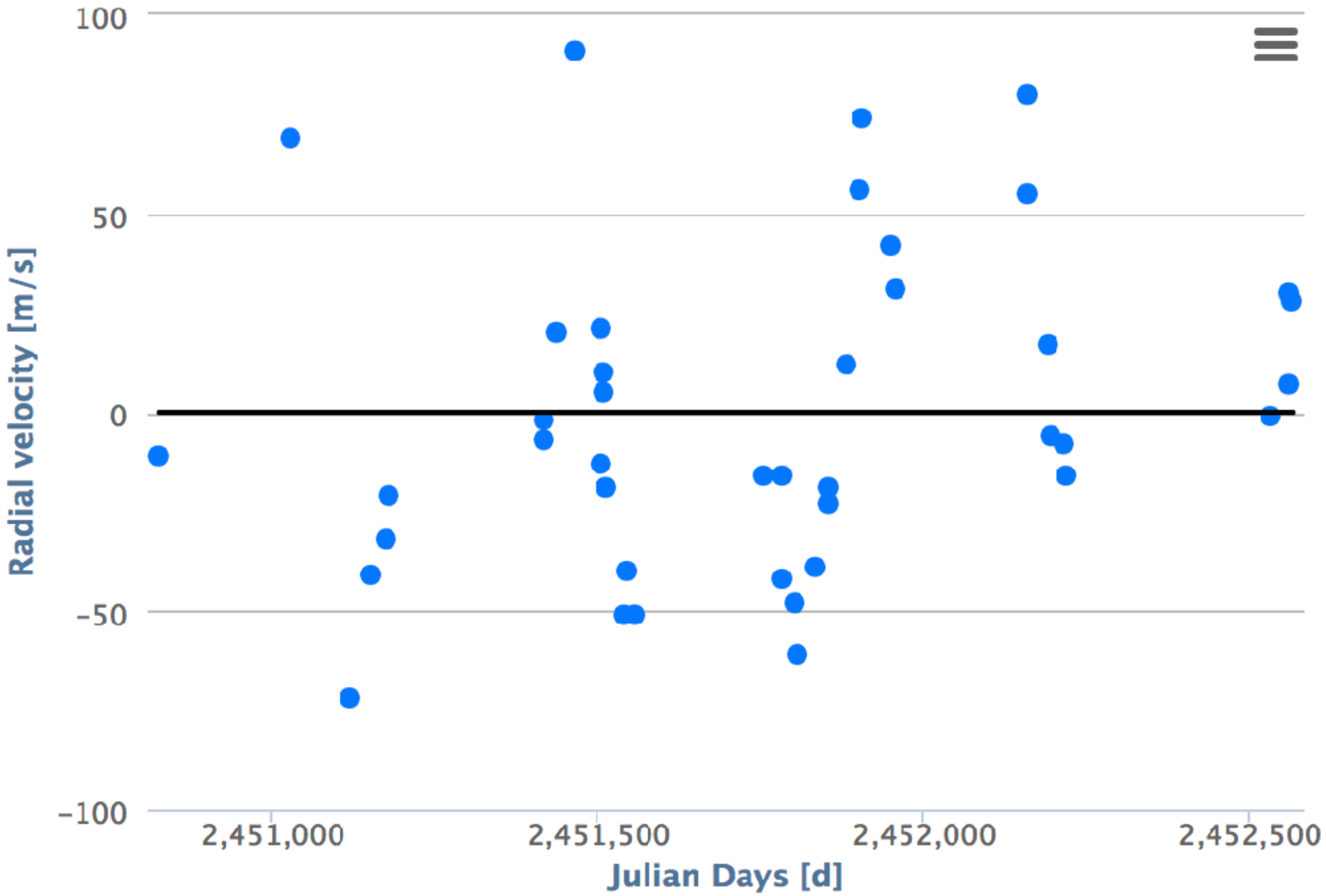


Vogt 2014

Fitting orbital parameters : HD 8574

http://www.stefanom.org/systemic-online/?sys=HD8574.sys

		HD 8574
HIP		6643
$Sp.Type$		F8
m_V		7.12
$B-V$		0.577 ± 0.011
π	(mas)	22.65 ± 0.82
$Distance$	(pc)	$44.2 \pm ^{1.7}_{1.5}$
$\mu_\alpha \cos(\delta)$	(mas yr^{-1})	252.59 ± 0.76
μ_δ	(mas yr^{-1})	-158.59 ± 0.55
M_V		3.89
$B.C.$		-0.034
L	($L_\odot$)	2.25
$T_{\rm eff}$	(K)	6080 ± 50
$\log g$	(cgs)	4.41 ± 0.15
ξ_t	(km s^{-1})	1.25 ± 0.10
[Fe/H]		0.05 ± 0.07
$W_{\lambda,\rm Li}$	(mÅ)	52.1
$\log n$(Li)		2.56 ± 0.09
$v \sin i$	(km s^{-1})	4.04 ± 0.61
M_*	($M_\odot$)	1.17



Perrier 2003

Fitting orbital parameters : HD 8574

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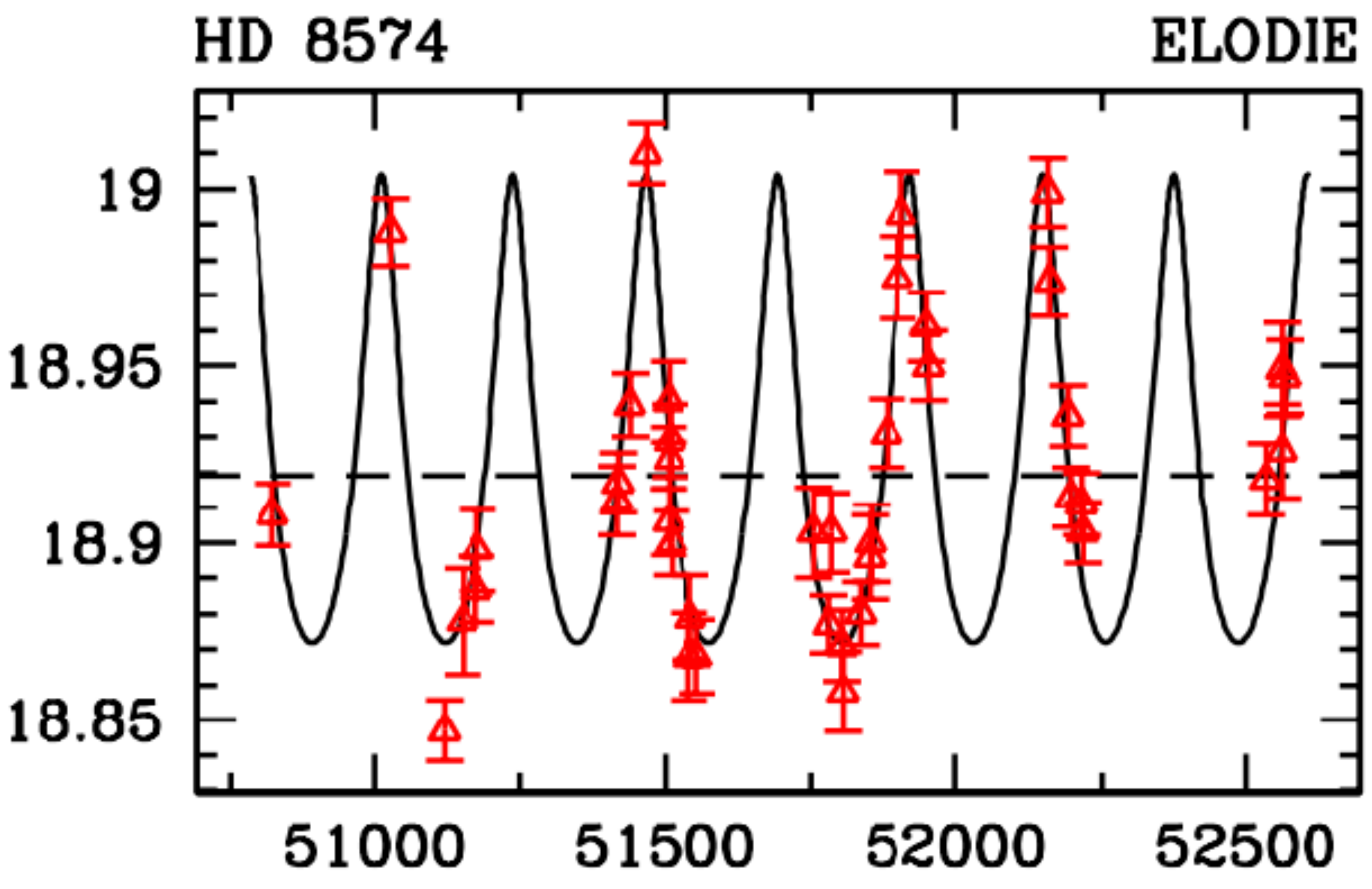


		HD 8574
P	(days)	227.55 ± 0.77
T	(JD)	$2\,451\,467.5 \pm 6.6$
e		0.288 ± 0.053
γ	(km s^{-1})	18.919 ± 0.002
w	(°)	3.6 ± 10.9
K_1	(m s^{-1})	66 ± 5
$a_1 \sin i$	(10^{-3}AU)	1.32
$f_1(m)$	($10^{-9}M_\odot$)	5.97
$m_2 \sin i$	($M_{\rm Jup}$)	2.11
a	(AU)	0.77
$a_{\rm min}$†	(AU)	0.55
$a_{\rm max}$‡	(AU)	0.99
$N_{\rm meas}$		41
$\langle\epsilon_{\rm RV}\rangle$	(m s^{-1})	10.2
$\sigma(O-C)$	(m s^{-1})	13.1
$\chi^2_{\rm red}$		1.99

Perrier 2003

Orbit determination : outline

1) Get RVs from your favorite observer (or better yet, go observing!)

2) Feed the RV data into some sort of RV analysis package [ExoFast2, Radvel, Systemic, etc]

3) Look at the periodogram for the RV data, identify peaks that are:
> Above your threshold for being a significant signal
> Well separated from other peaks

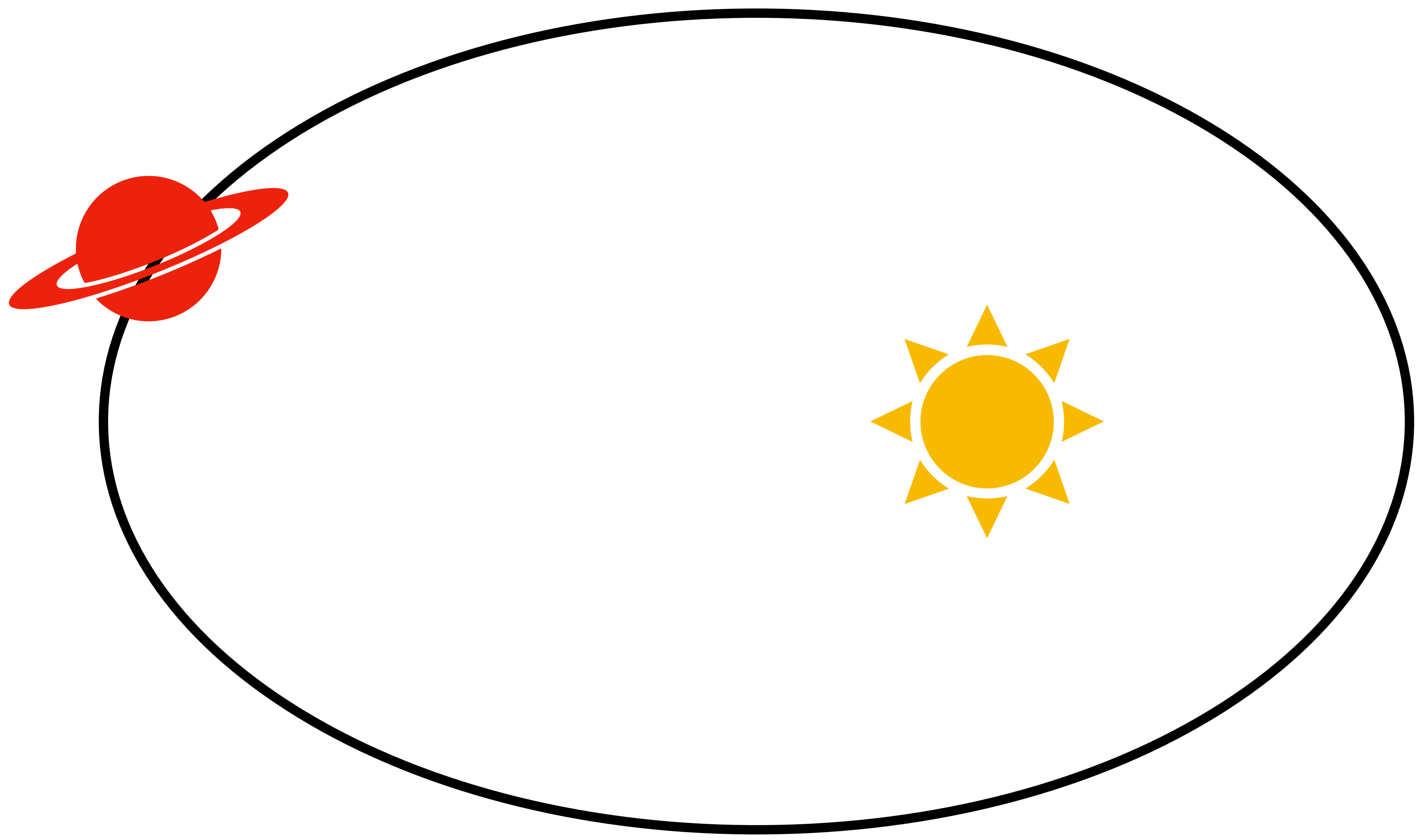
4) Fold the data to that period, and use the software to determine the orbital parameters

5) Look at the residuals periodogram, if there are additional peaks repeat steps 3/4

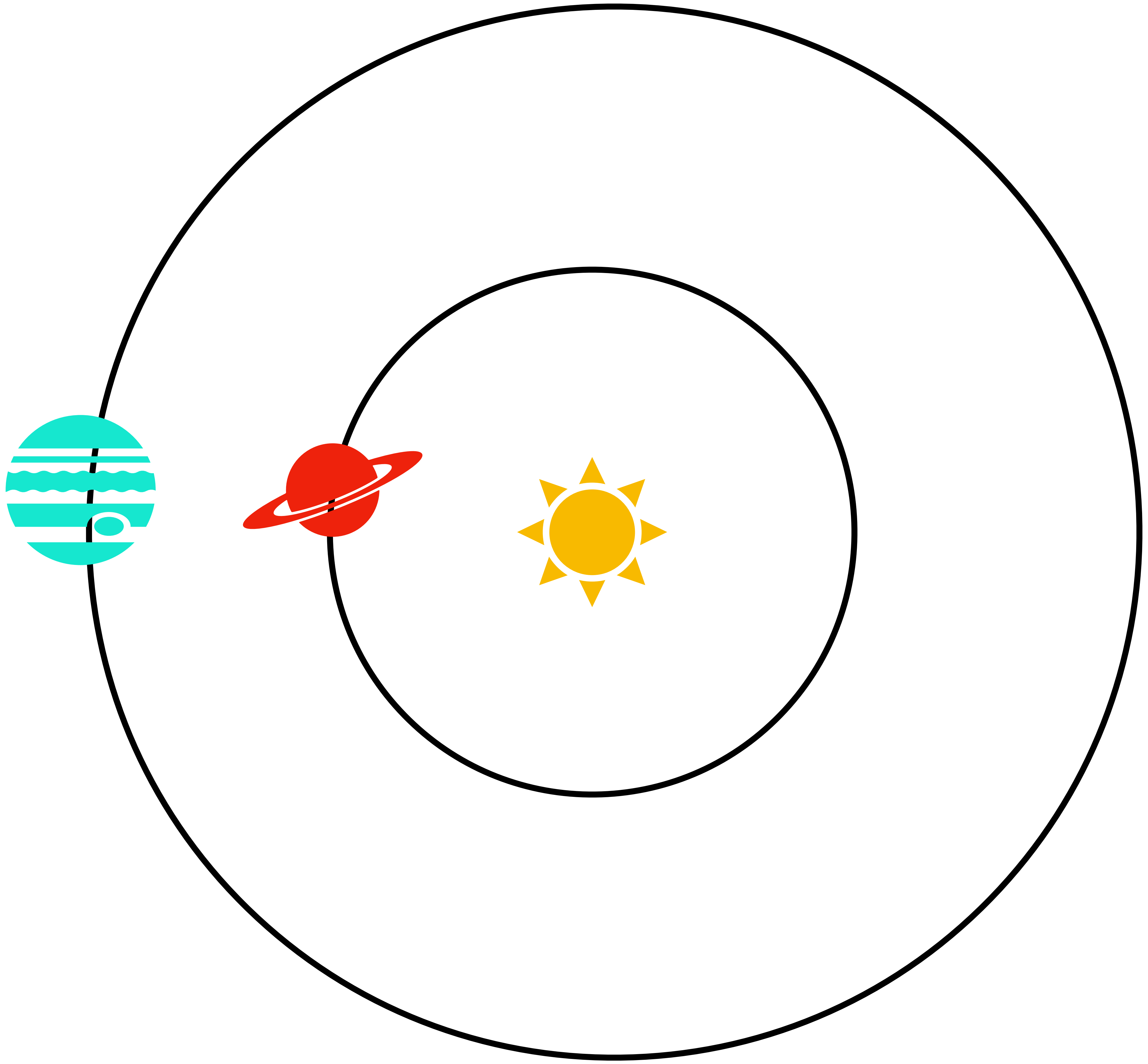
Then need to make sure that these signals are actually [the correct] planets…

