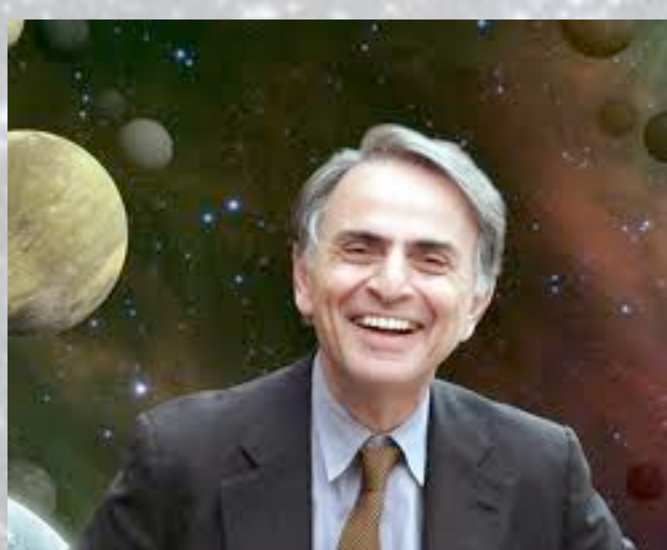
Planetary Dynamics: From Birth to Death

Kaitlin M. Kratter
ITC Harvard-Smithsonian Center for Astrophysics

Sagan Workshop, July 2012



Goals

1) Provide an overview of role of dynamics in formation and evolution of the systems you will observe

2) Provide basic tools to help analyze and validate new systems

3) Keep theory in mind in choosing project and writing papers*

*speaker may be biased...

Outline

- Quantifying the dynamical influence of a planet
- How dynamics shape planetary growth
- Stability of multi-planet systems (2, >3)
- Stability of binary (multi) planetary systems
- The dynamical fate of planets after stellar evolution

Overviews and primary references:

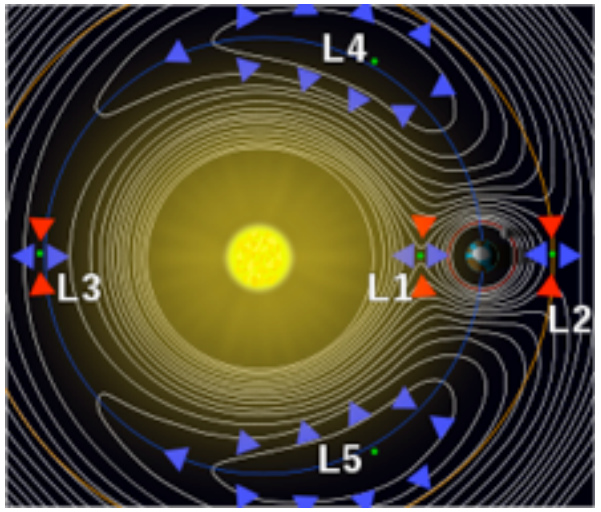
Peale 1976
Gladman 1993
Chambers, Wetherill, & Boss 1996
Holman & Wiegert 1999
Goldreich, Lithwick & Sari 2004
Smith & Lissauer 2009
Armitage 2010
Fabrycky et al 2011
Youdin & Kenyon 2012

Who's in Charge: The Hill Radius

- Def: Where the Planet's Gravity Dominates over Tidal gravity due to the star

$$\frac{GM_p}{\Delta R^2} \approx \frac{GM_*}{a^3}\Delta R$$

$$\Delta R \to R_H \approx \left(\frac{M_p}{M_*}\right)^{1/3} a$$



Hill Radius

$$R_H = \left(\frac{m_p}{3M_*}\right)^{1/3} a$$

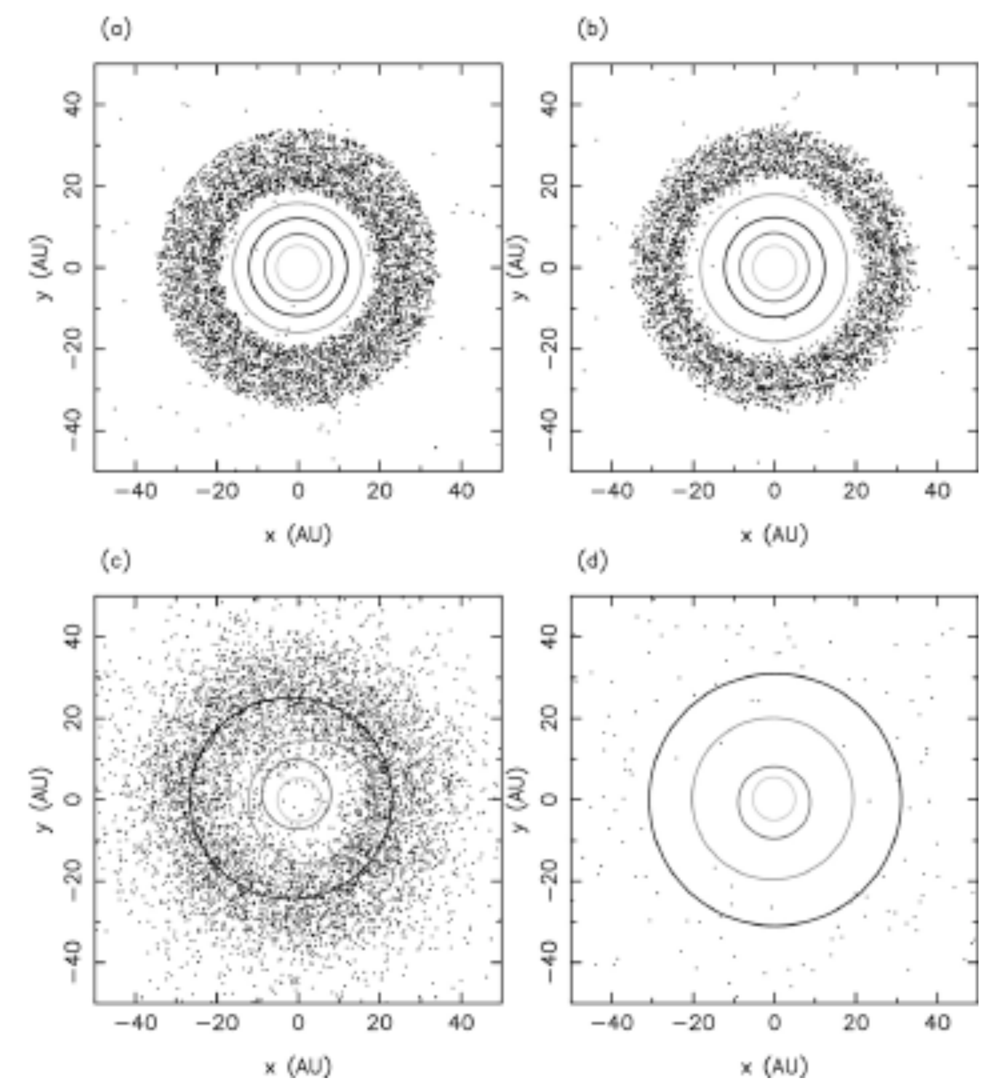
Dimensionless planet(esimal) size

$$\psi \equiv \frac{r_p}{R_H} = \left(\frac{3\rho_*}{\rho_p}\right)^{1/3} \frac{R_*}{a}$$

Roche Radius/Limit: size of a body that will be tidally disrupted

Roche Lobe: defined by equipotential surfaces, more appropriate for $\mu \sim 1$

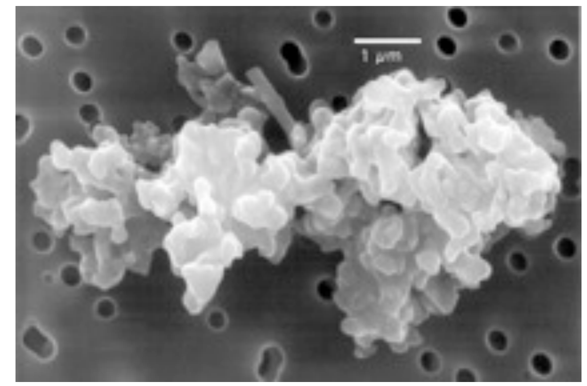
Part I: Birth

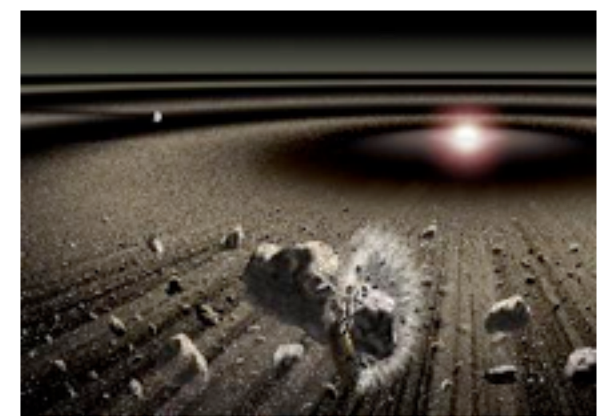


Proplyd (credit: Hubble) → Solar System (e.g. Nice Model)

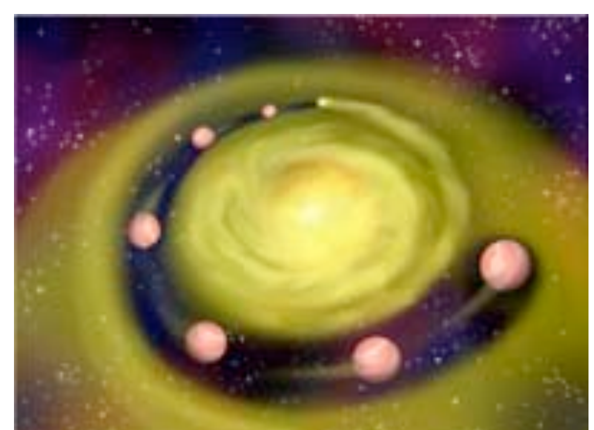
Core Accretion Theory of Planet Formation

- **Step 1**: Planetesimal formation via coagulation, collisions, gravitational instability of solids: from µm - km (100 km?)