A couple of things to watch out for

Eccentric planets can masquerade as resonances

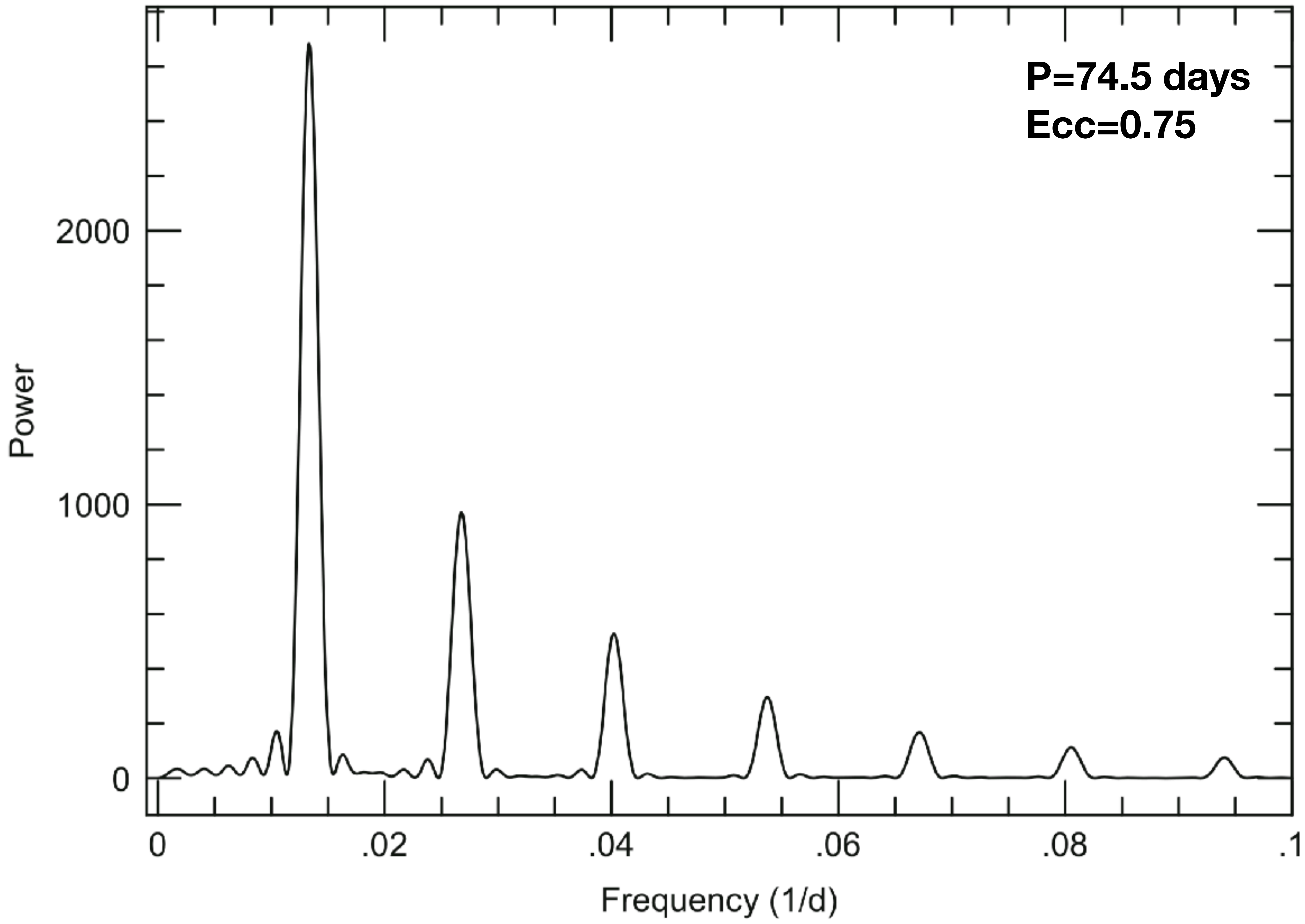


# of planets = 1
Eccentricity = 0.75



# of planets = 2
Eccentricity = 0
P1/P2 = 0.5

Eccentric planets can masquerade as resonances



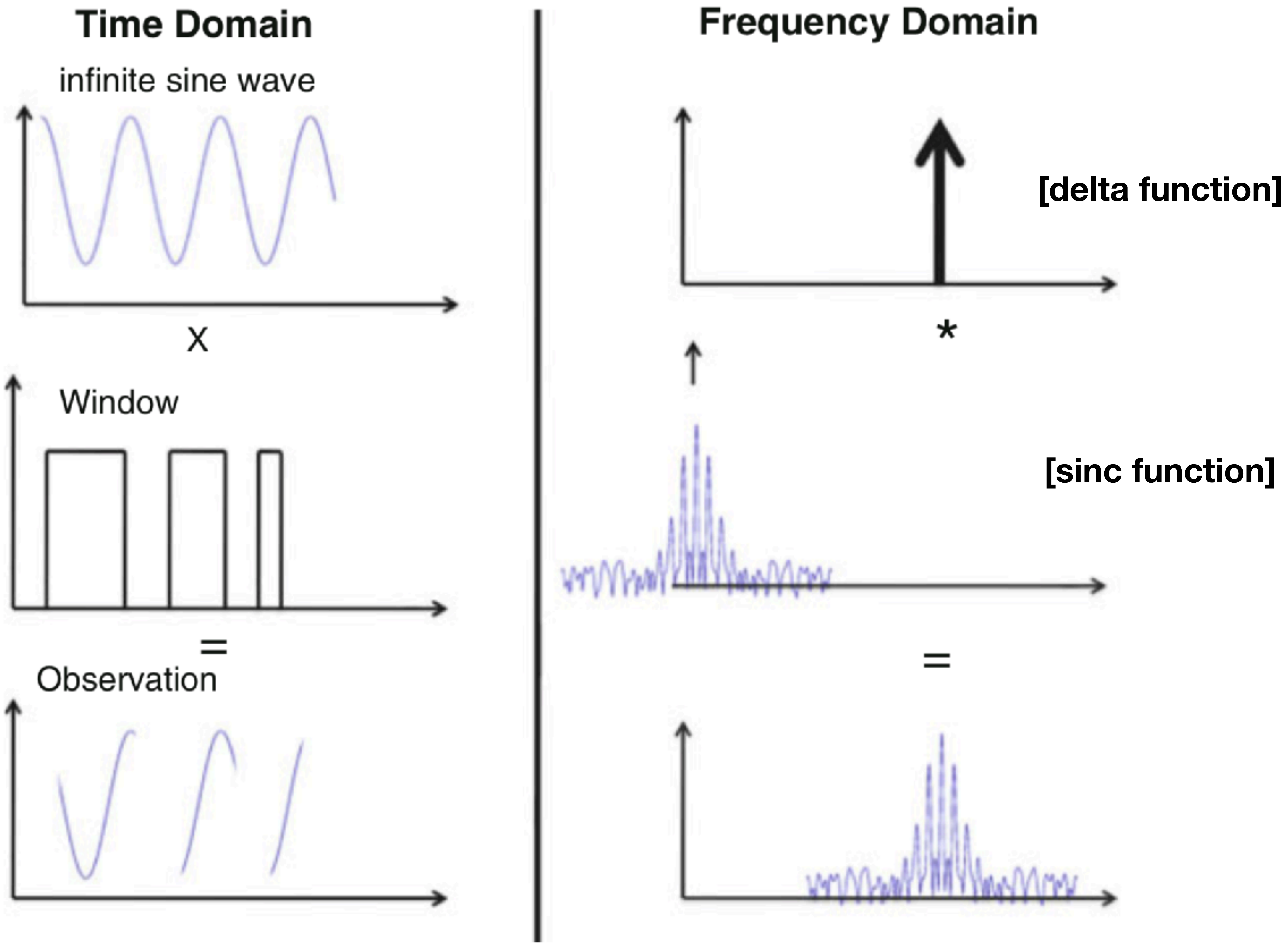
Fourier transform of an eccentric orbit shows both the peak at the true frequency & its harmonics

If you see a peak in your periodogram and at least one of Its harmonics, try fitting an eccentric planet to see if that

Bozza 2016

Window functions add periodicities to your data

Window functions add periodicities to your data



Bozza 2016

Window functions add periodicities to your data

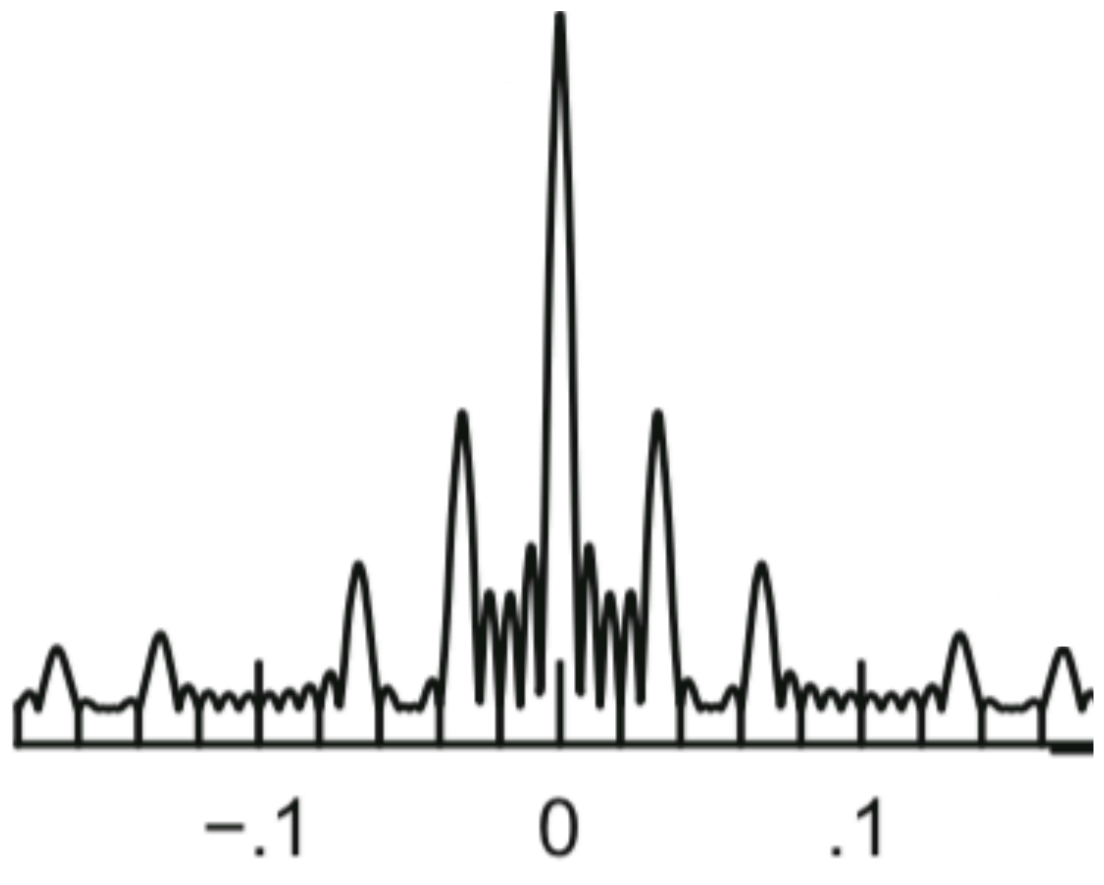
8 planet system?