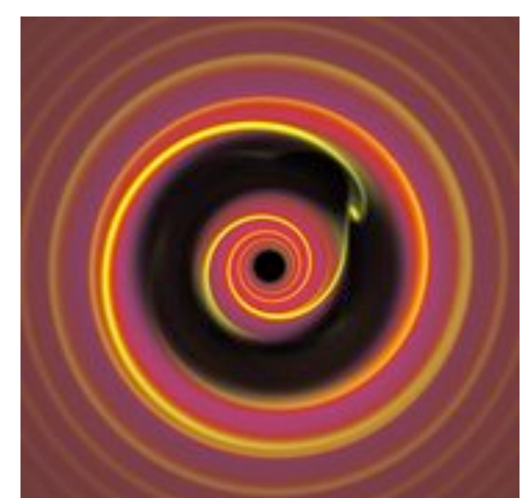




- **Step 2**: Terrestrial planet growth via gravity assisted collisional accretion: from ~km to Earth mass cores



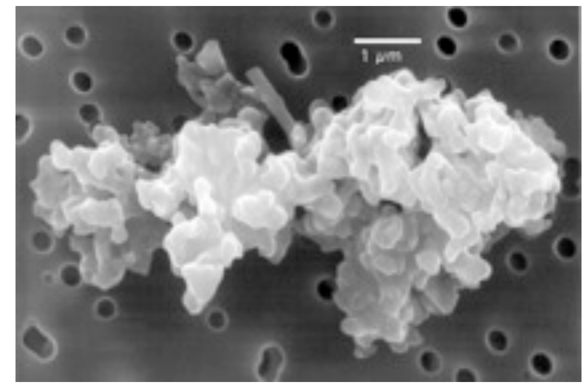
- **Step 3**: Core Accretion: Solid cores gather gas until disk disappears



- **Step 4**: Migration and Scattering (in/out) to new location within stellar system

Dynamics-Driven

~~Core Accretion Theory of~~ Planet Formation



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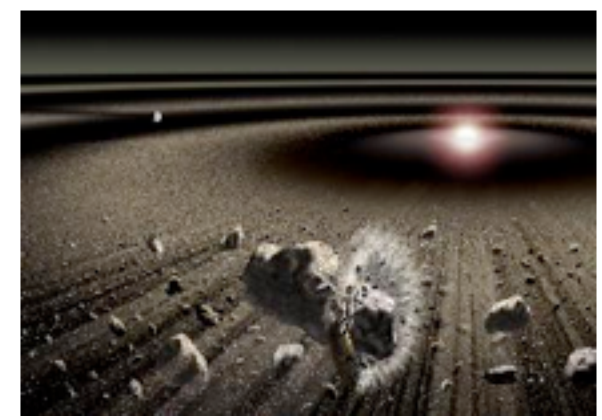

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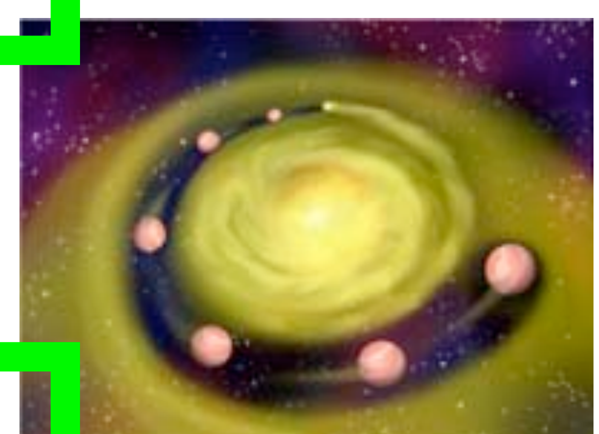

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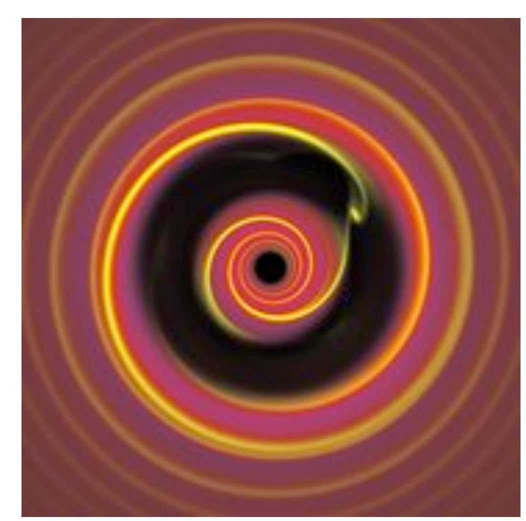

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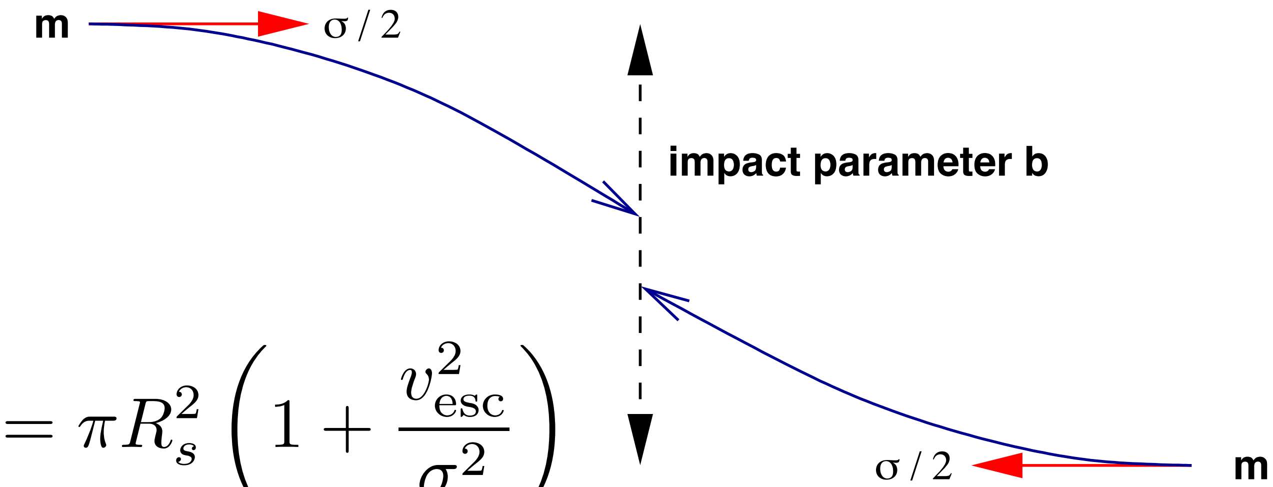
Dynamics in Planetesimal Growth

Armitage 2010



$$\Gamma = \pi R_s^2 \left(1 + \frac{v_{\rm esc}^2}{\sigma^2}\right)$$

- Compare to Hill velocity:

$$v_H = \Omega R_H = \left(\frac{m_p}{3M_*}\right)^{1/3} v_K$$

$$\sigma_{\rm rel} > v_H$$

“Dispersion-Dominated”

(2 body problem)