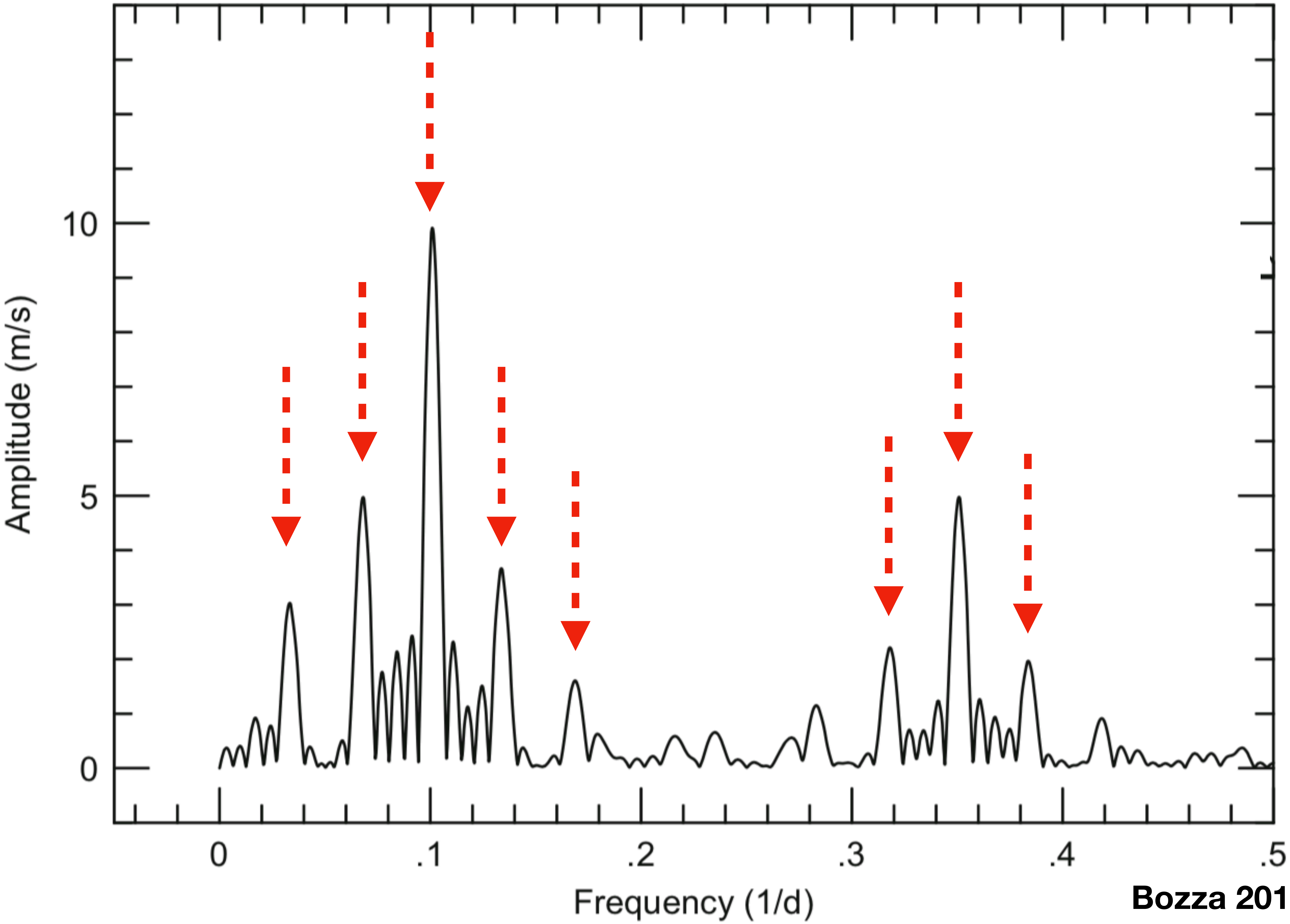
no…

2 planets

+

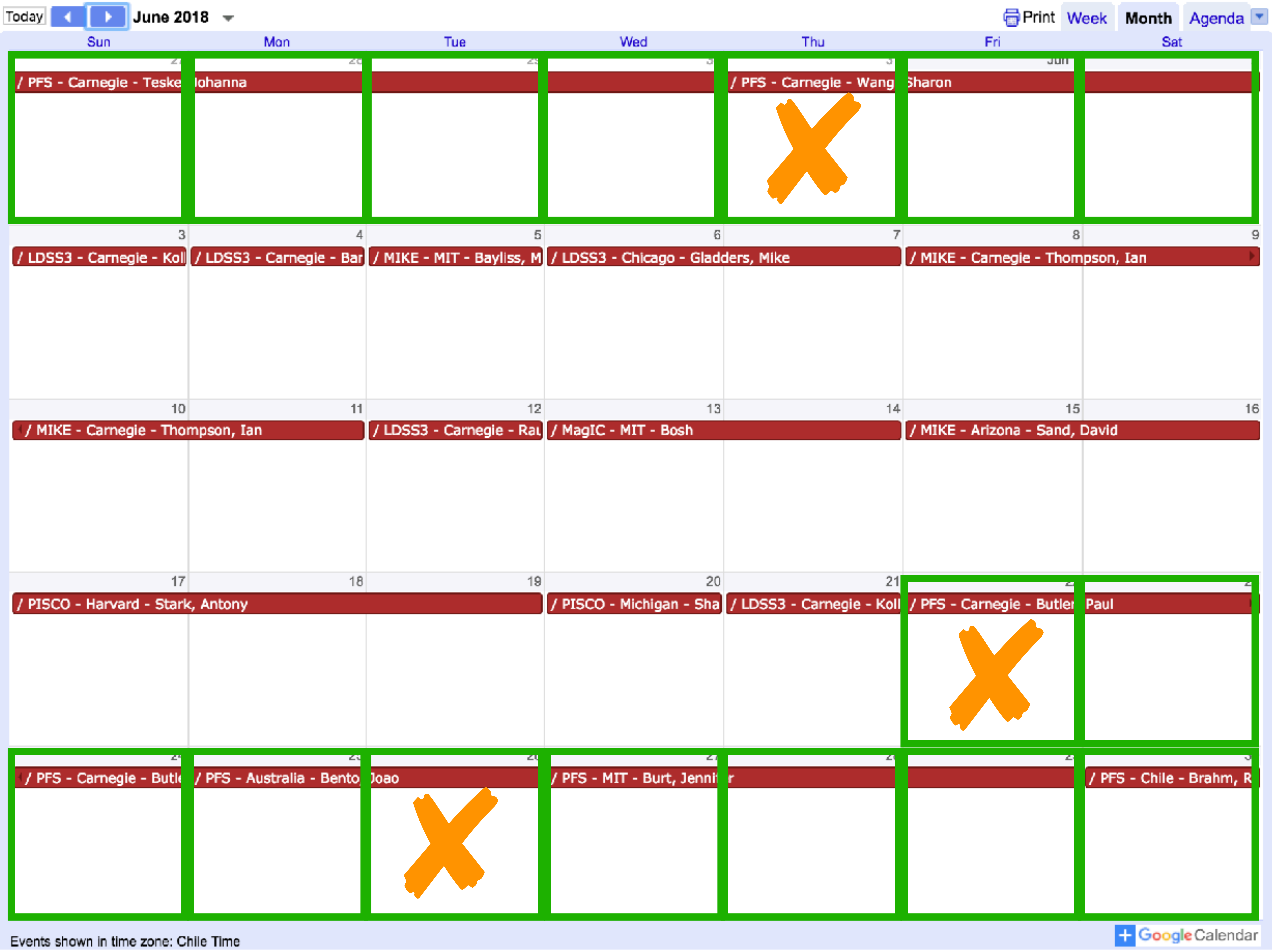
1 window function





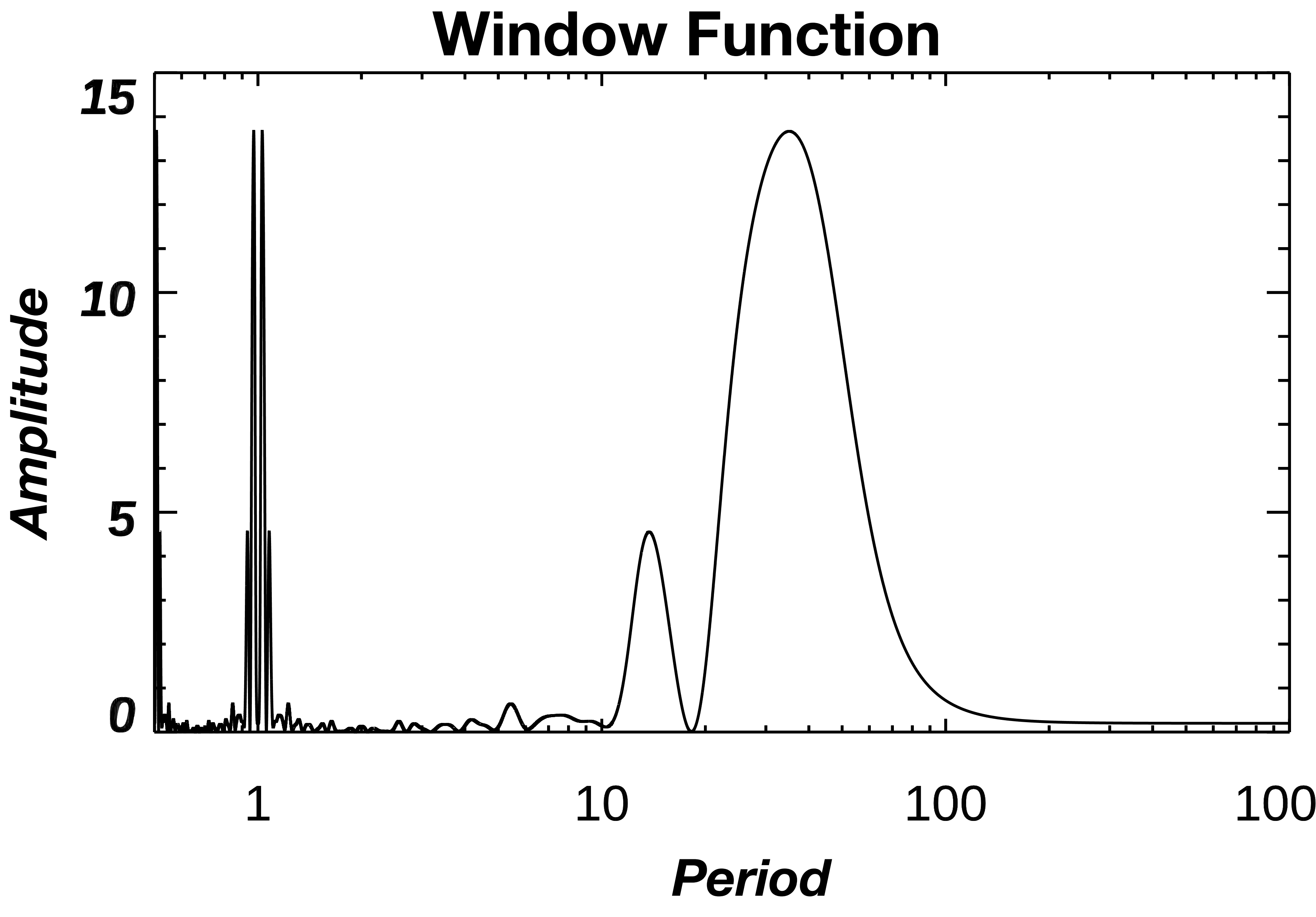
Bozza 2016

Window functions: observing example



obs=[1,1,1,1,0,1,1,0,0,0,0,0,0,0,
0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,0,
1,1,1,1]

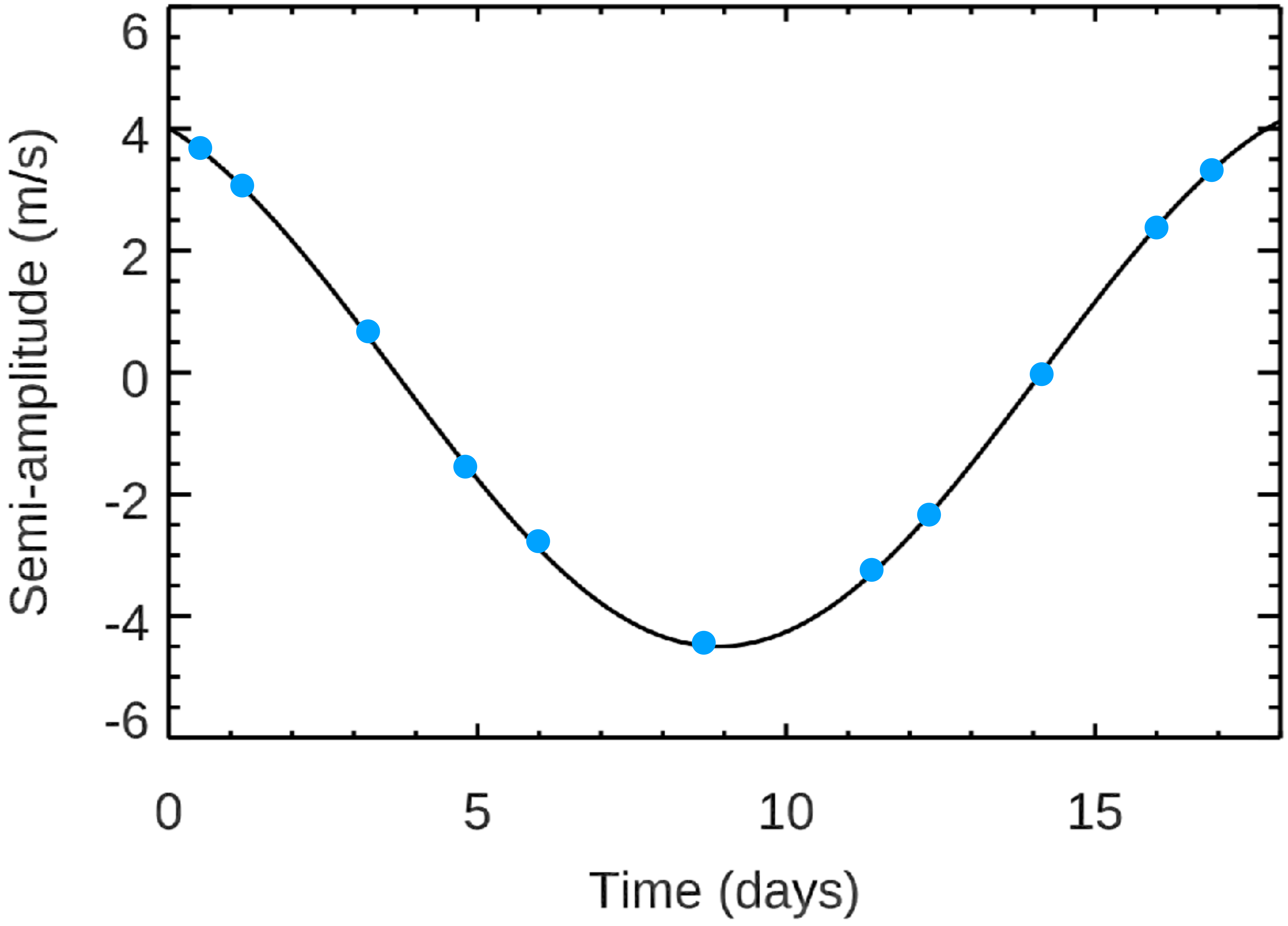
Lomb-Scargle(jd,obs)



Aliasing can make you select the wrong period

Aliasing can make you select the wrong period

Alias period : an incorrect (longer) period that is due to under sampling the correct (shorter) orbital period



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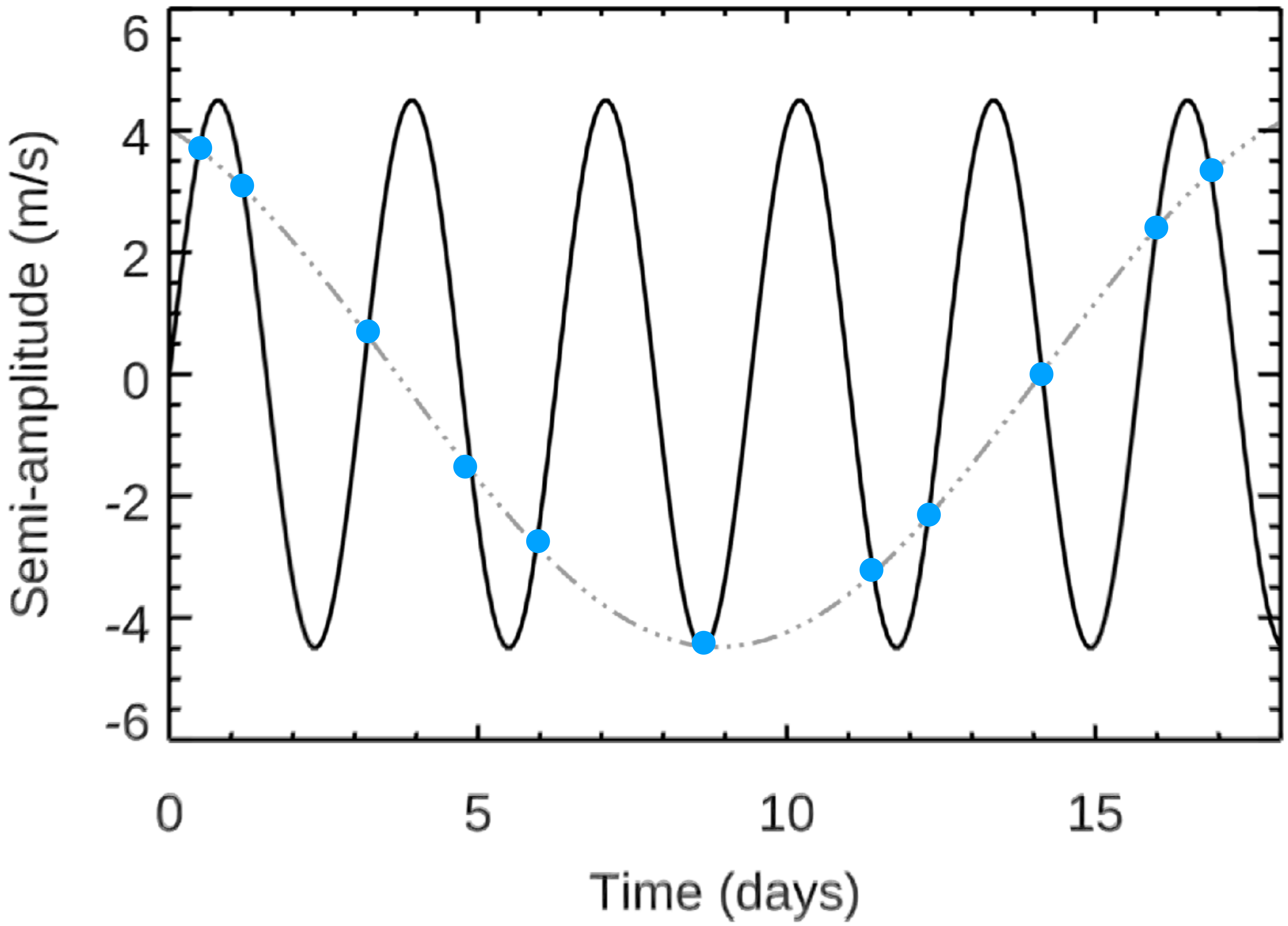
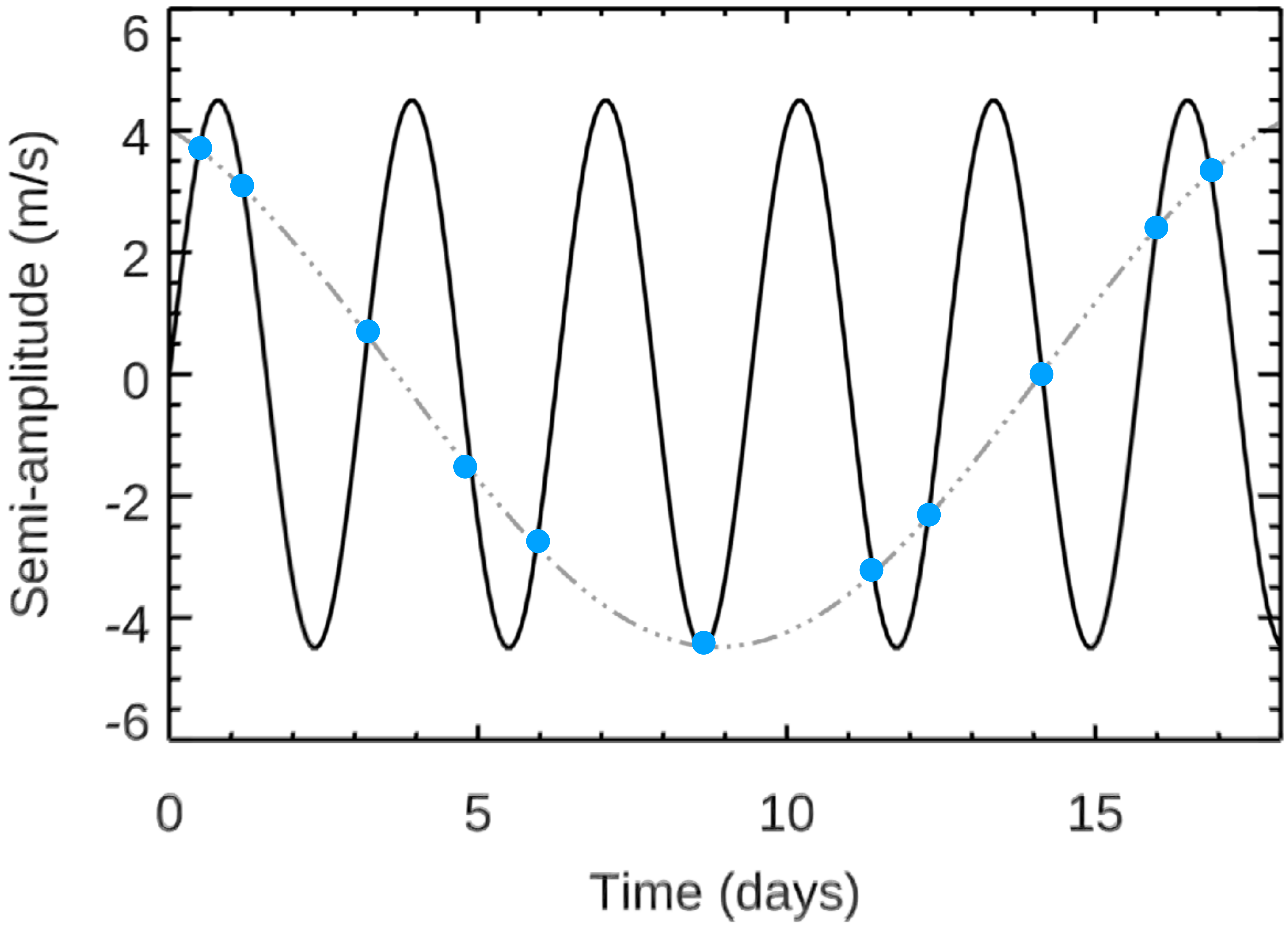


Aliasing can make you select the wrong period

Alias period : an incorrect (longer) period that is due to under sampling the correct (shorter) orbital period