- Growth rates depend on the relative velocities within the disk:
  - heating: scattering, collisions, fragmentation
  - cooling: dy friction, co effects

$$\sigma_{\rm rel} <$$

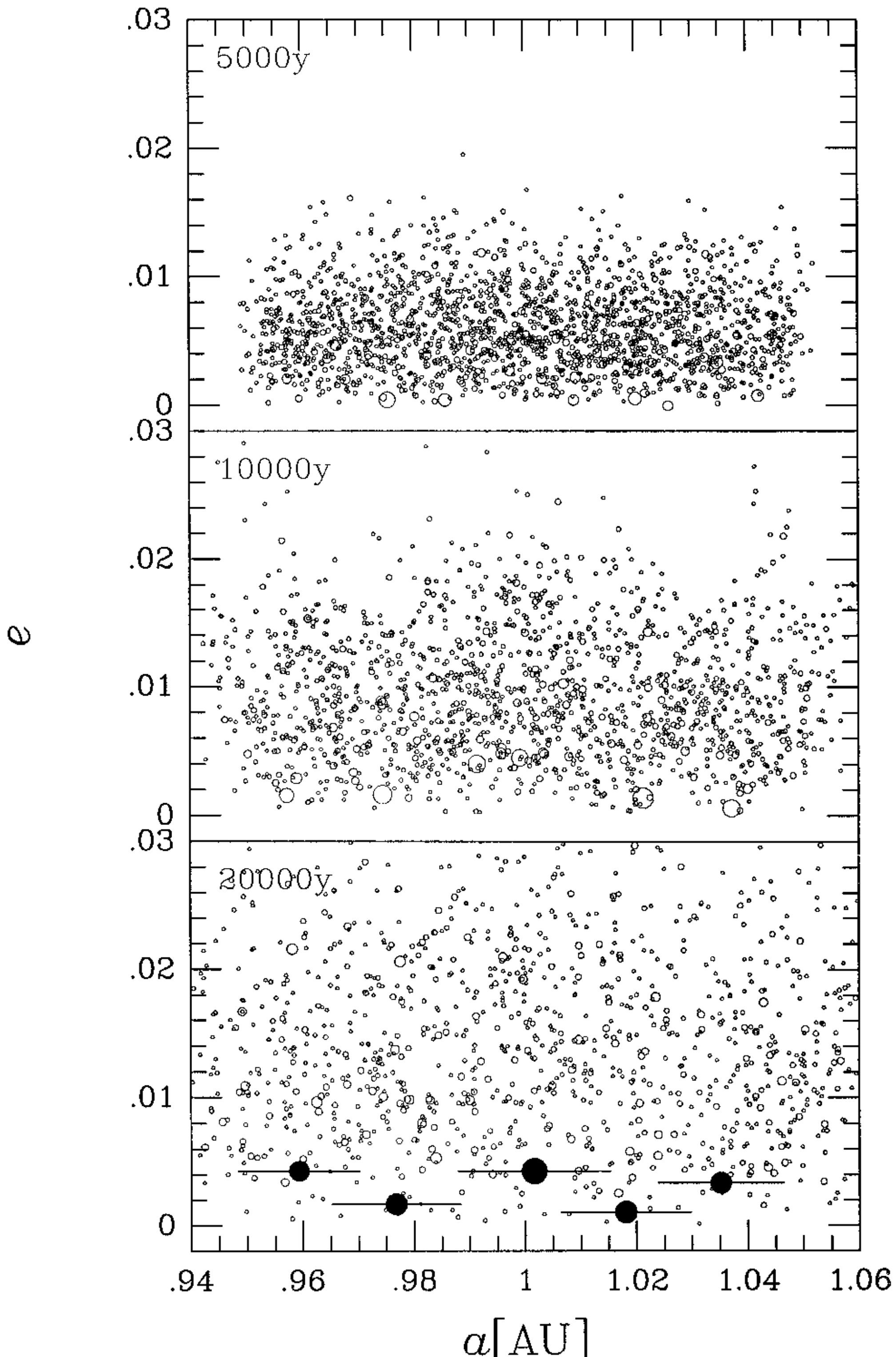
“Shear-Don

(3-body problem)

consider finding better diagram

direct collision cross section lower than gravitational

From Planetesimals to Protoplanets

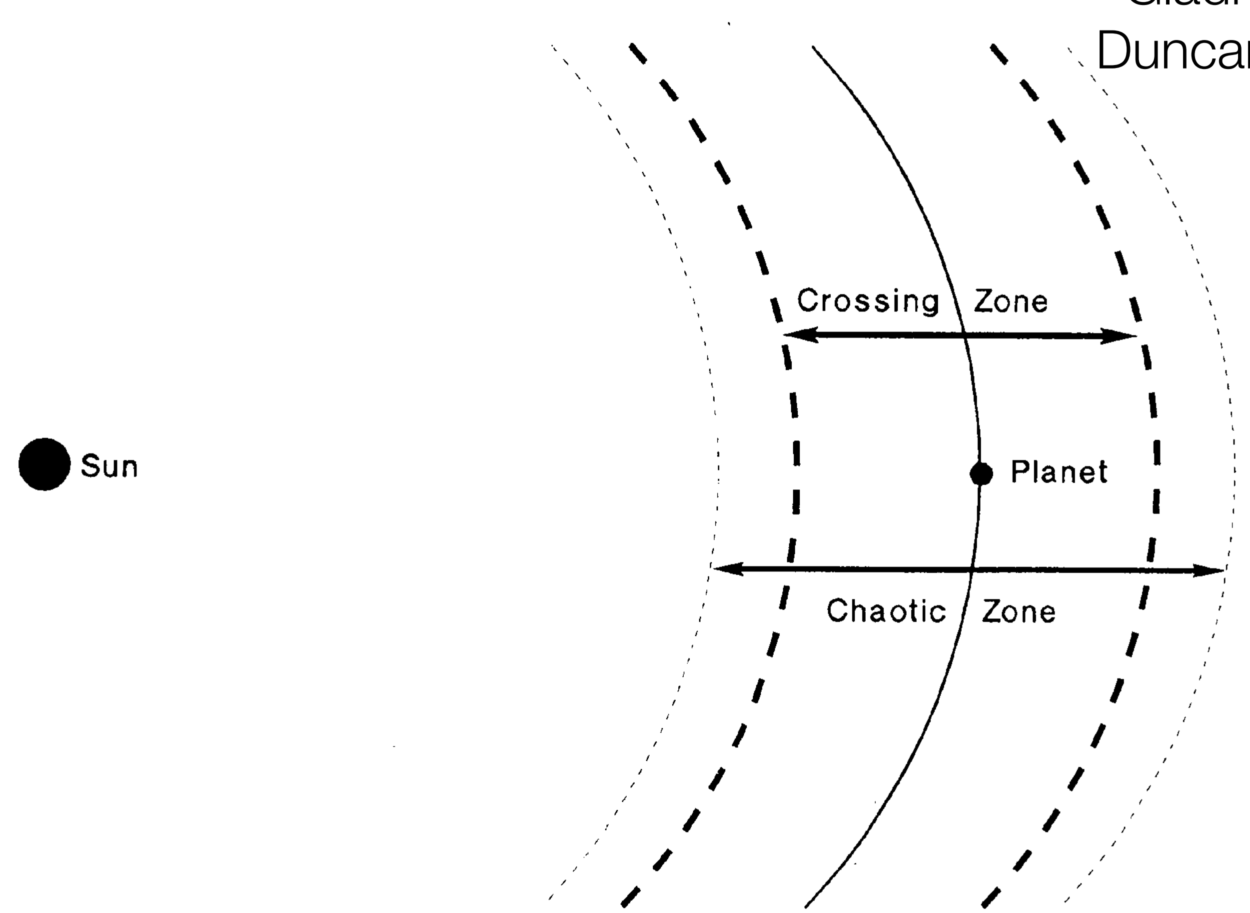


(Kokuba & Ida, 2000)

- Growth depends on location, velocity dispersion, mass, surface density
- Larger planetesimals grow fast due to gravitational focusing / dynamical friction
- Large bodies stir small ones
- Large bodies compete with each other repel to separations of ~5 R_H
- Eventually leads to well-known Oligarchic Growth where smaller proto-planets grow faster

The end of growth: Isolation Mass

- Protoplanet can only feed from a limited zone comparable to Hill radius
- Not a runaway process because feeding zone increases more slowly than mass
- Simulations show $C \sim 3.5$



$$M_{\rm iso} = 4\pi a \cdot C \left(\frac{M_{\rm iso}}{3M_*}\right)^{1/3} a \cdot \Sigma_p$$