To avoid aliasing, need to Nyquist sample your data

$$f_{data} \geq 2f_{Nyq}$$

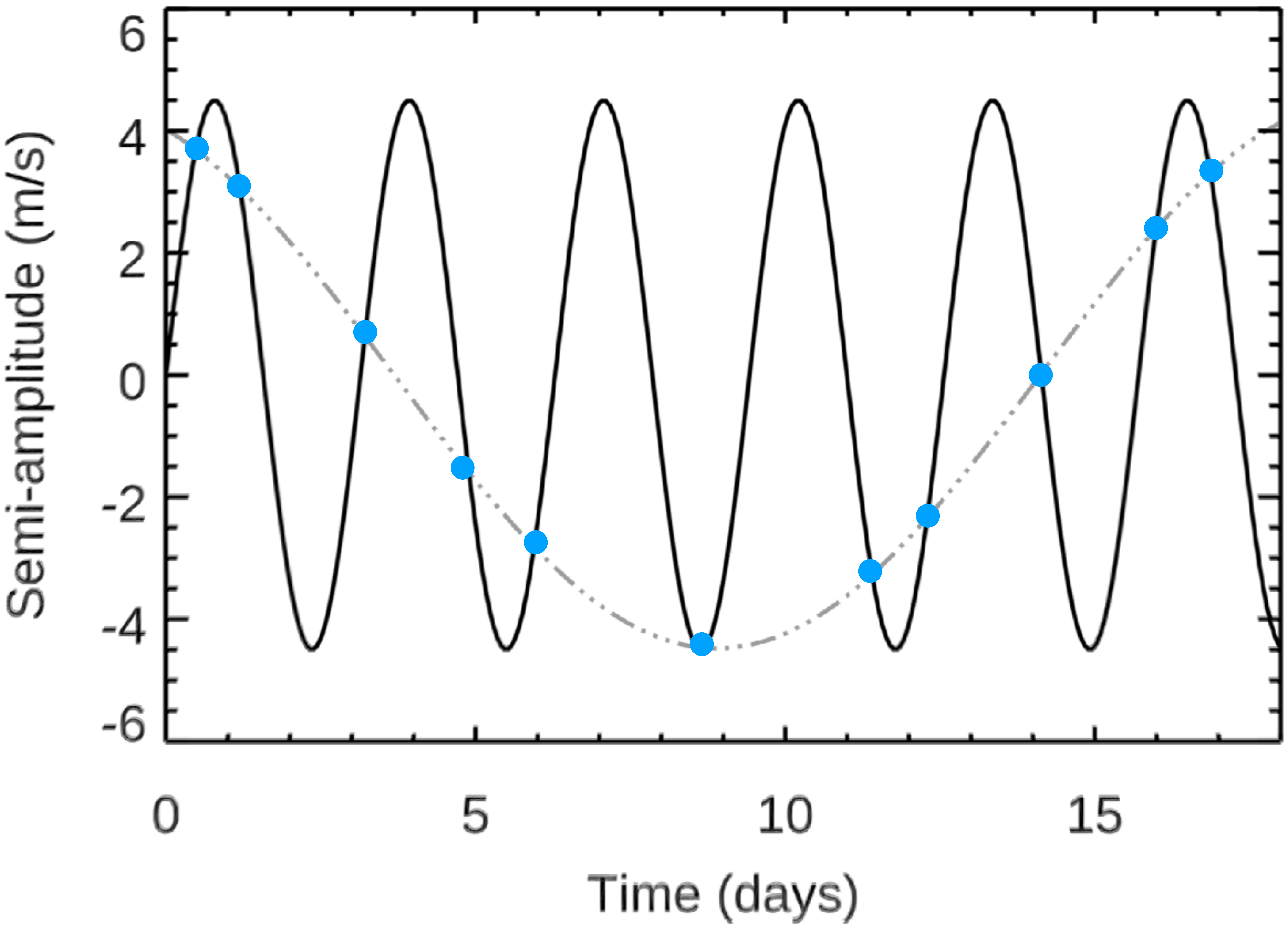


Aliasing can make you select the wrong period

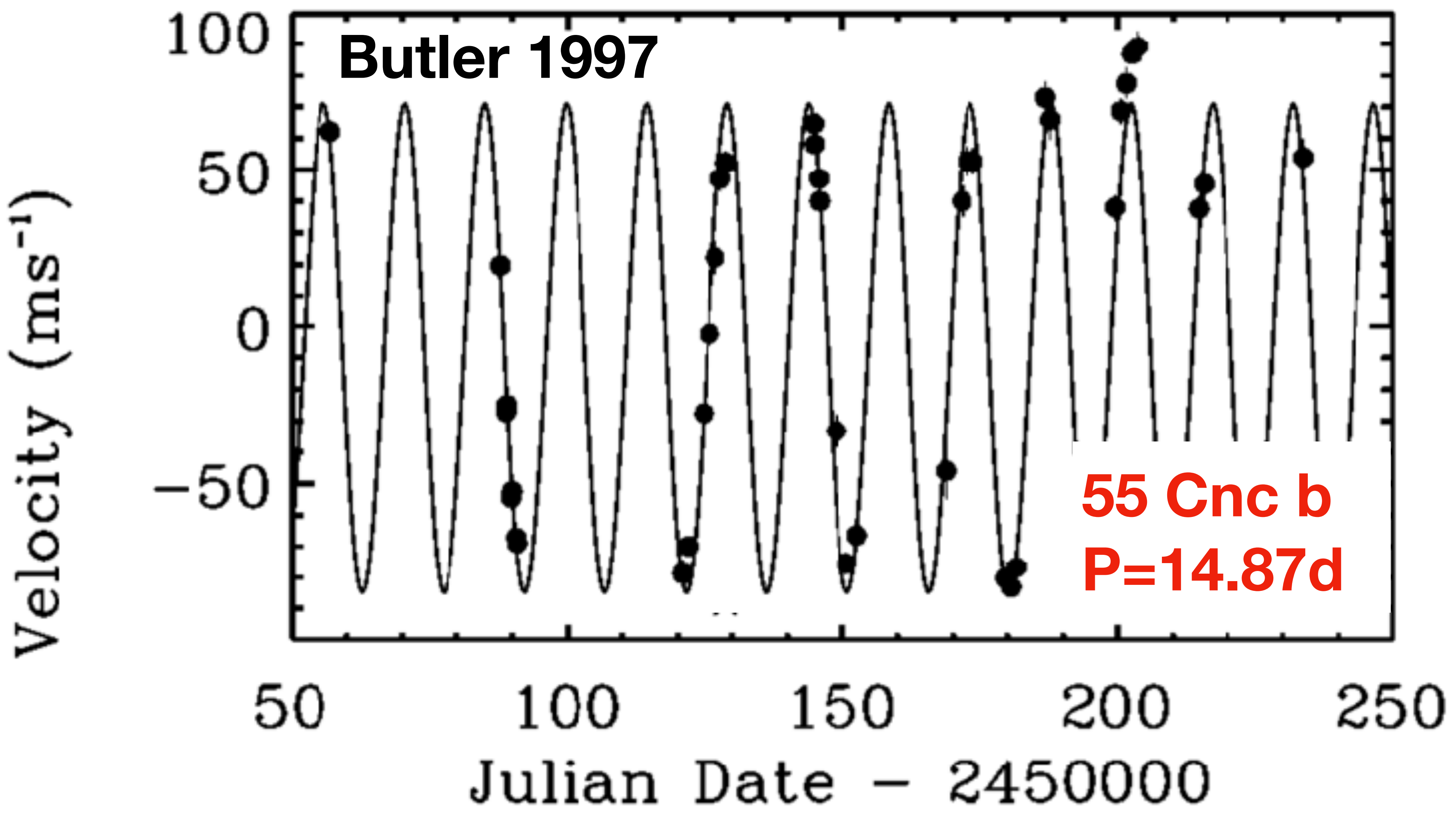
$$f_{data} \geq 2f_{Nyq}$$

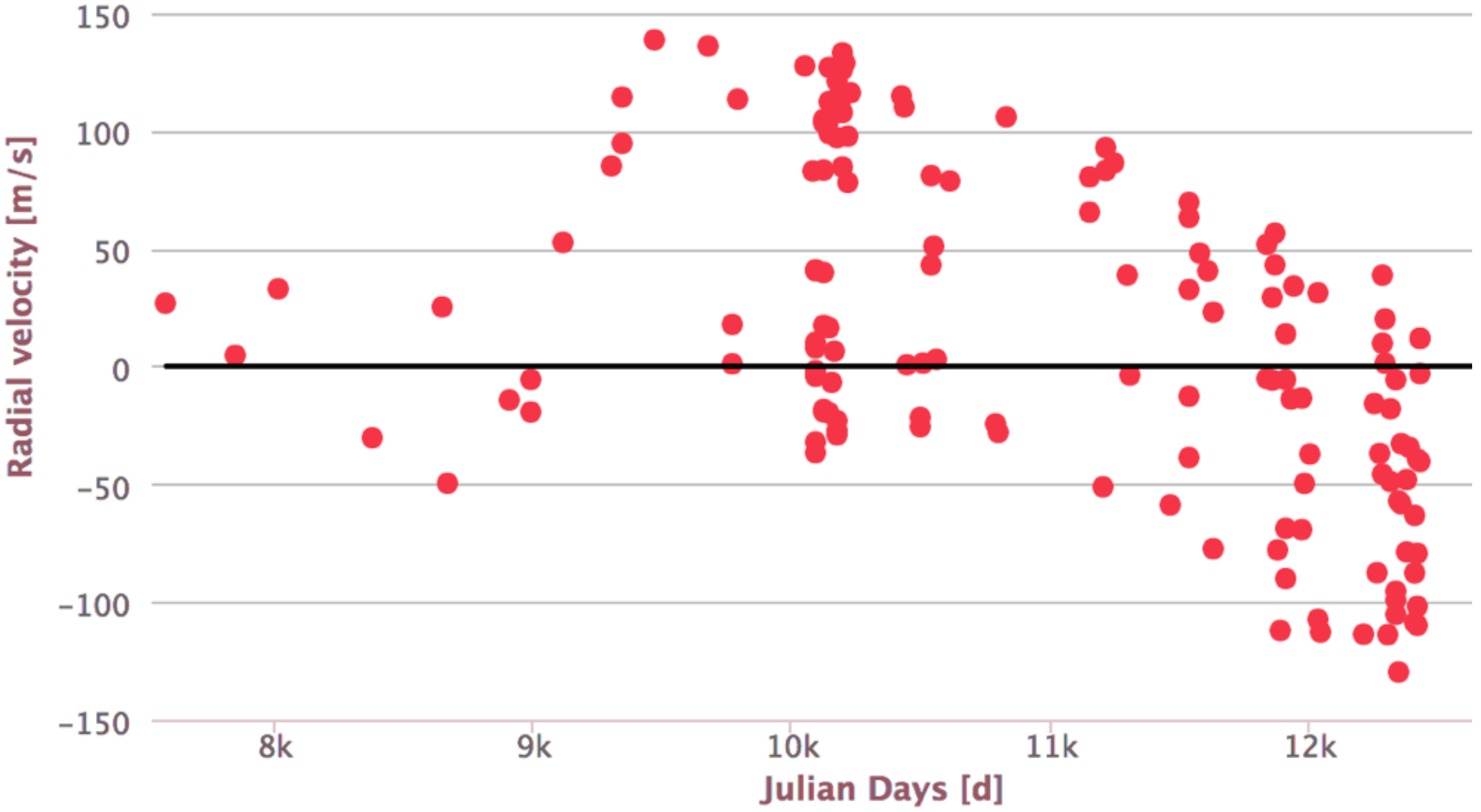
So if you observe once per night (f_{data} = 1d^{-1}) then the limiting Nqyuist frequency is then: f_{Nyq} = 0.5 d^{-1}

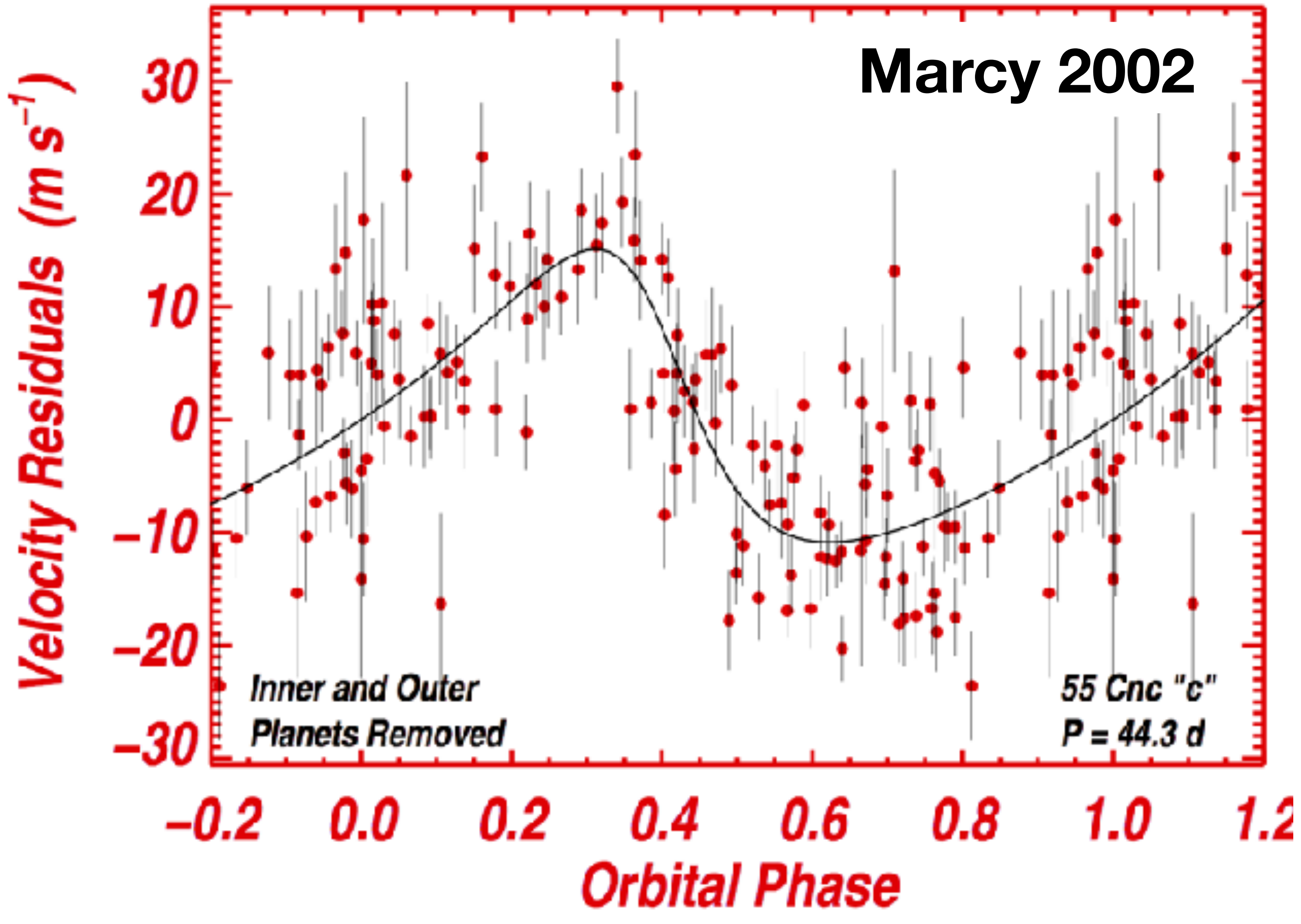
This means you can't detect any signals with frequencies longer than 2*f_{data} (i.e. any planets with P < 2 days)

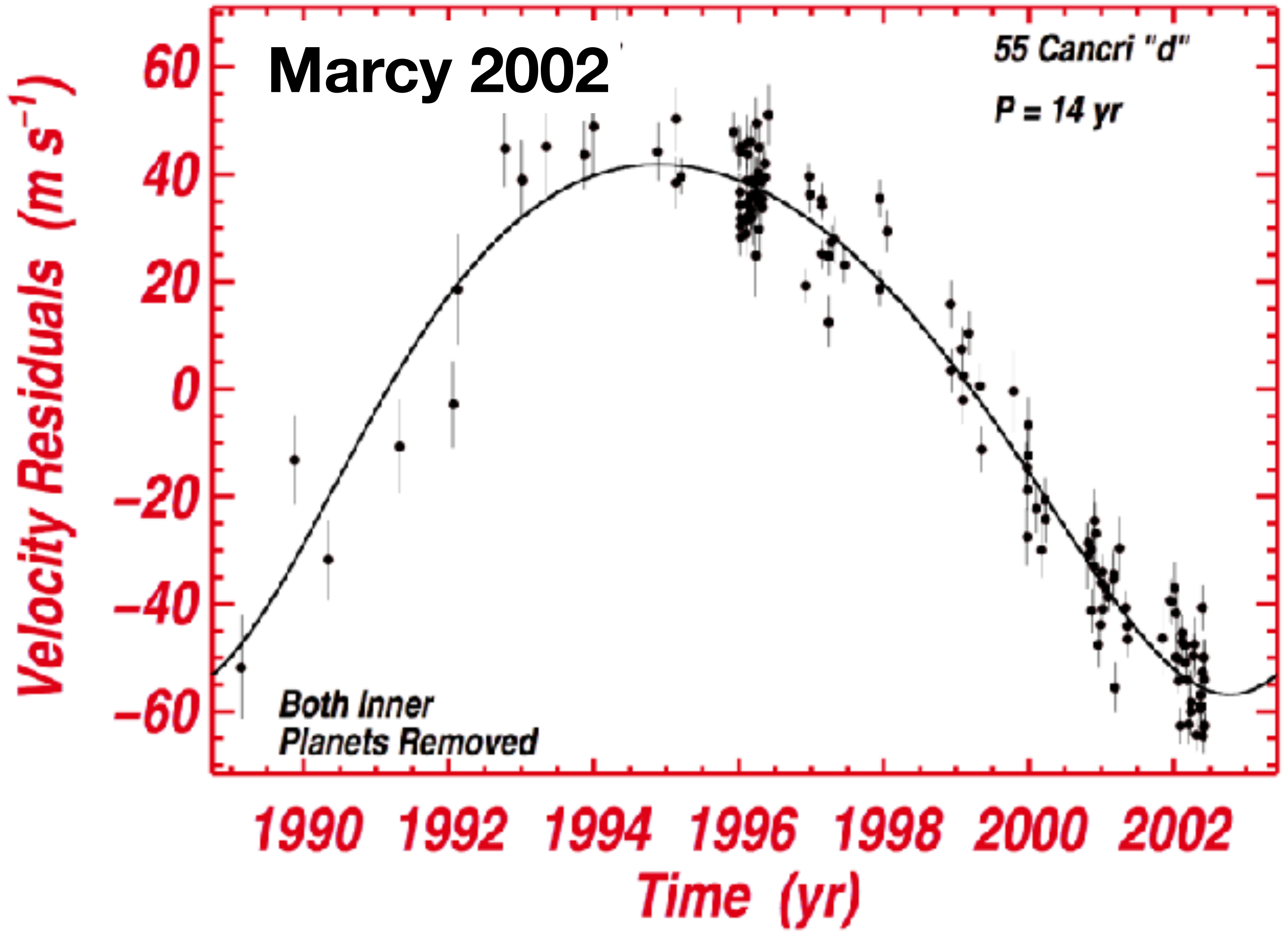


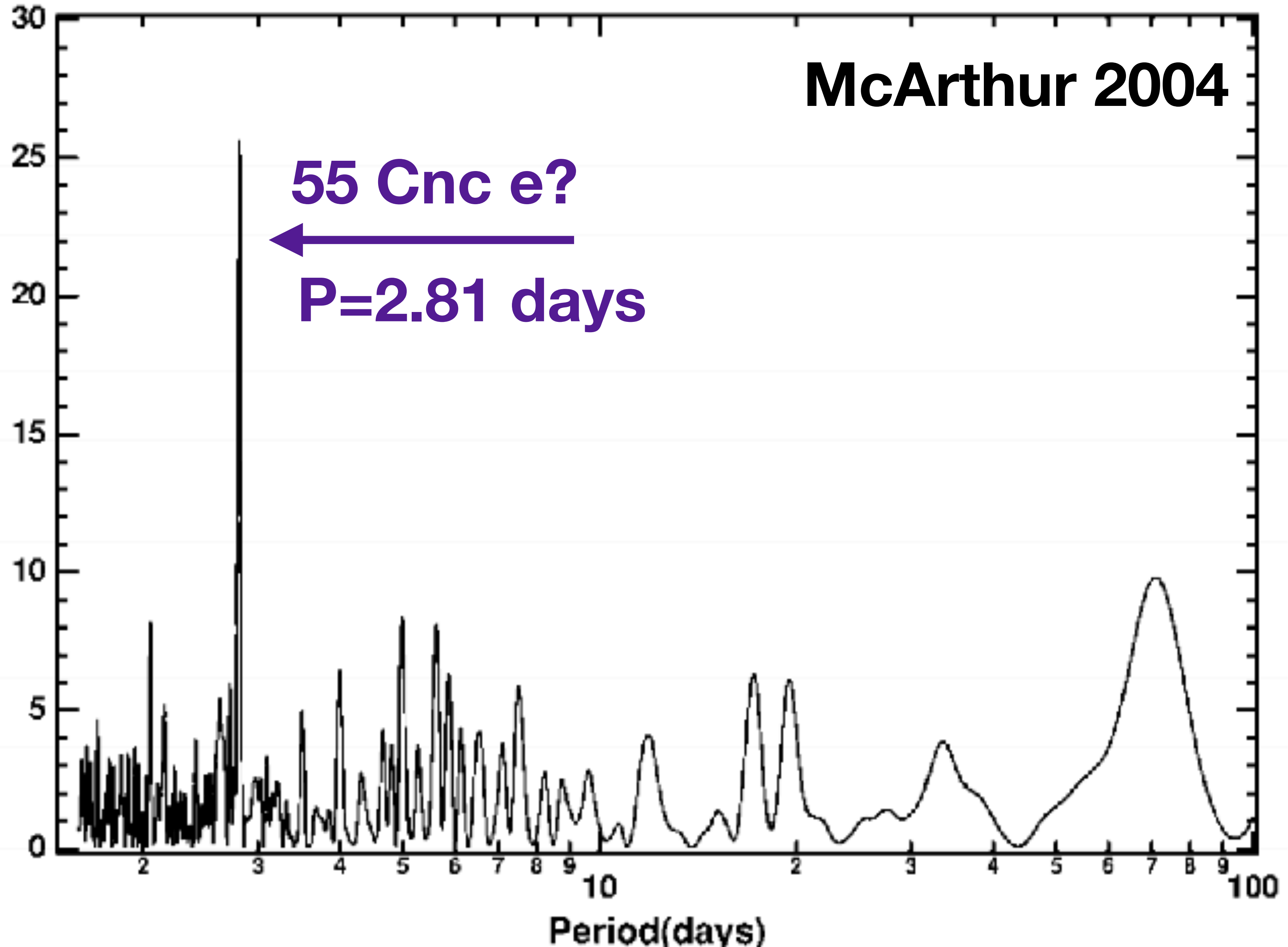
Aliasing: 55 Cancri



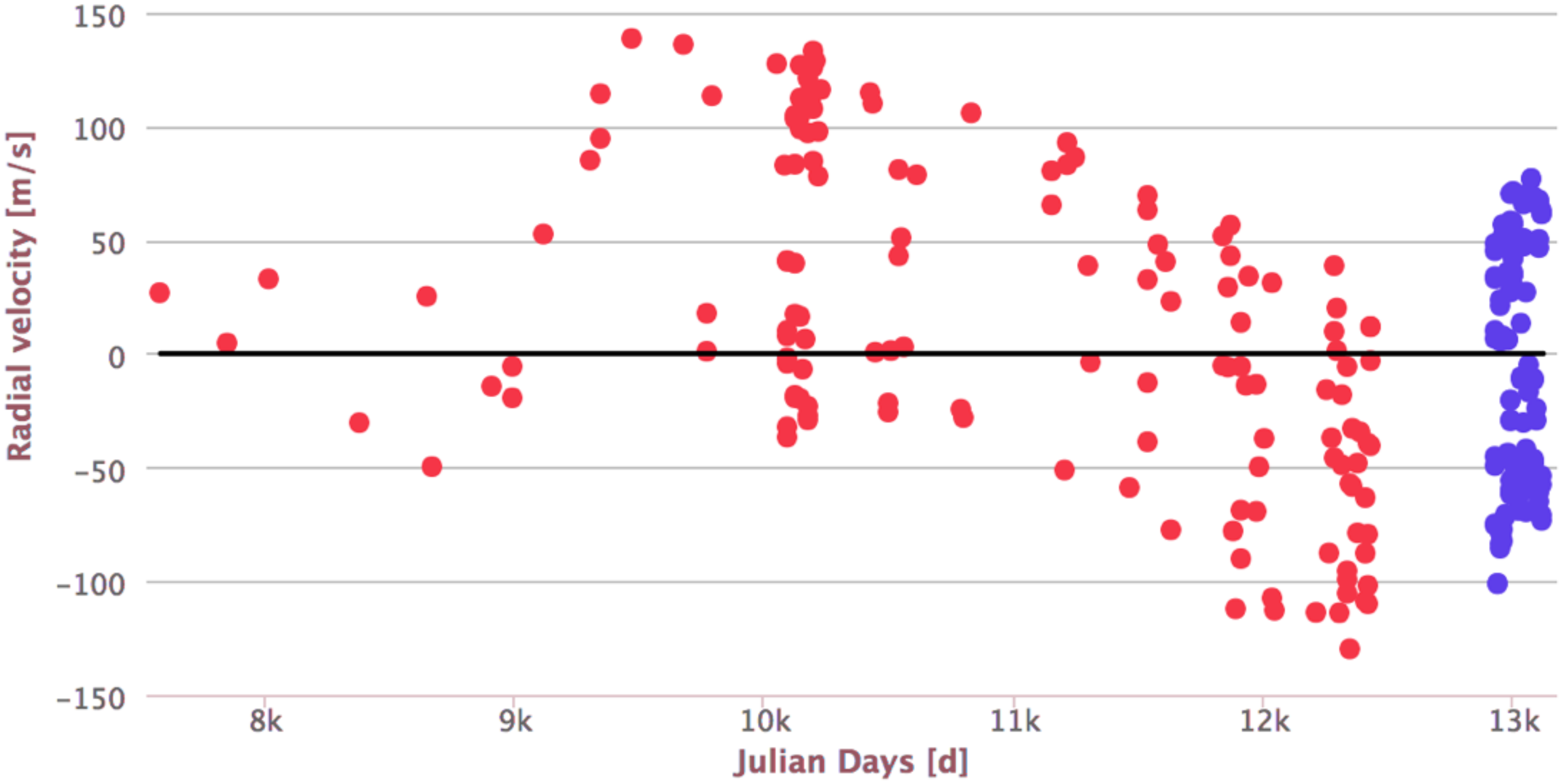


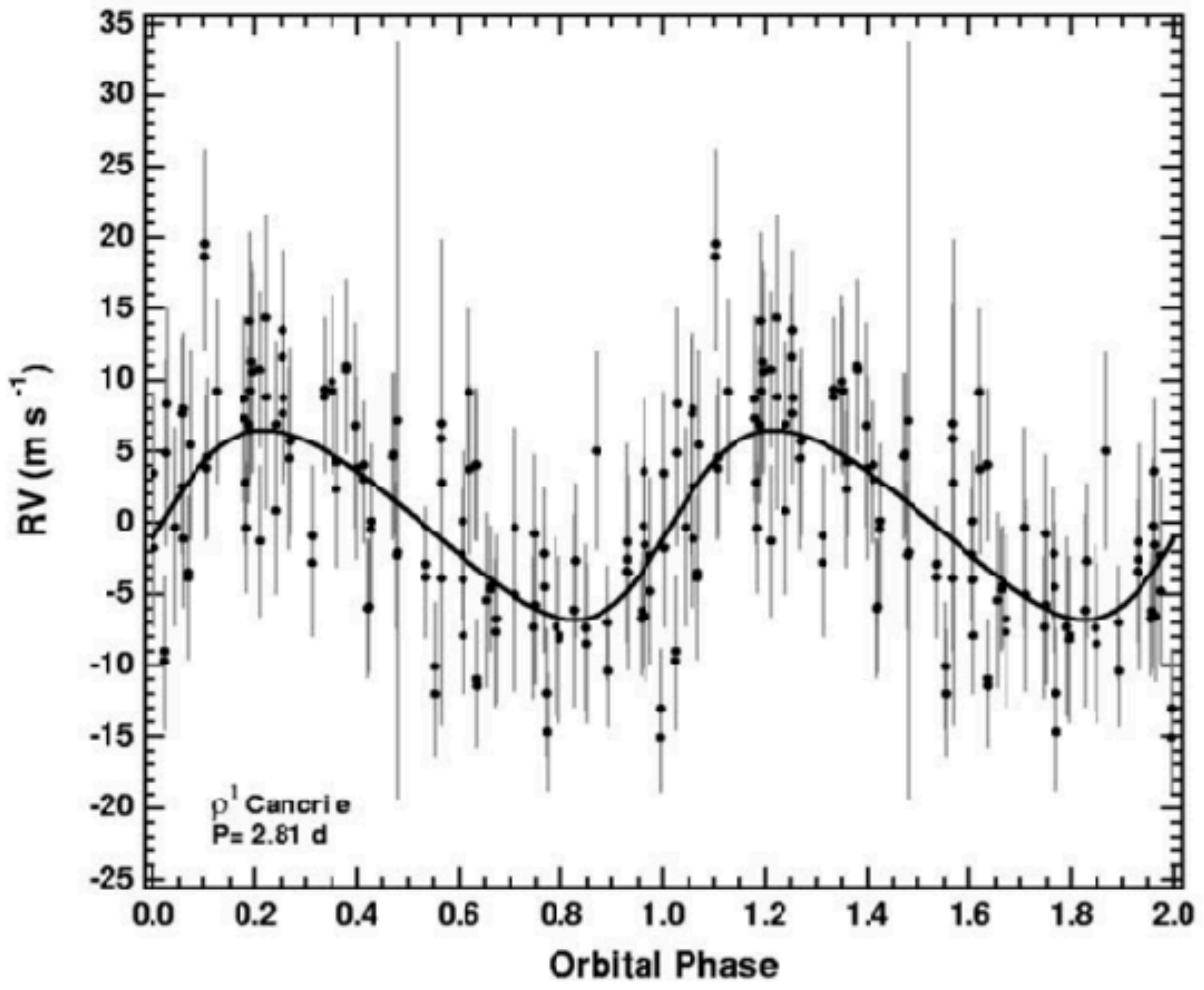






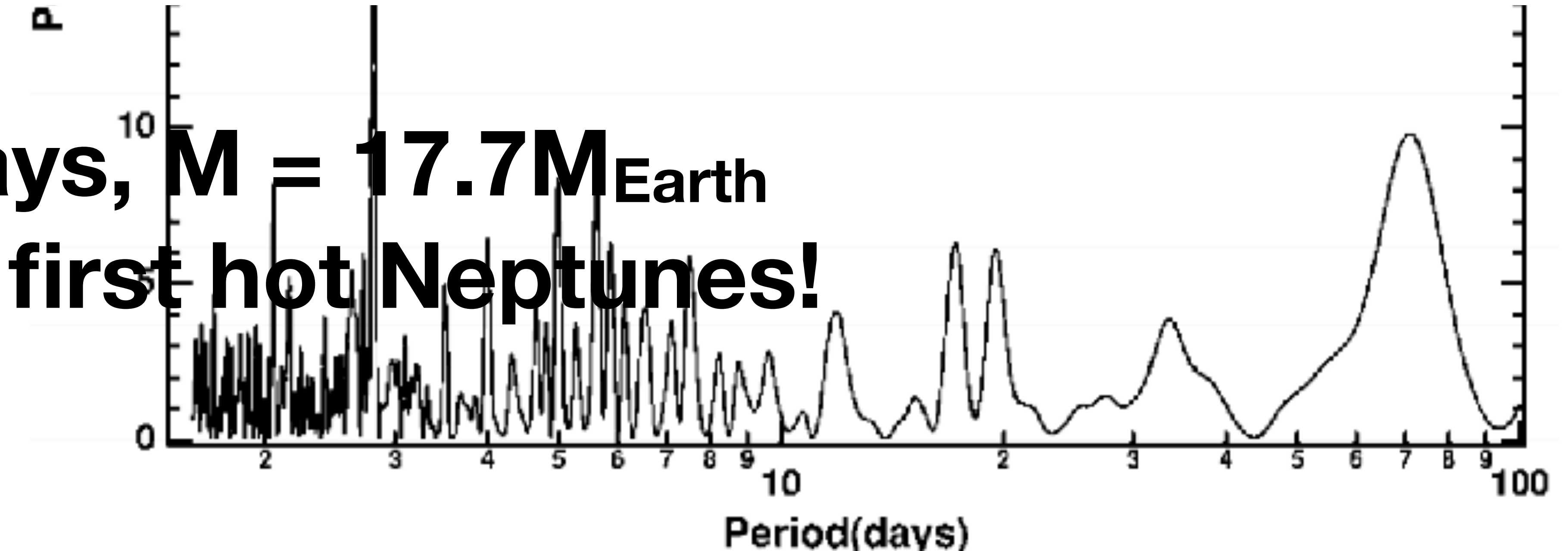
Aliasing: 55 Cancri





Quad-Keplerian Orbital Elements of ρ^1 Cancri

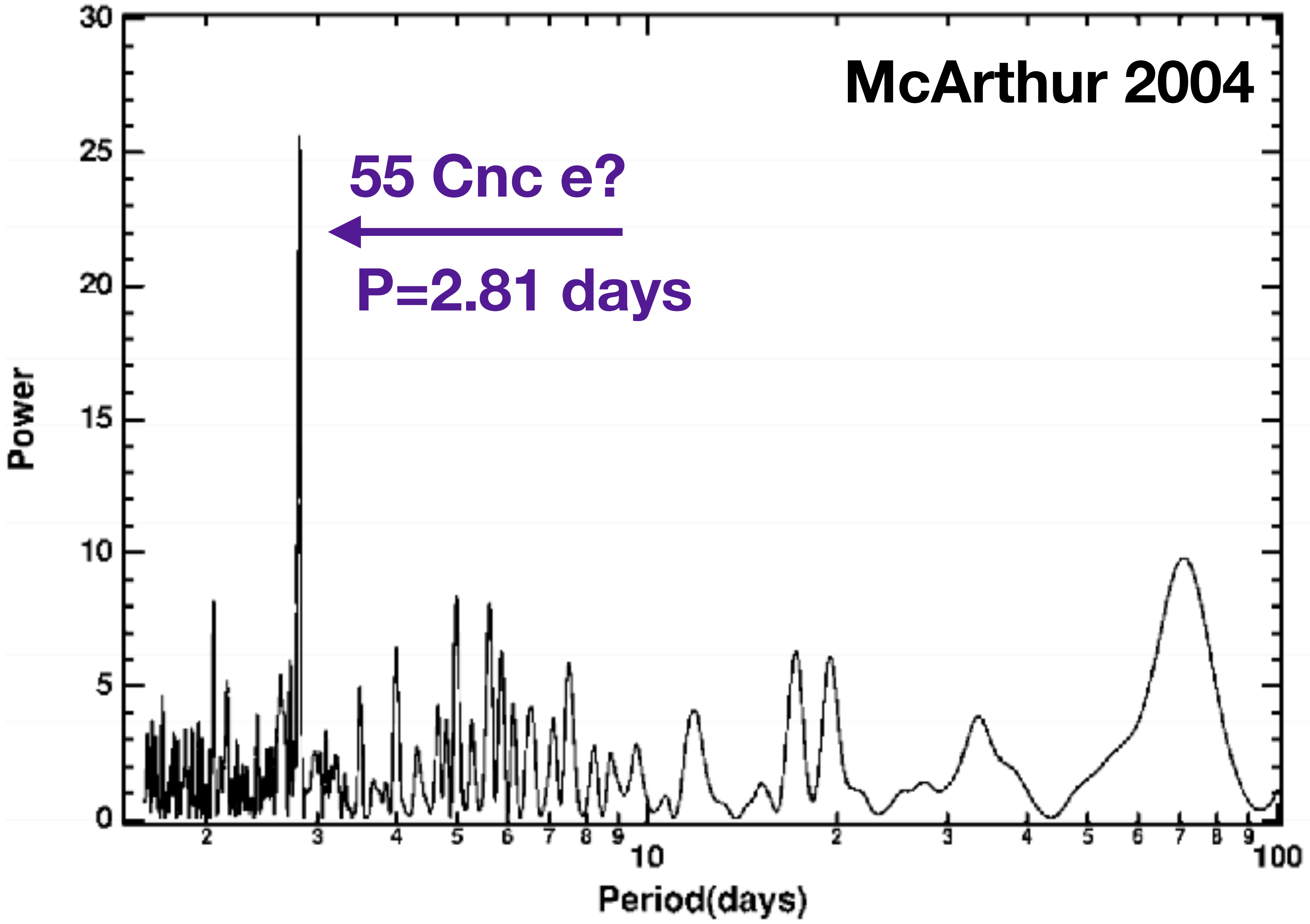
Element	ρ^1 Cancri e	ρ^1 Cancri b	ρ^1 Cancri c	ρ^1 Cancri d
Orbital period P (days)	2.808 ± 0.002	14.67 ± 0.01	43.93 ± 0.25	4517.4 ± 77.8
Epoch of periastron T [a]	3295.31 ± 0.32	3021.08 ± 0.01	3028.63 ± 0.25	2837.69 ± 68.87
Eccentricity e	0.174 ± 0.127	0.0197 ± 0.012	0.44 ± 0.08	0.327 ± 0.28
ω (deg)	261.65 ± 41.14	131.49 ± 33.27	244.39 ± 10.65	234.73 ± 6.74
Velocity amplitude K (m s^{-1})	6.665 ± 0.81	67.365 ± 0.82	12.946 ± 0.86	49.786 ± 1.53