most
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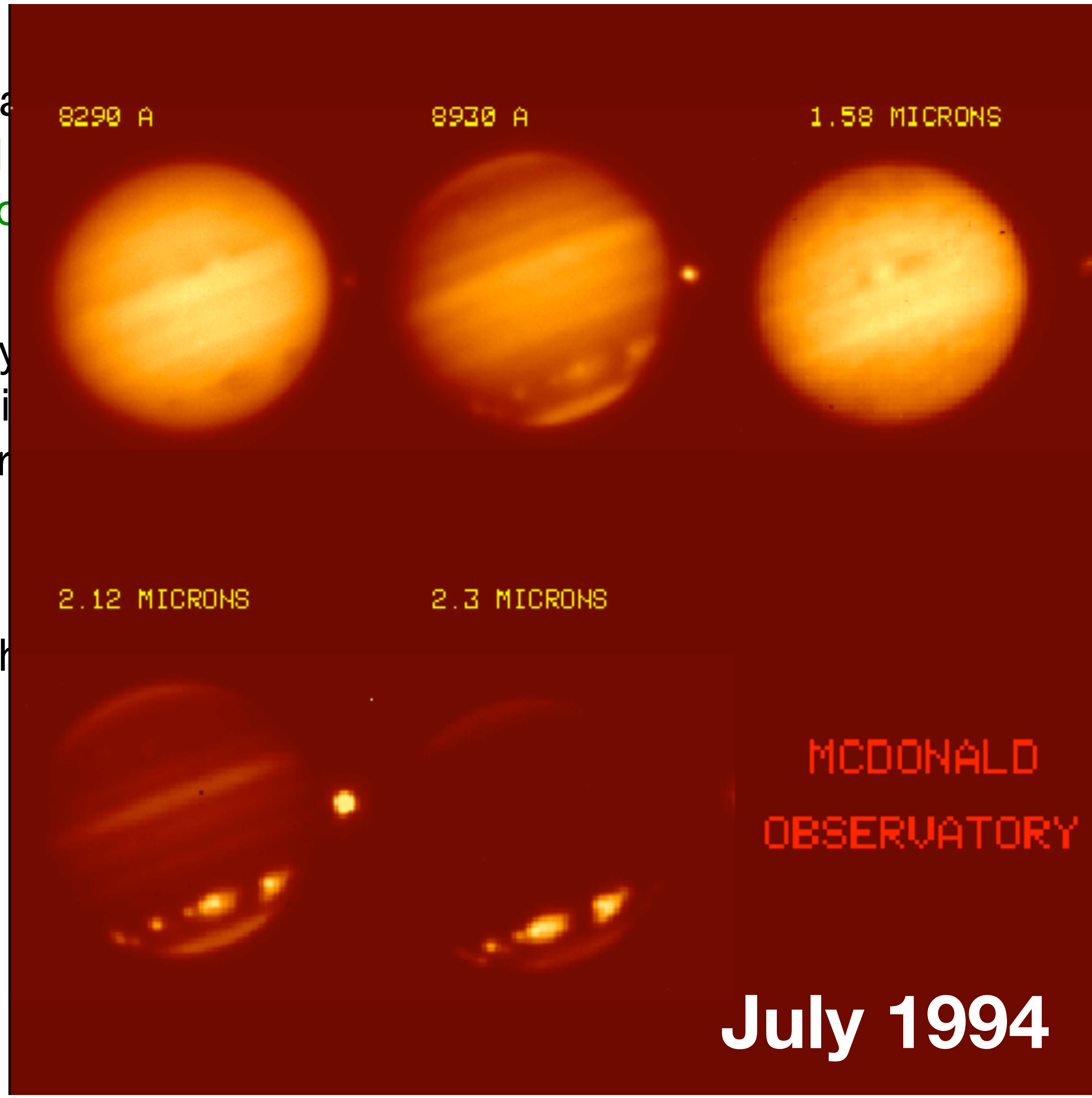
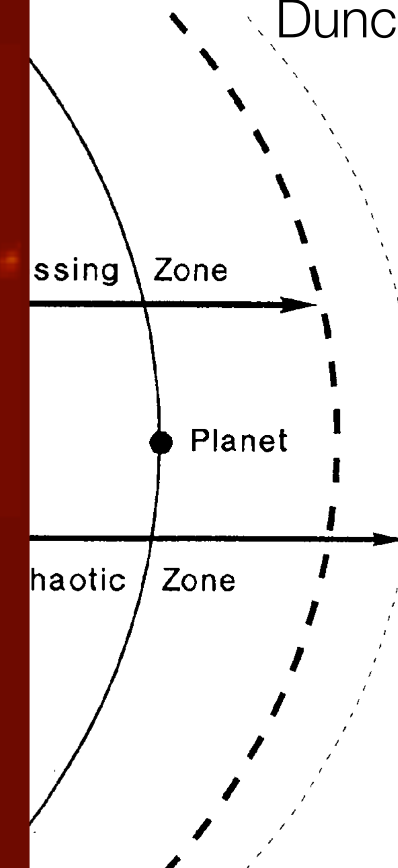
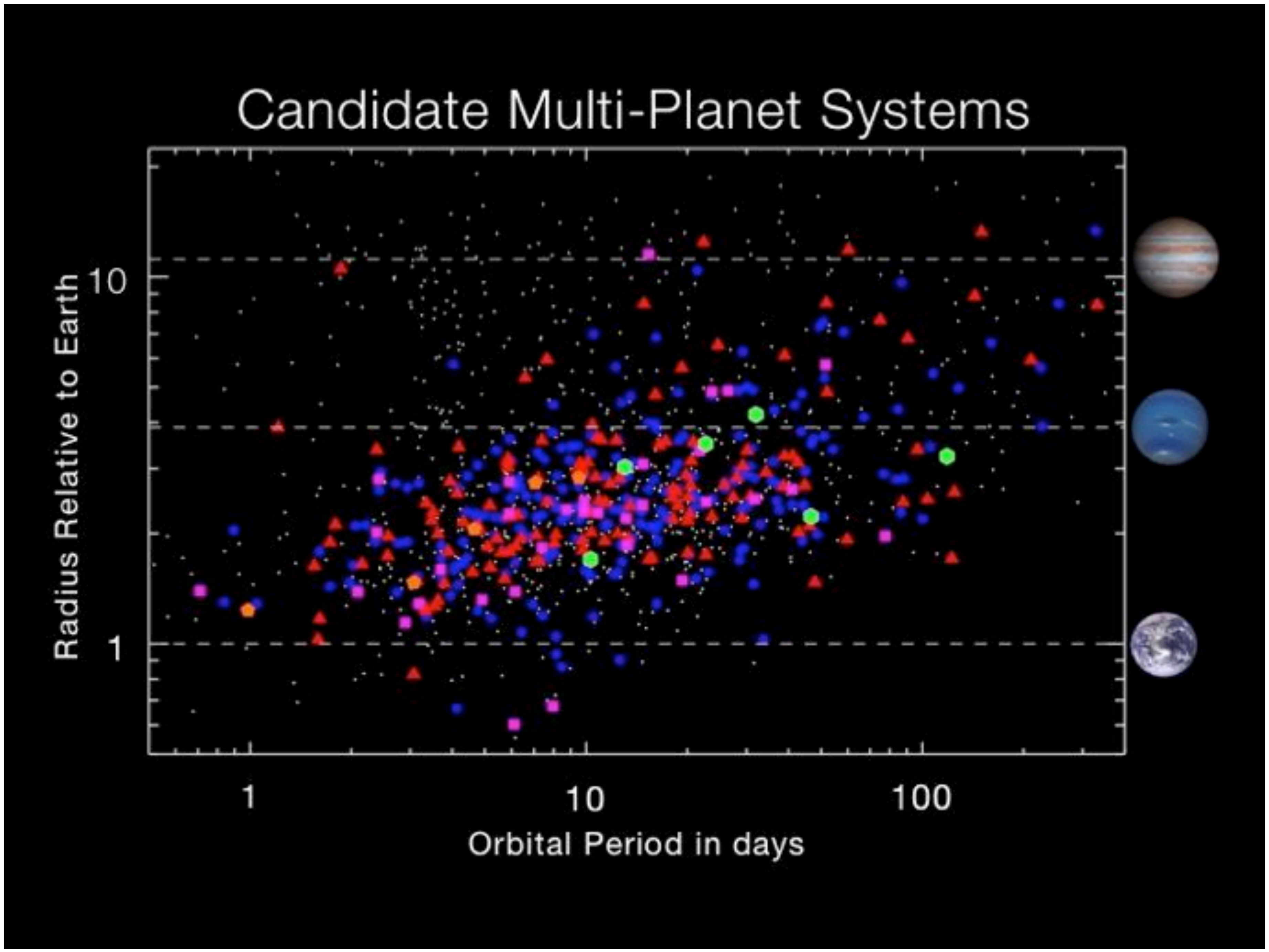


Gladman & Duncan 1990



Part II: Planetary System Architecture



Multi-Planet Stability: Restricted 3 body problem

- Two massive bodies in orbit generate a potential in which test particles move
- Equation of motion are simple in dimensionless coordinates, in the rotating frame
- There is one integral of motion, known as the Jacobi constant
- Explore orbital behavior using zero velocity curves, and Poincare surface of section
- Planetary limit, curves at L1 open when:

$$\Delta > 3.5 R_H$$

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} x + 2\dot{y} - \frac{1-\mu}{r_1^3}[x+\mu] - \frac{\mu}{r_2^3}[x-(1-\mu)] \\ y - 2\dot{x} - \frac{1-\mu}{r_1^3} y - \frac{\mu}{r_2^3} y \\ -\frac{1-\mu}{r_1^3} z - \frac{\mu}{r_2^3} z \end{bmatrix}$$

$$r_1 = \sqrt{(x+\mu)^2 + y^2 + z^2}$$

$$r_2 = \sqrt{(x-(1-\mu))^2 + y^2 + z^2}$$

$$C_J = n^2(x^2 + y^2) + 2\left(\frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}\right) - \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$$