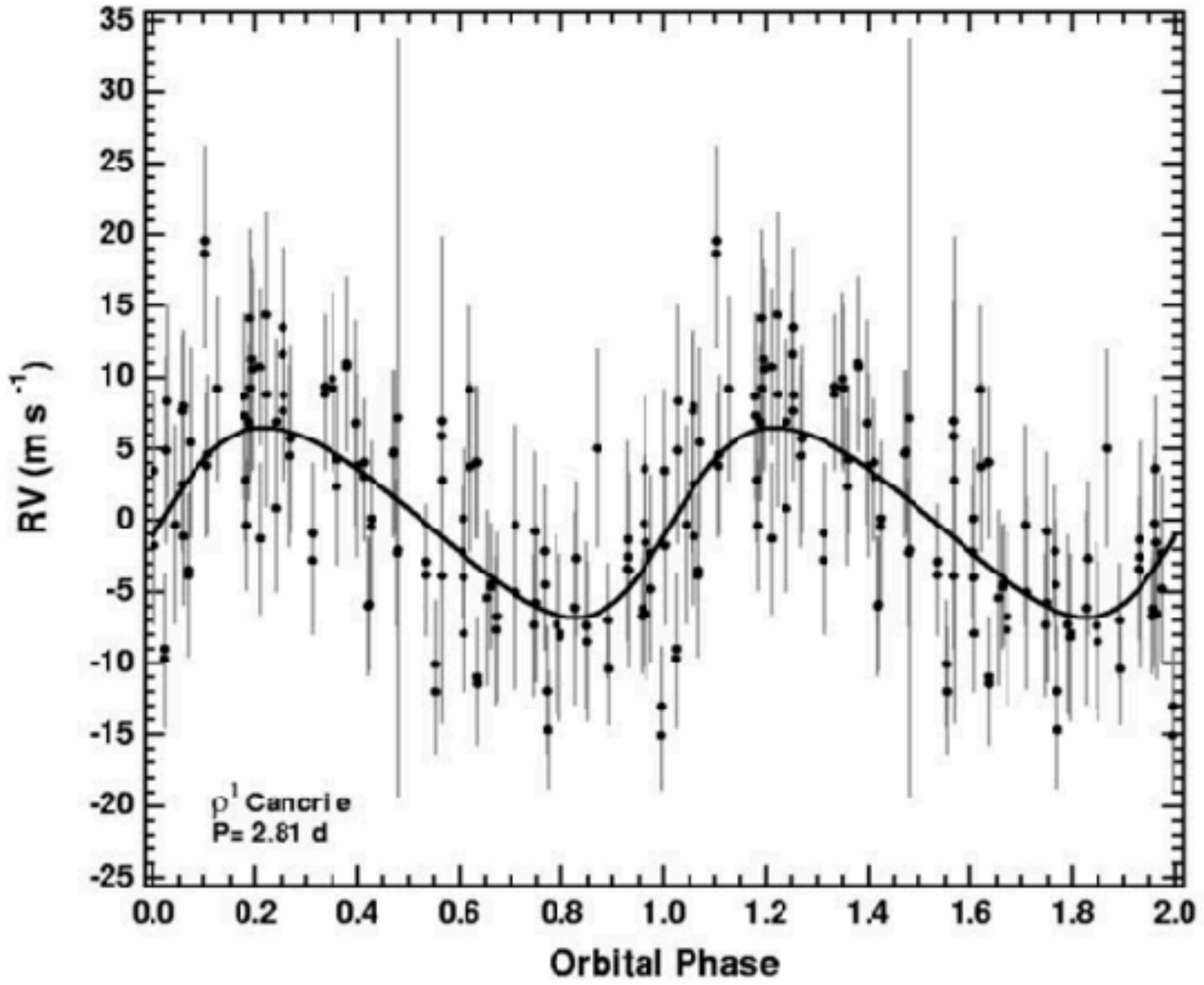


P = 2.8 days, M = 17.7M$_{Earth}$

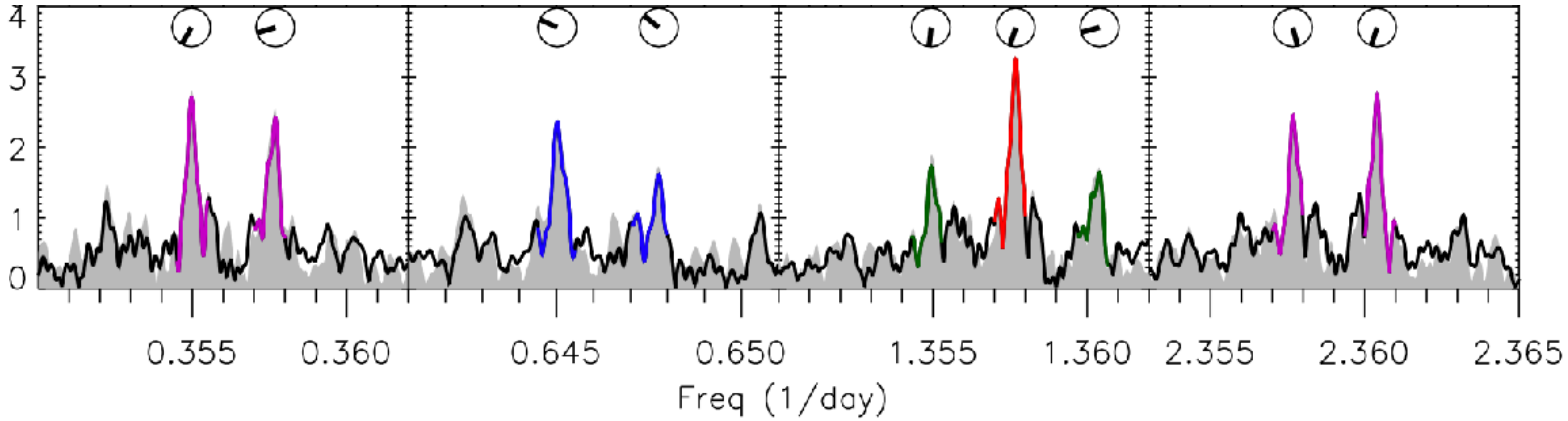
One of the first hot Neptunes!

Aliasing: 55 Cancri



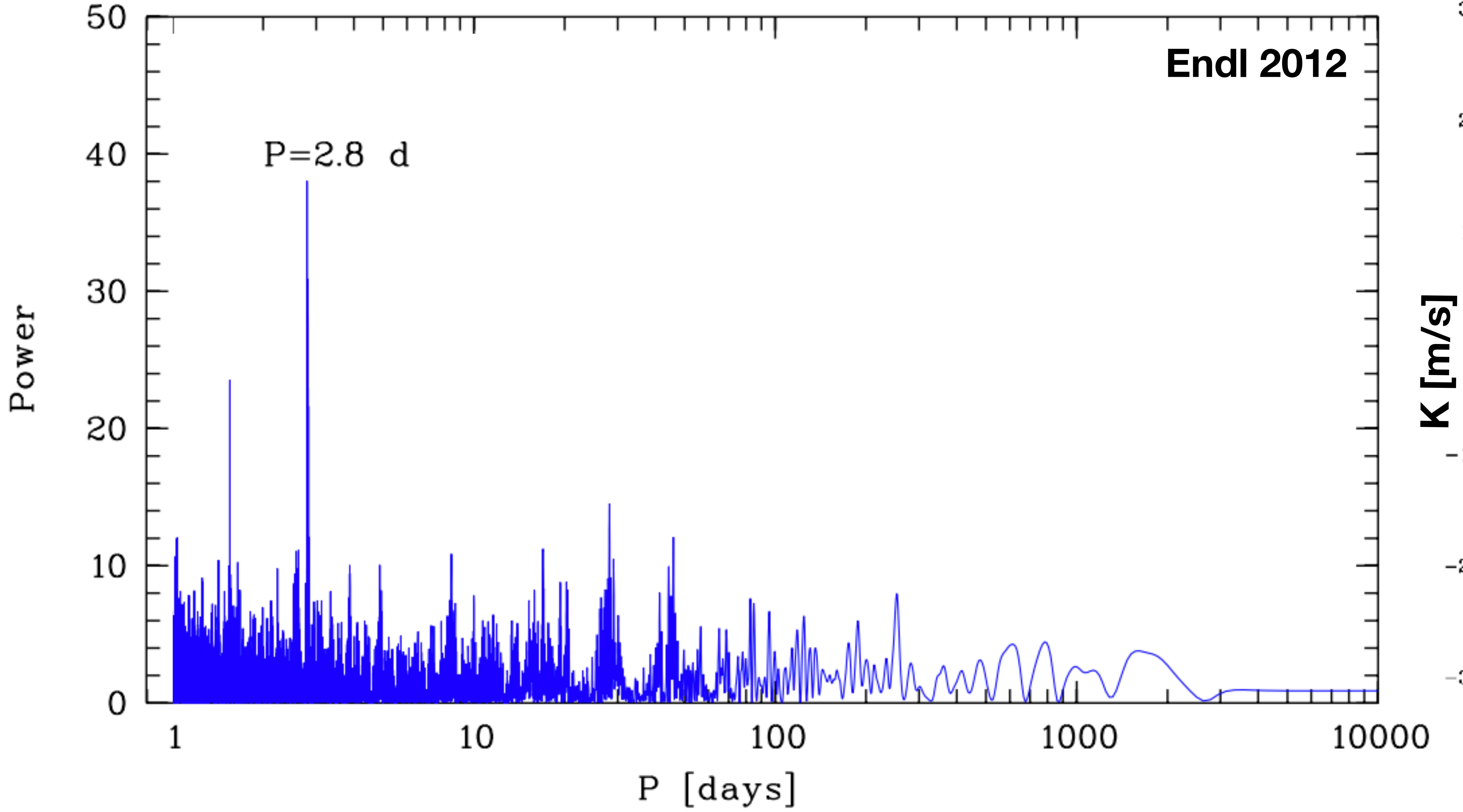


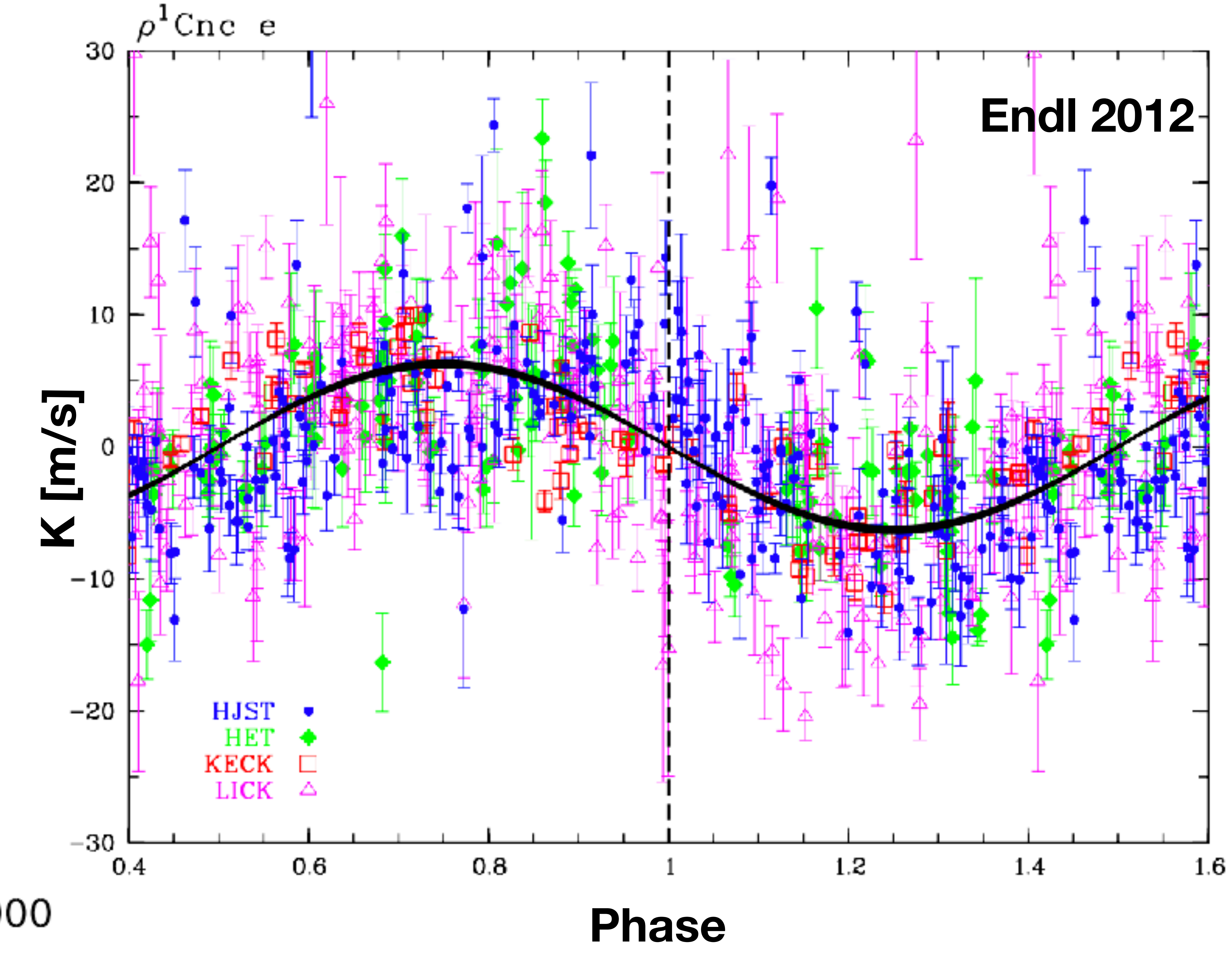
But then, Dawson & Fabrycky 2010 applied a new approach to aliasing analyses to the 55 Cancri data…



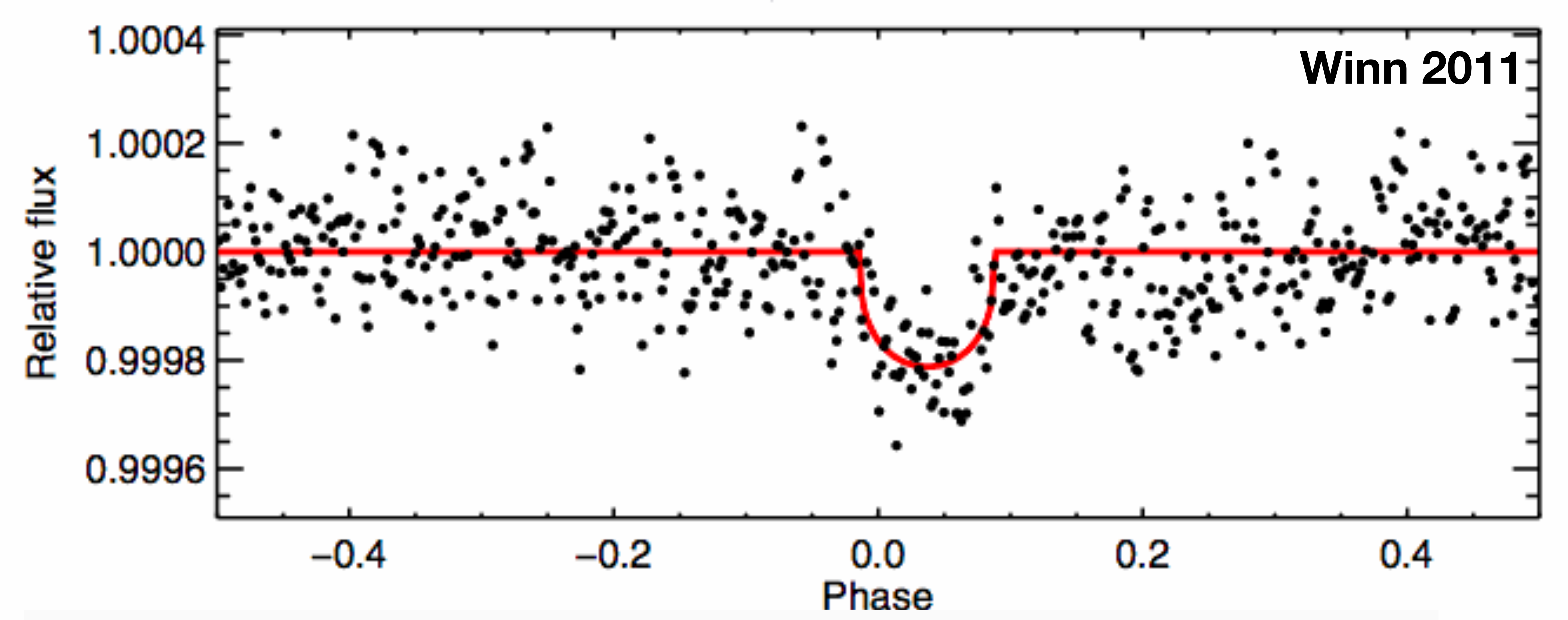
A planet with P < 1 day was unheard of in 2004, so the original paper didn't search that part of parameter space and instead found the 2.8 day alias. Dawson & Fabrycky re-examined the data, and looked below P = 1 day to find the true signal at 0.74 days