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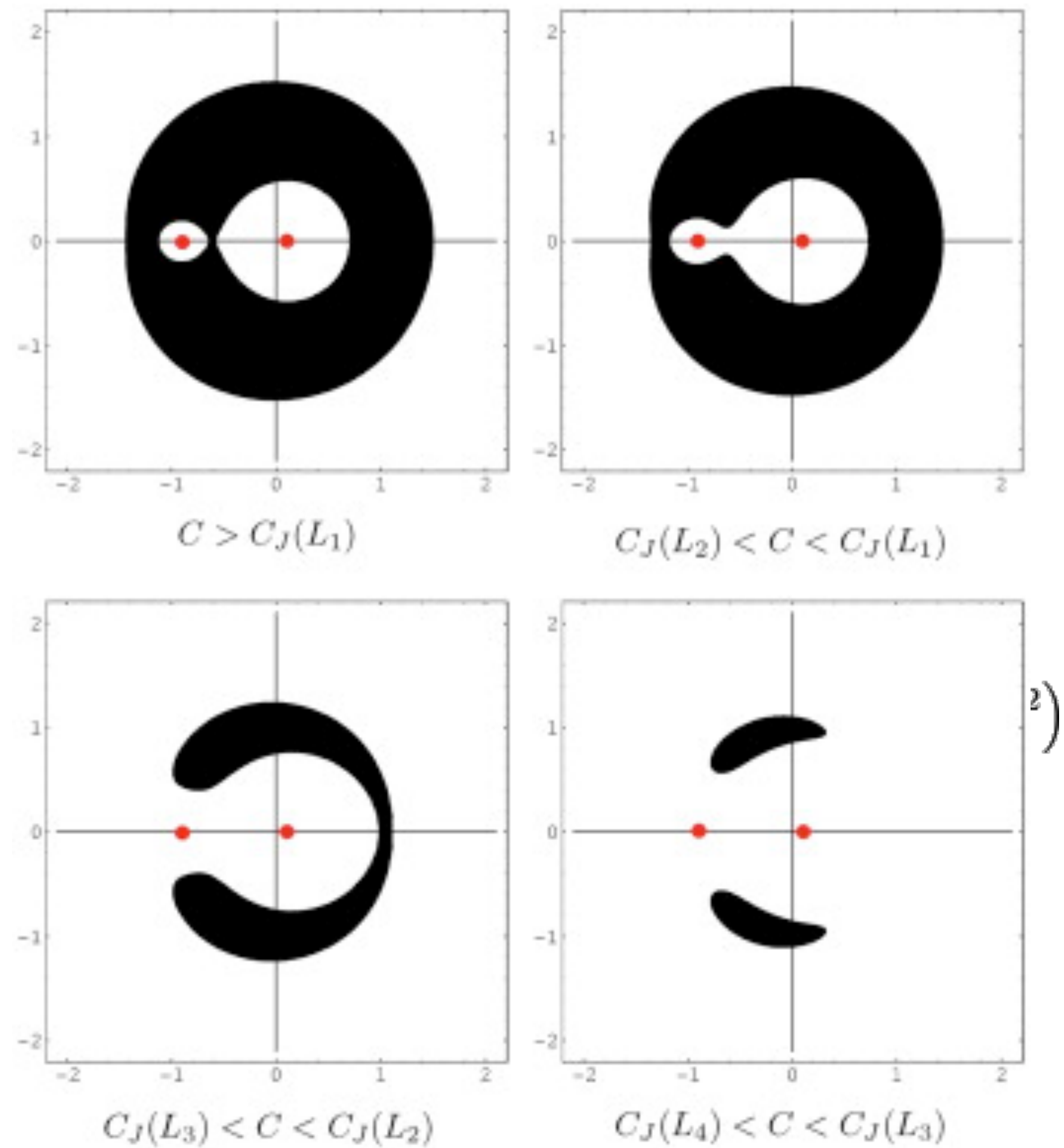

Alessi 2011

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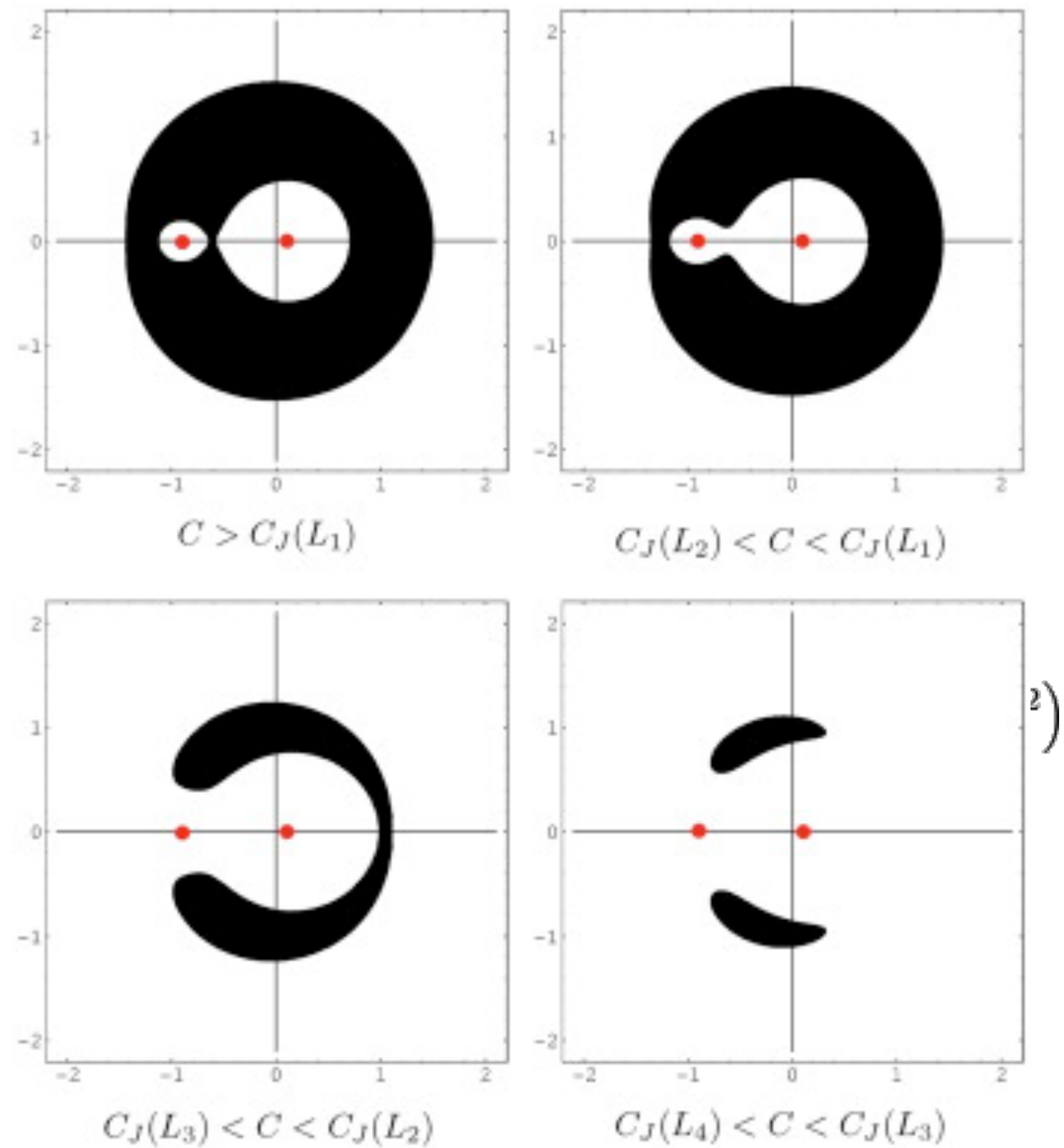
isolation mass!



Alessi 2011

Stability of Two Planet Systems (3 body problem)

- Lagrange Stable: Semi-major axes are bounded

- **Hill Stable:** **No close interactions allowed**

  Topological Stability in the (general) three body problem:

  $$L^2 E > (L^2 E)_{\rm crit}$$

  - initial conditions dictate stability for all time

  - equivalent of zero-velocity curves in restricted three-body problem (not as easily visualized)

  - useful to define mutual Hill Radius:

  $$R'_H = \left(\frac{\mu_1 + \mu_2}{3}\right)^{1/3} \frac{(a_1 + a_2)}{2}$$

Marchal and Bozis 1982 Gladman 1993

Stability of Multi Planet Systems: Two Close Planets

- Compute the critical value in terms of $L^2 E$ for circular two planets:

$$\Delta \approx 2.4(\mu_1 + \mu_2)^{1/3}$$

$$\mu_1 = \mu_2 \rightarrow \Delta \approx 3\mu^{1/3}$$

$$G = 1, \mu = m_p/m_*, a_1 = 1, e = 0, i = 0$$

- Non-circular orbits:

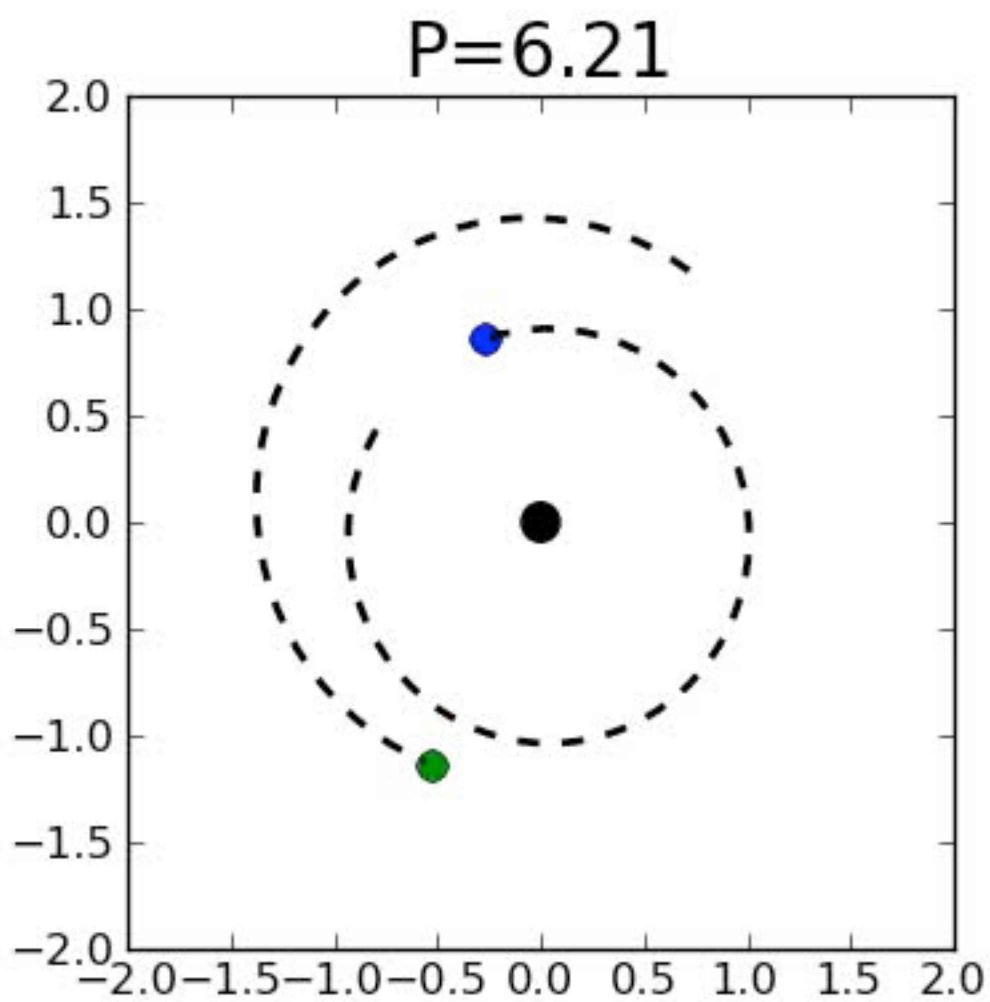
$$\Delta^2 > 12 + 4/3 \left(\frac{\mu_1 + \mu_2}{3} \right)^{2/3} (e^2 + i^2)$$

Gladman 1993, Hasegawa & Nakazawa 1990)

Stability Check

System 1:

$$\Delta \sim 3.2R'_H, m_* = 1.0,$$
$$a_1 = 1.0, m_2 = m_3 = 0.001$$



System 2:

$$\Delta \sim 3.5R'_H, m_* = 1.0,$$
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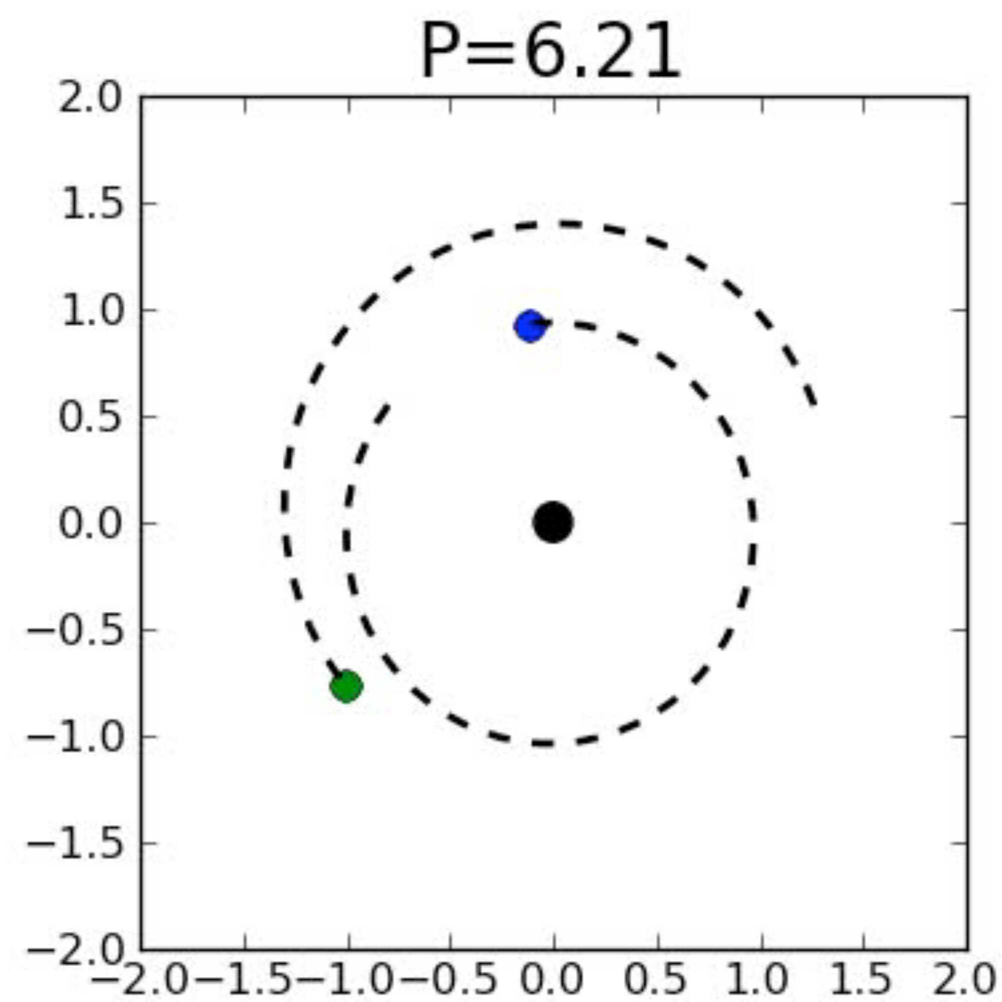
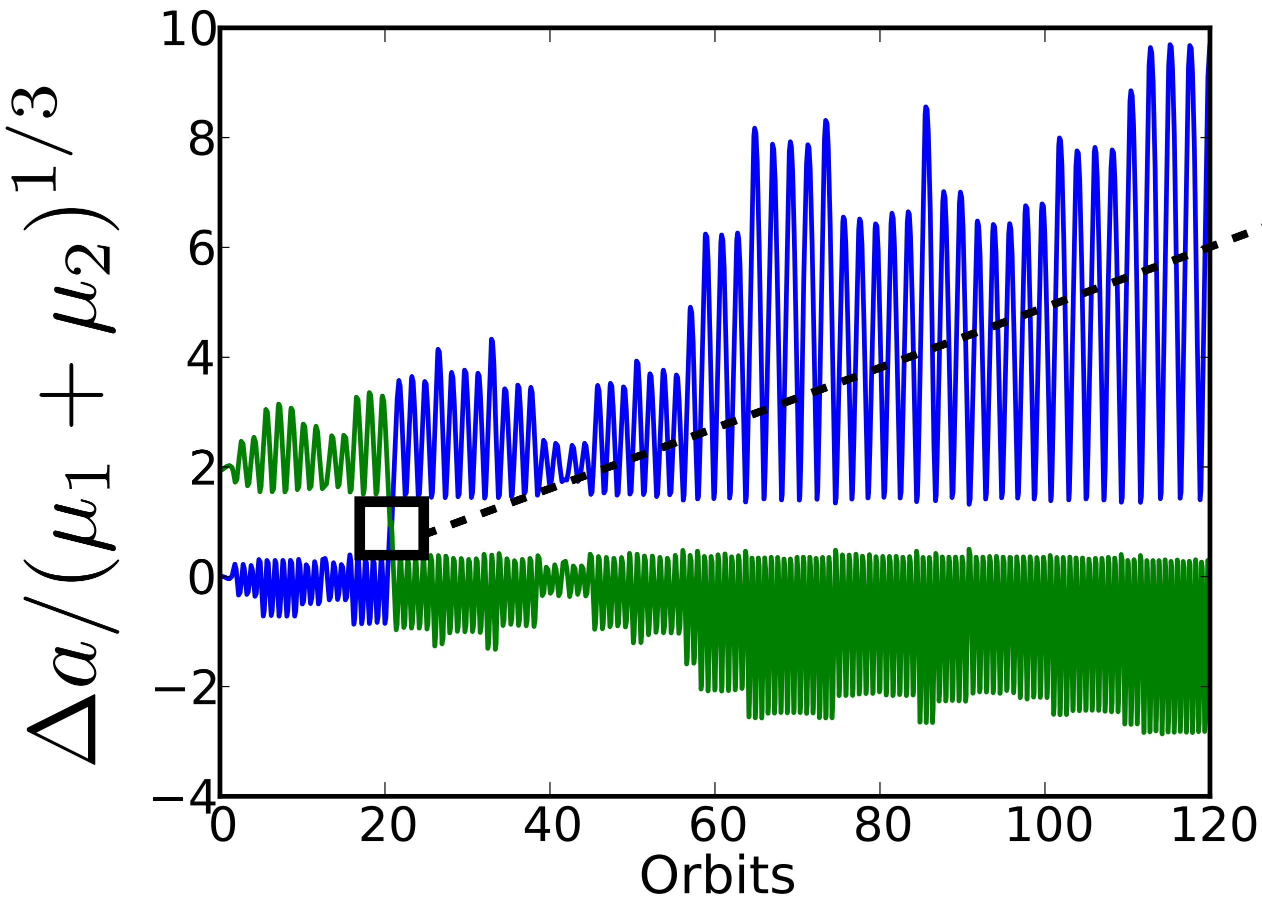
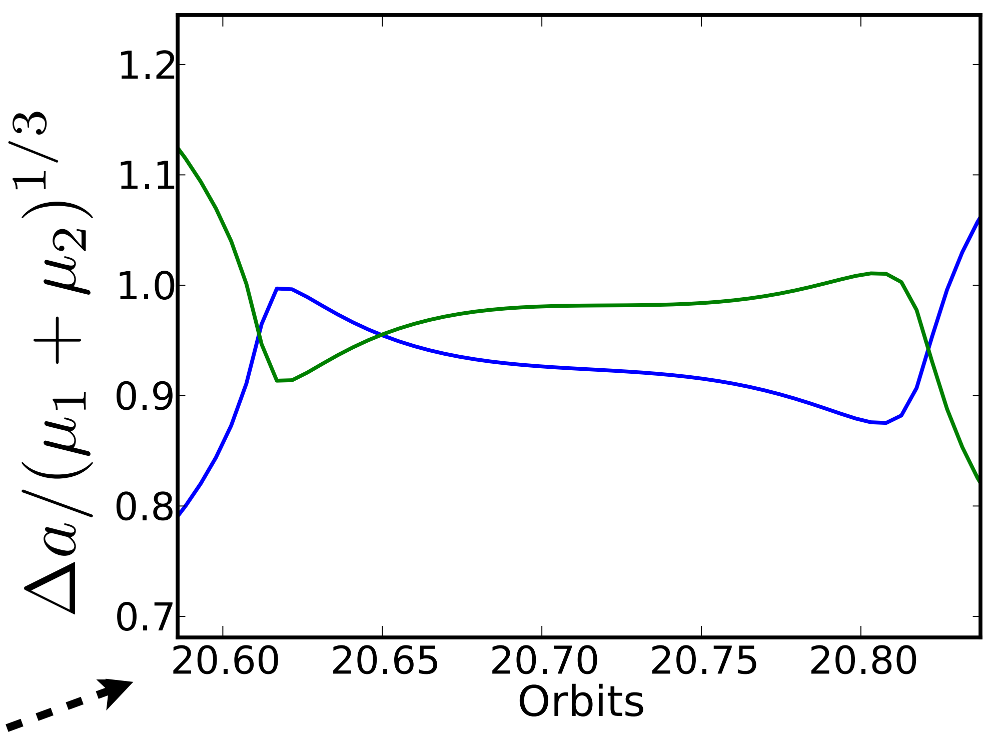


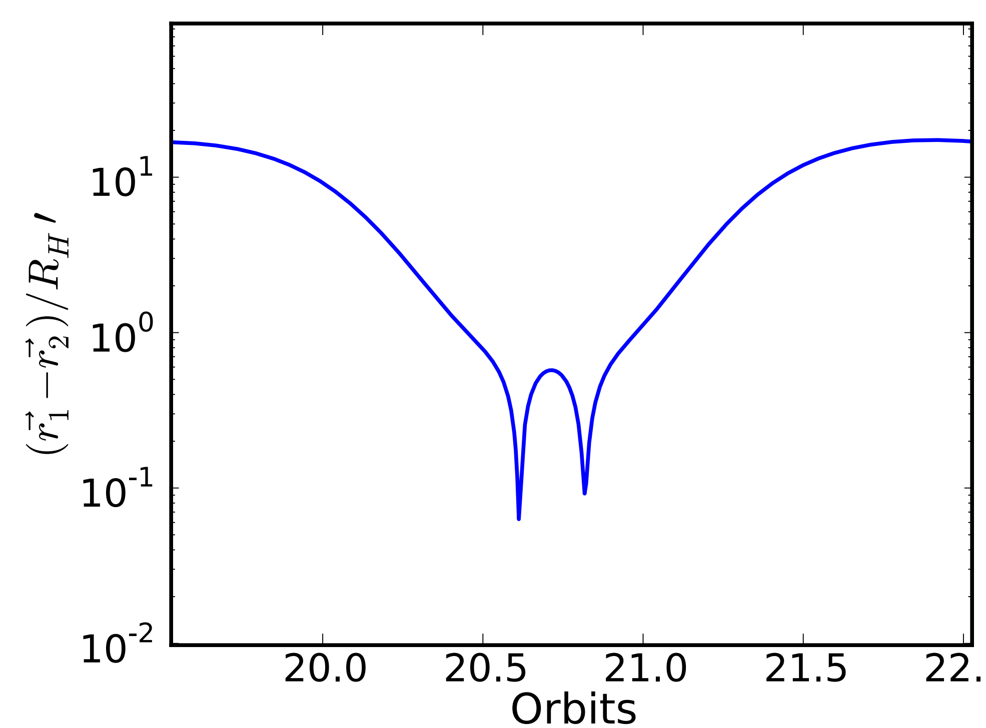
Hands on Dynamics Session: Why the tame reaction?

$$\frac{v_H}{v_{\mathrm{esc},*}}$$ O.O.M!

$$r_{\mathrm{ej}} \approx$$ O.O.M!!