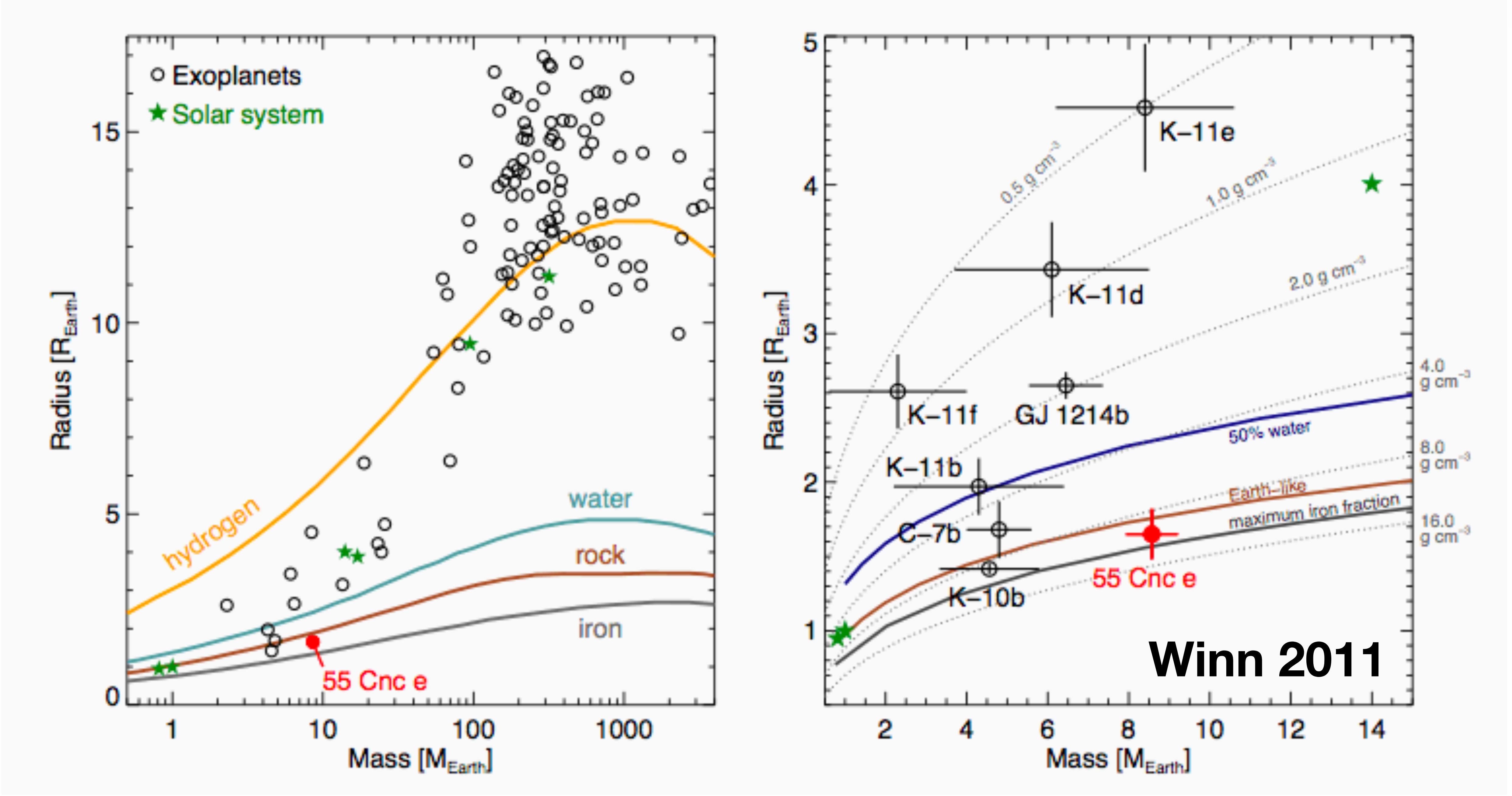
Aliasing: 55 Cancri

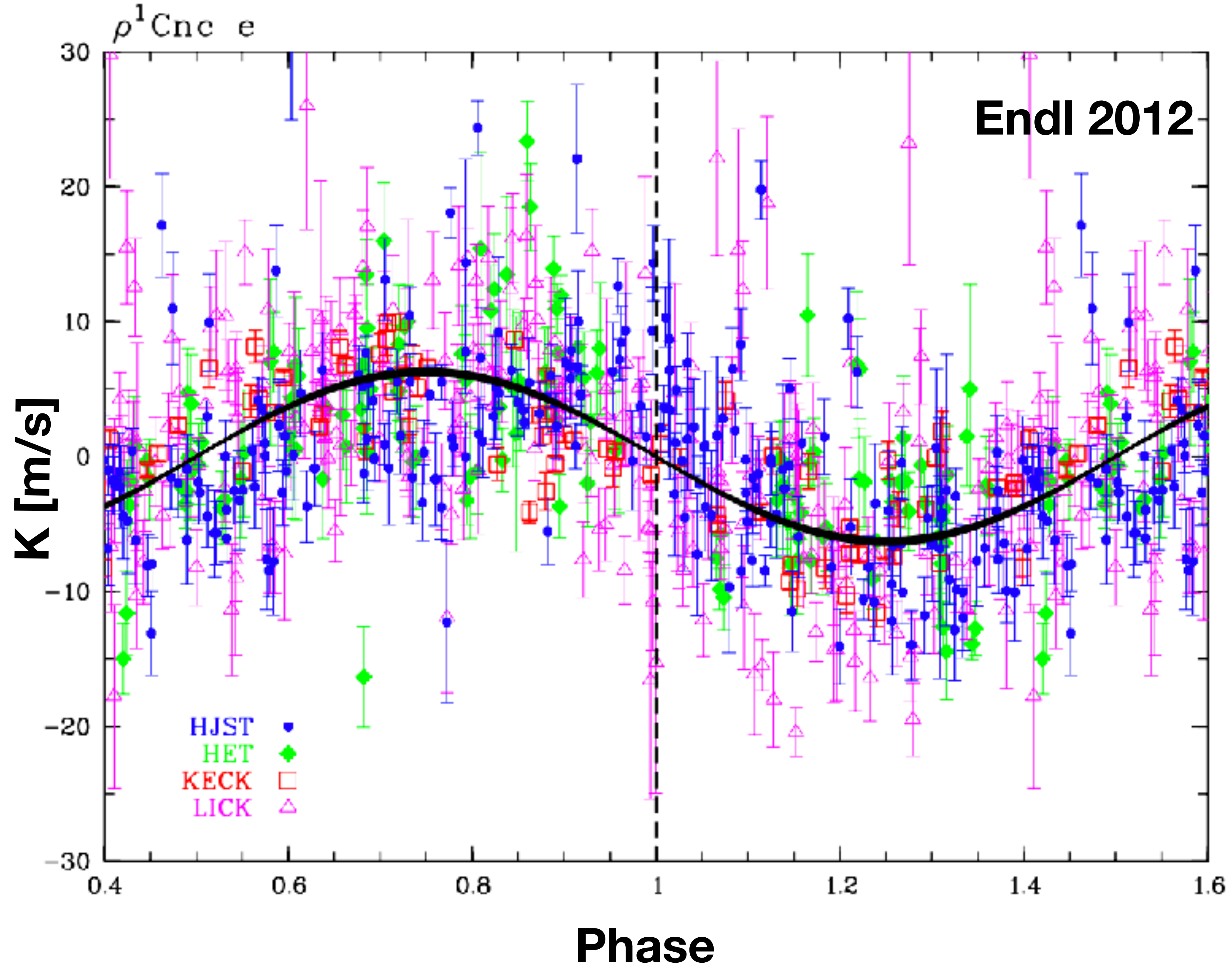




Aliasing: 55 Cancri







Take away: make sure you understand what periods your data are/are not sensitive to, and make sure to check for aliases when fitting planets!

Orbit determination : outline

1) Get RVs from your favorite observer (or better yet, go observing!)

2) Feed the RV data into some sort of RV analysis package [ExoFast2, Radvel, Systemic, etc]

3) Look at the periodogram for the RV data, identify peaks that are:
> Above your threshold for being a significant signal
> Well separated from other peaks

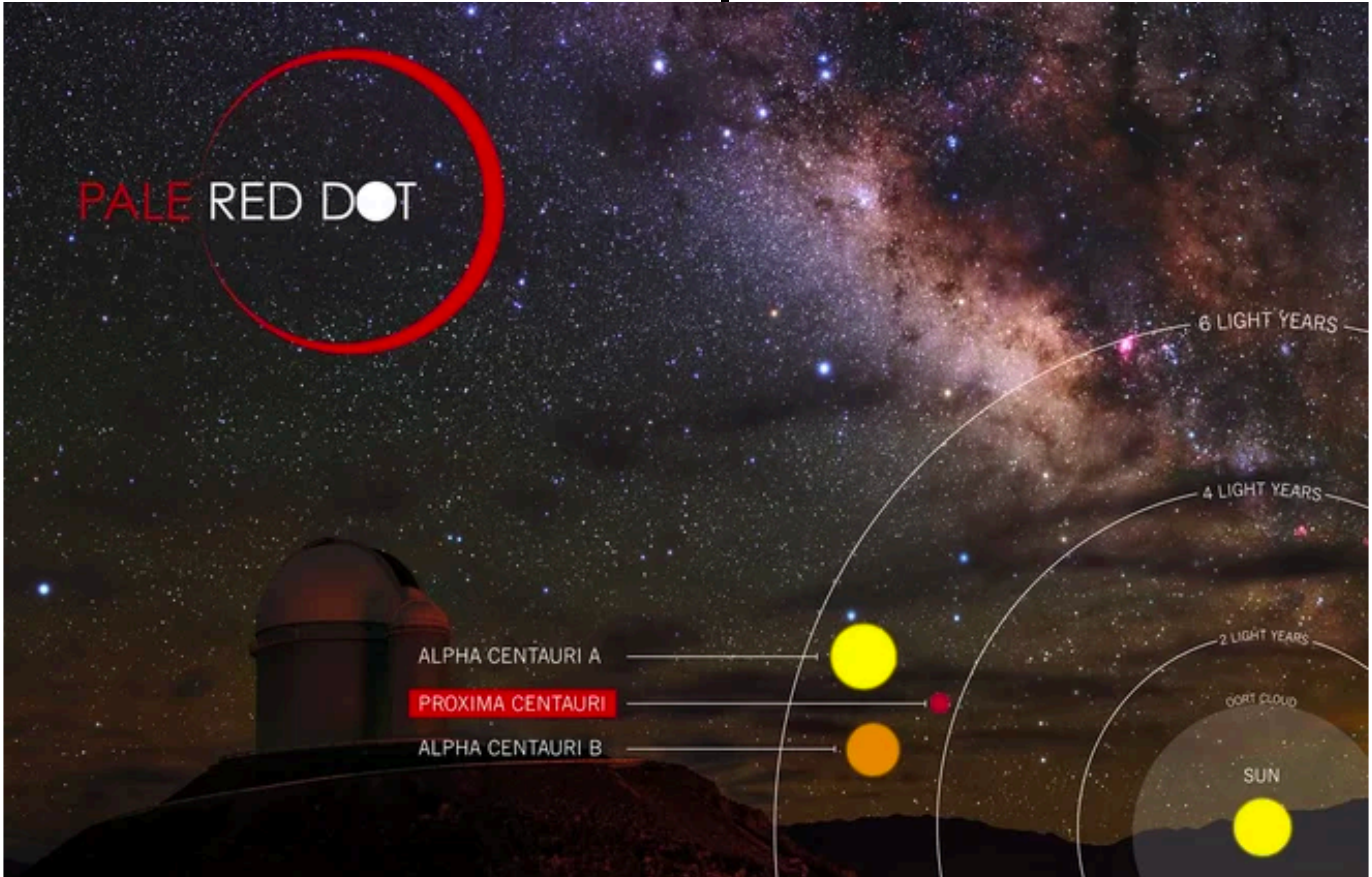
4) Fold the data to that period, and use the software to determine the orbital parameters

5) Look at the residuals periodogram, if there are additional peaks repeat steps 3/4

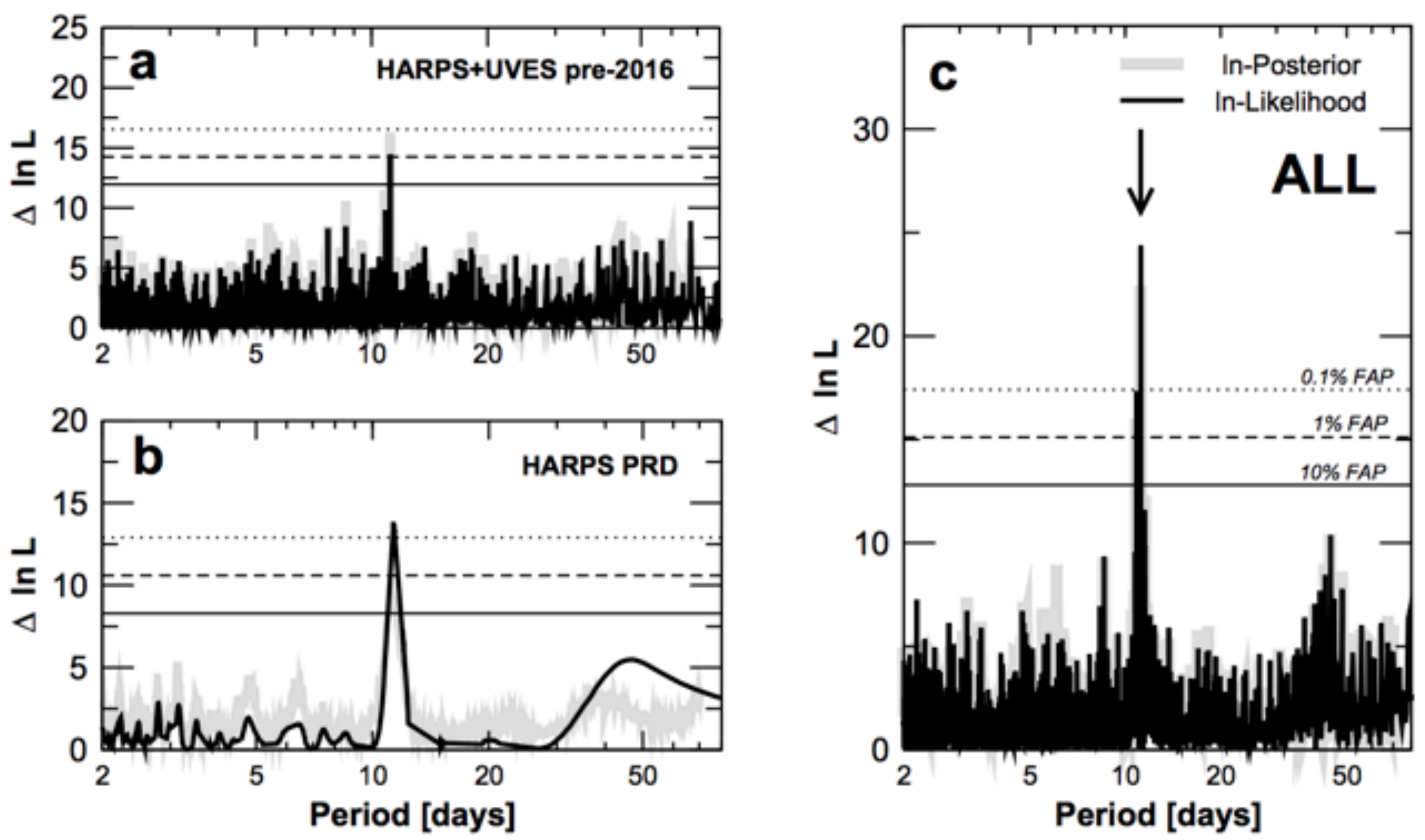
6) Check that none of your potential planet signals are on top of peaks in the window function, peaks from activity indicators, or aliases of one another

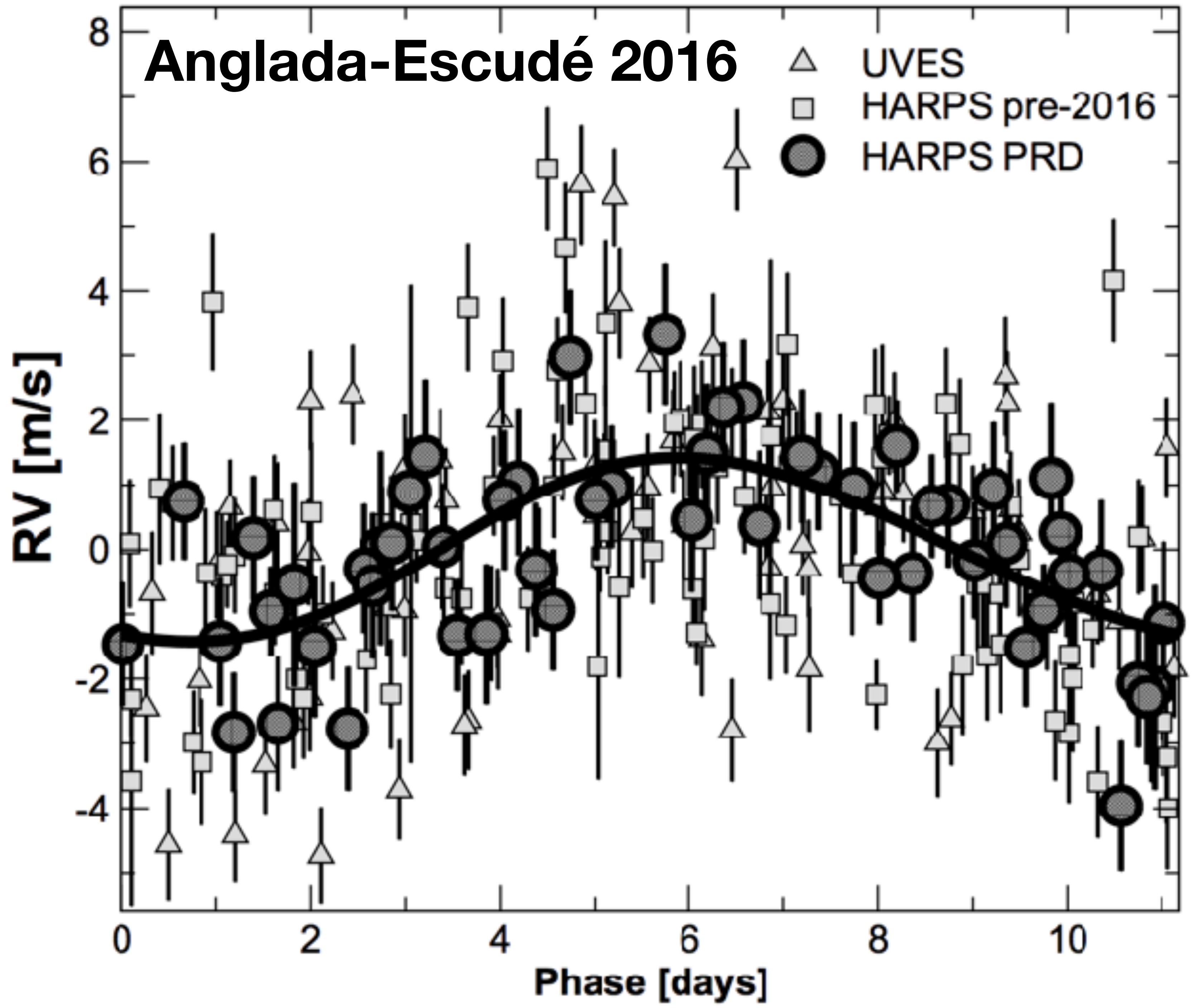
7) Spin up full MCMC analysis of your choice to get official error bars on your planet fit

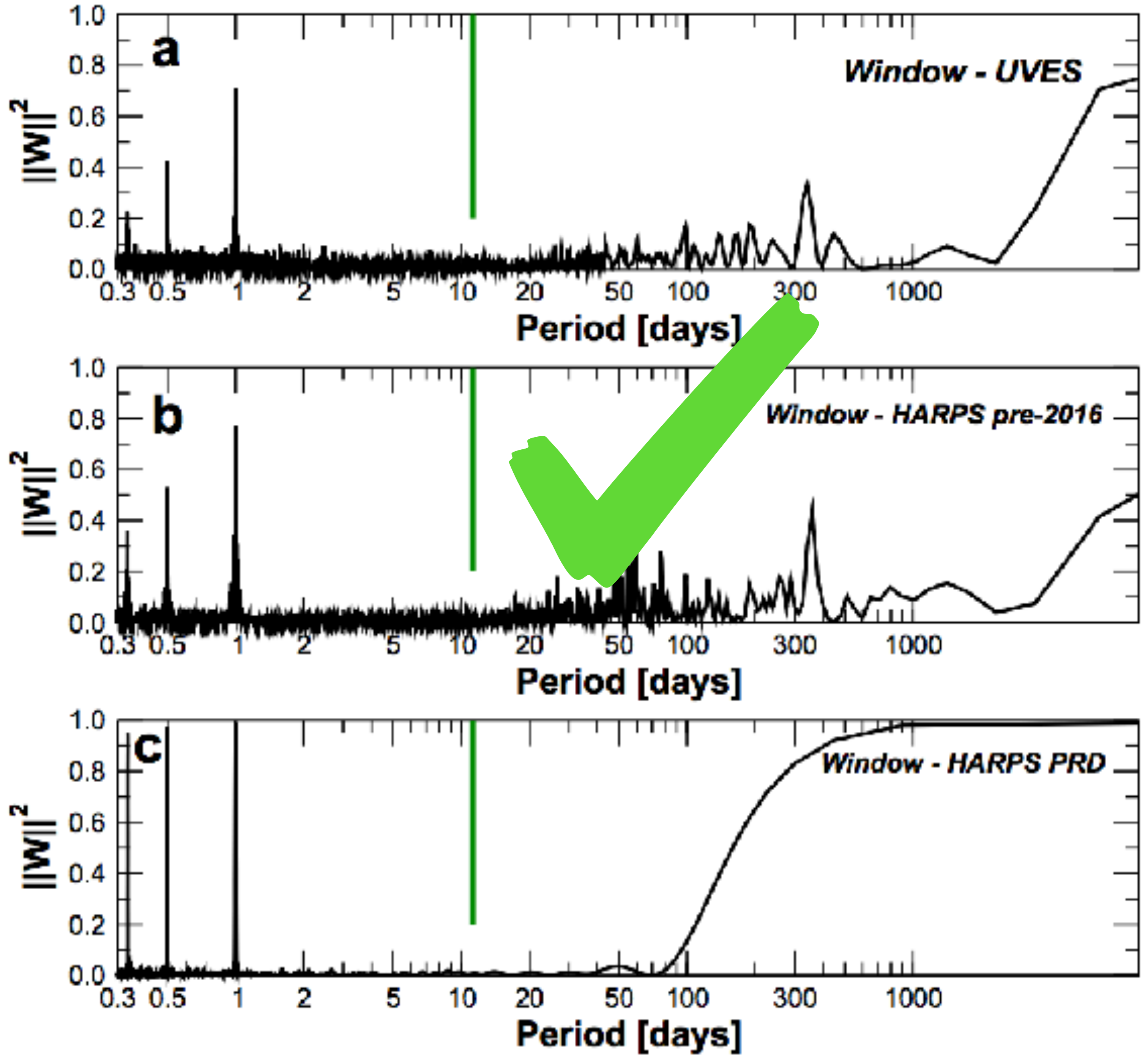
Orbit determination example: Proxima Cen b

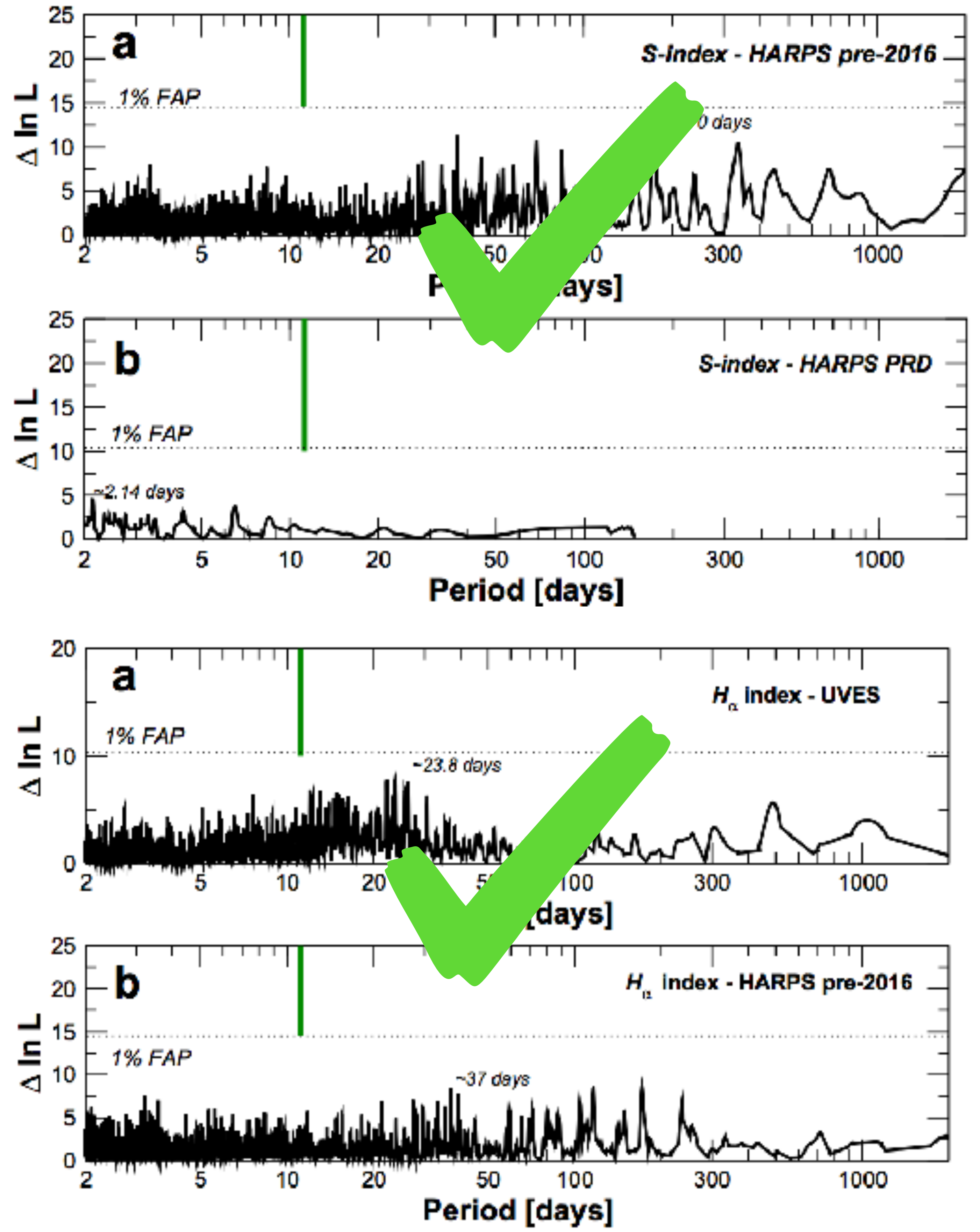


Orbit determination example: Proxima Cen b



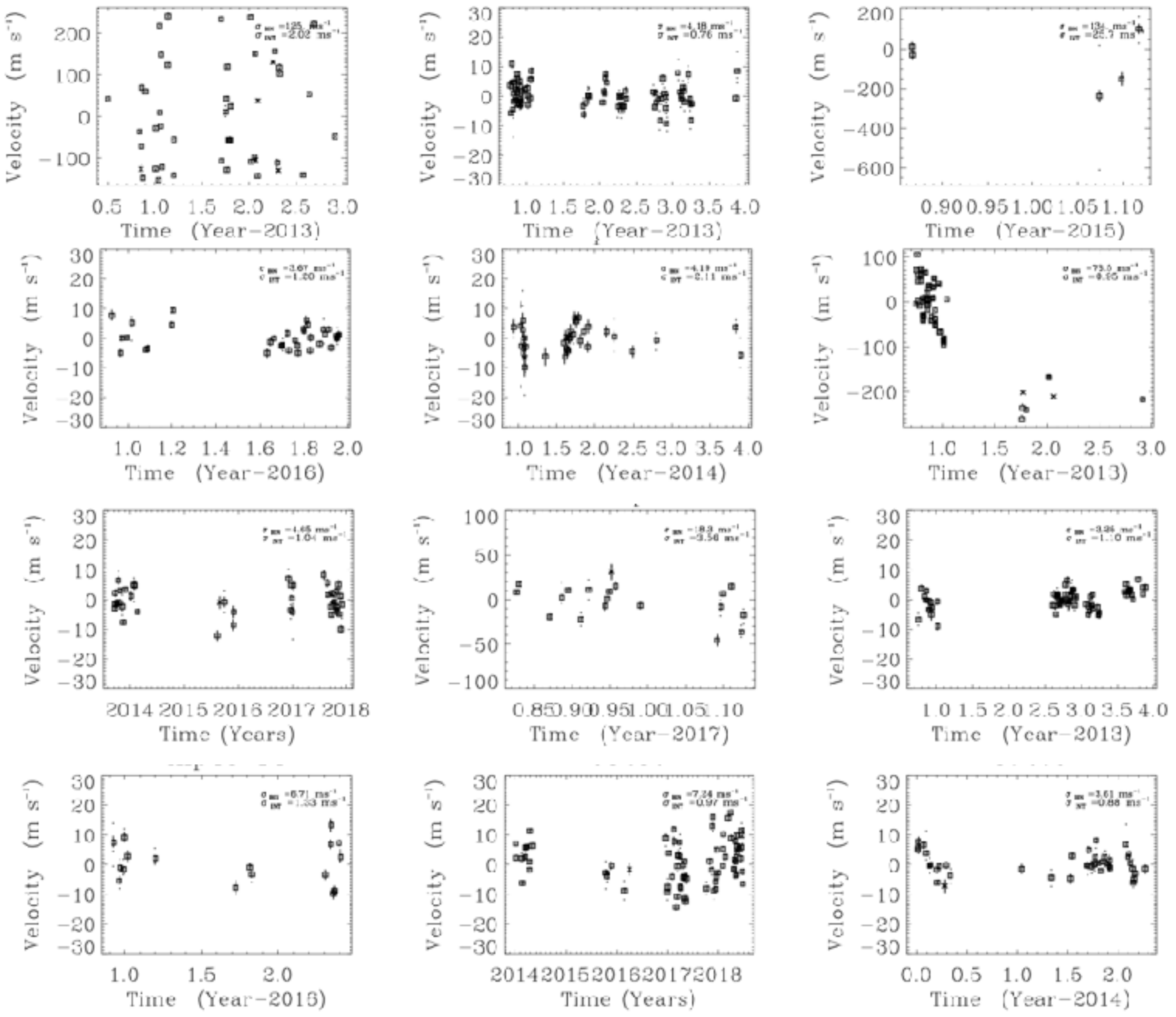


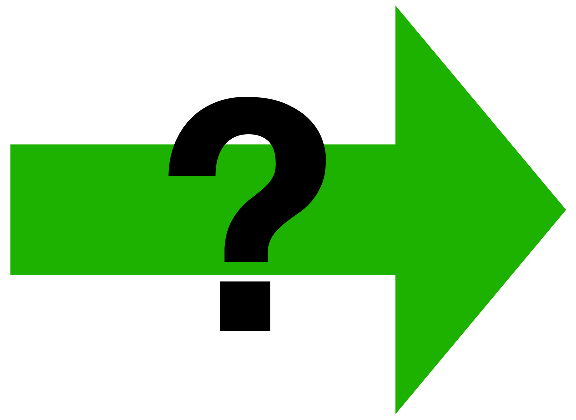


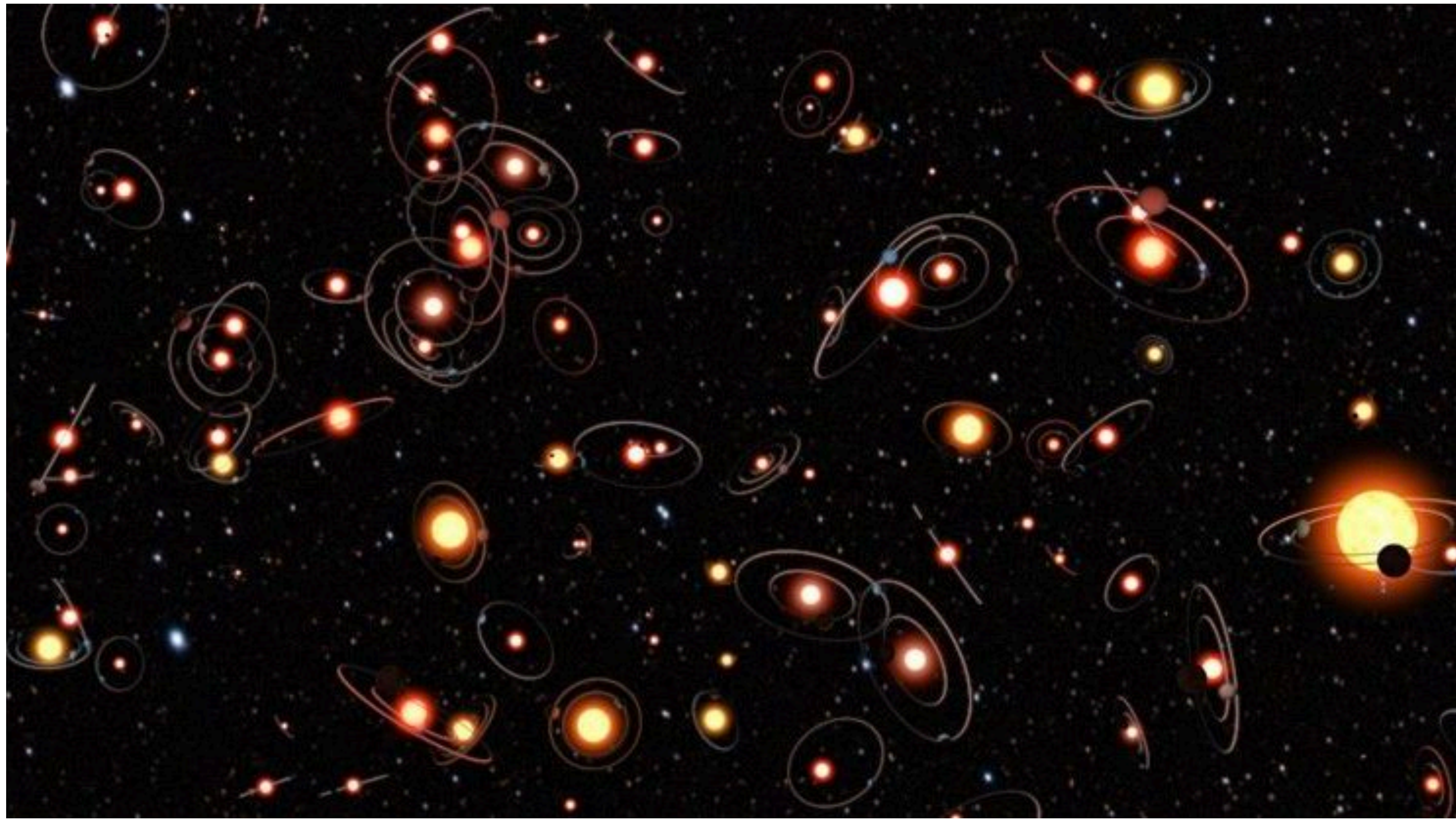


"Since the analysis of the activity data failed to identify any stellar activity feature likely to generate a spurious Doppler signal at 11.2 days, we conclude that the variability in the data is best explained by the presence of a planet (Proxima b, hereafter) orbiting the star"

Orbit Determination and Degeneracy of Models







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