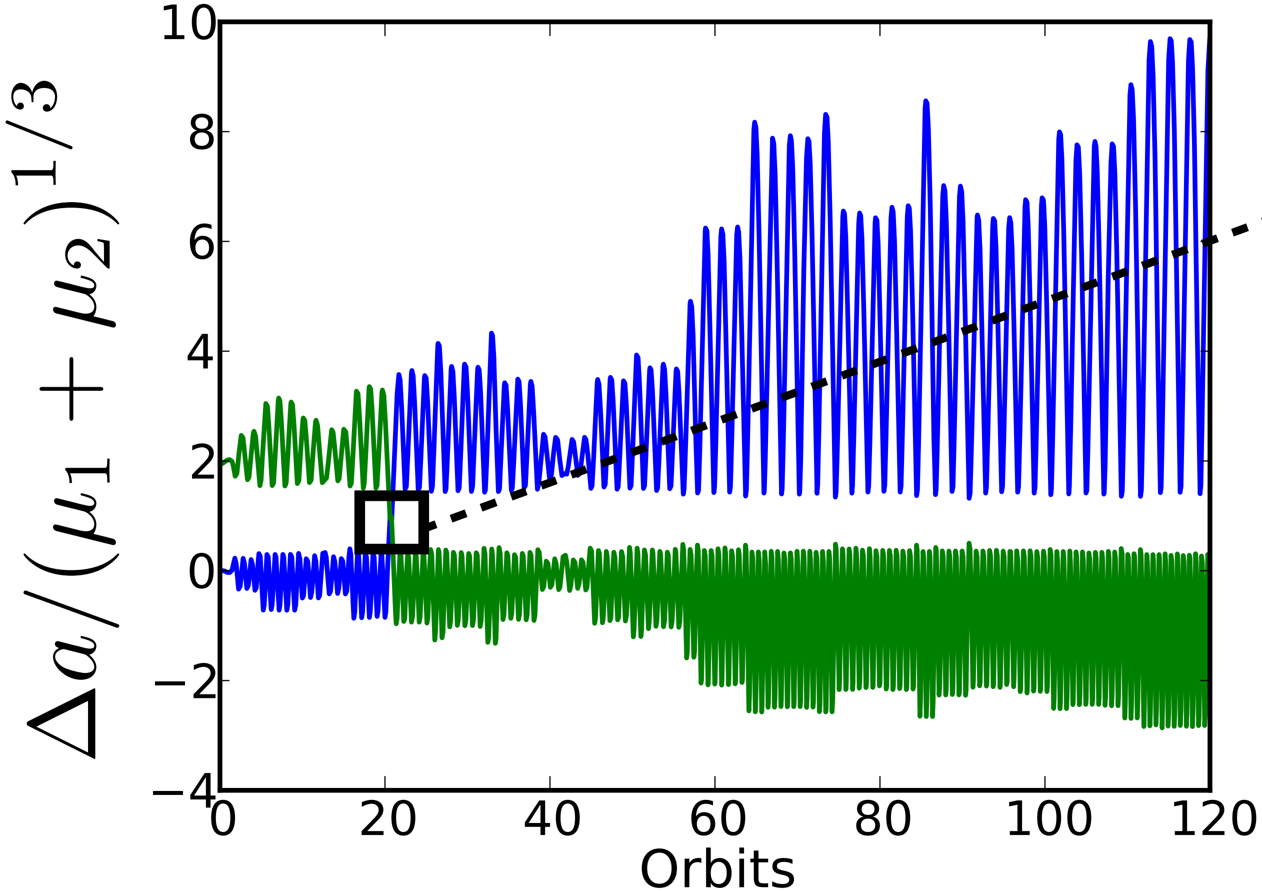


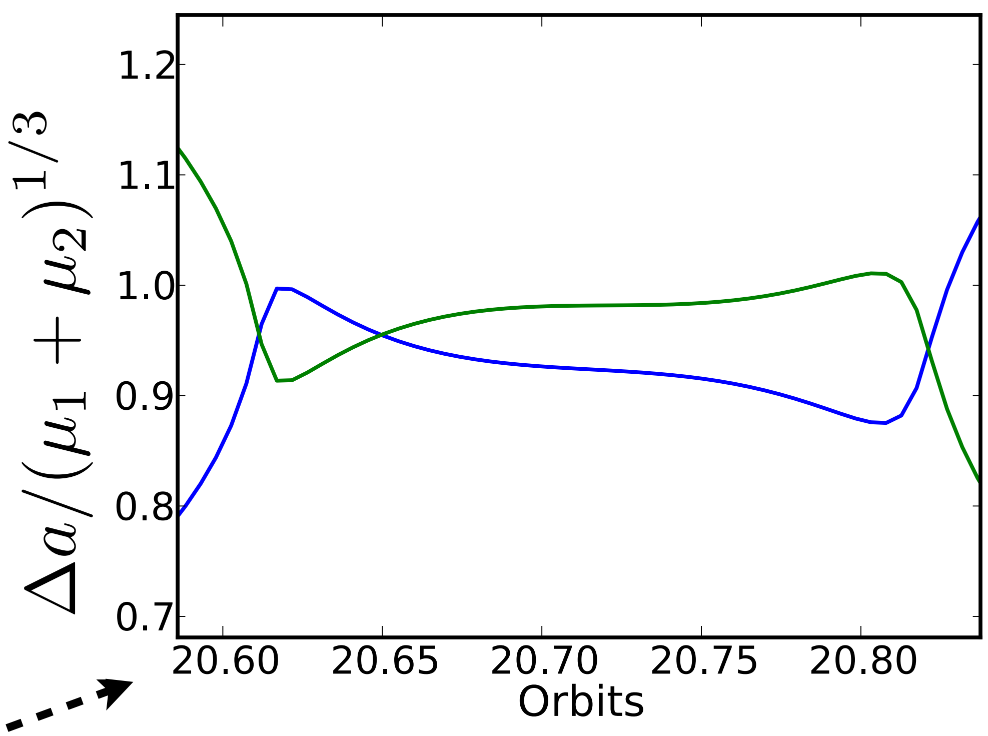


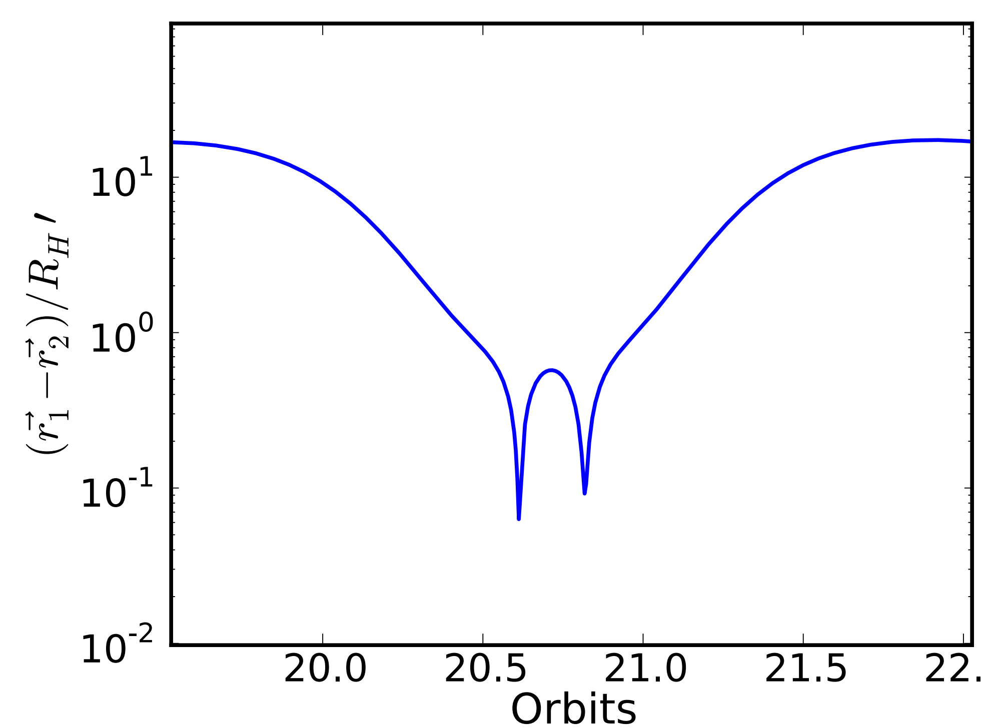
Hands on Dynamics Session: Why the tame reaction?

$$\frac{v_H}{v_{\mathrm{esc},*}} = \left(\frac{m_1}{3M_*}\right)^{1/3} / \sqrt{2} \approx 0.05$$

$r_{\mathrm{ej}} \approx$ O.O.M!!



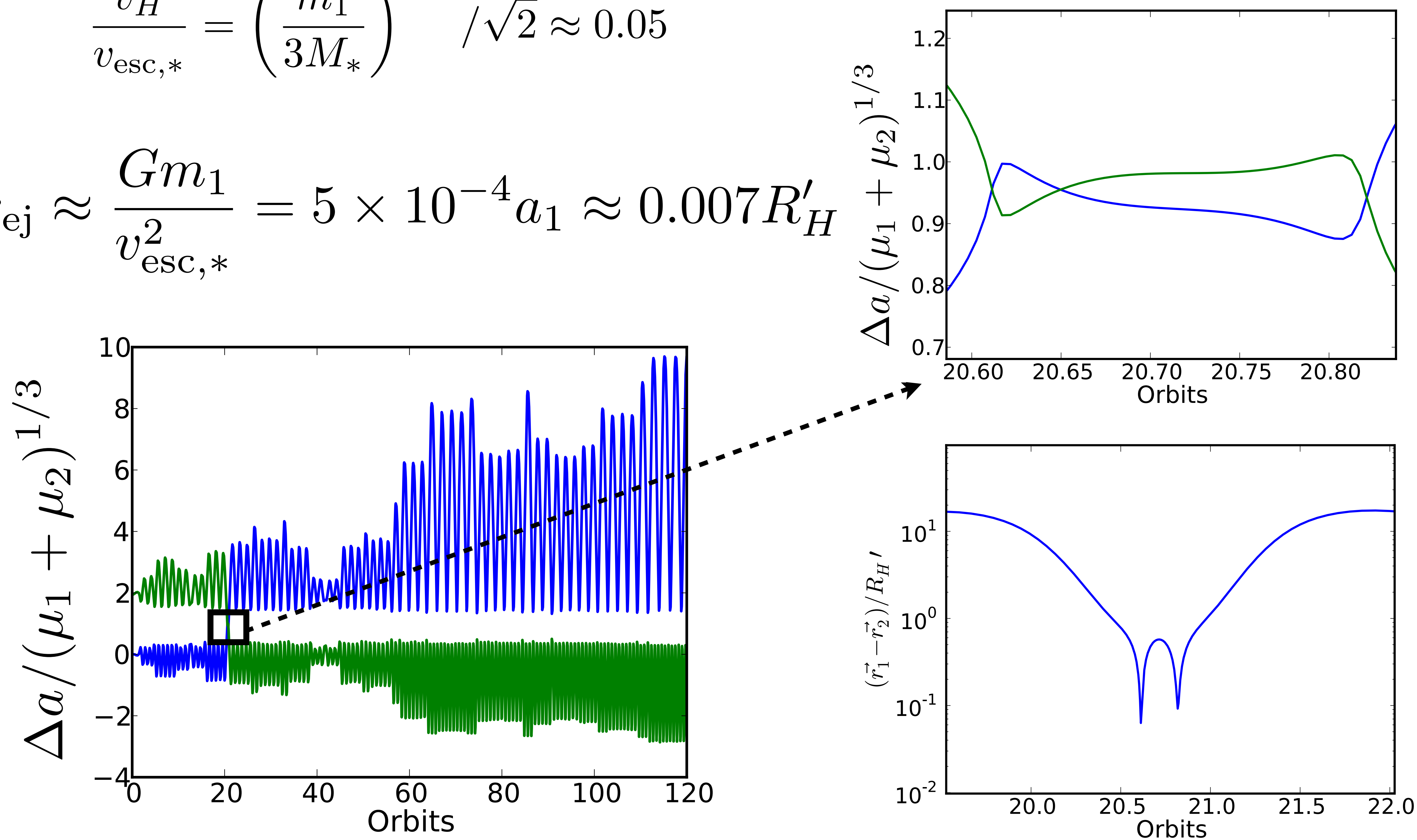




Hands on Dynamics Session: Why the tame reaction?

$$\frac{v_H}{v_{\rm esc,*}} = \left(\frac{m_1}{3M_*}\right)^{1/3} / \sqrt{2} \approx 0.05$$

$$r_{\rm ej} \approx \frac{Gm_1}{v_{\rm esc,*}^2} = 5\times 10^{-4} a_1 \approx 0.007 R_H'$$

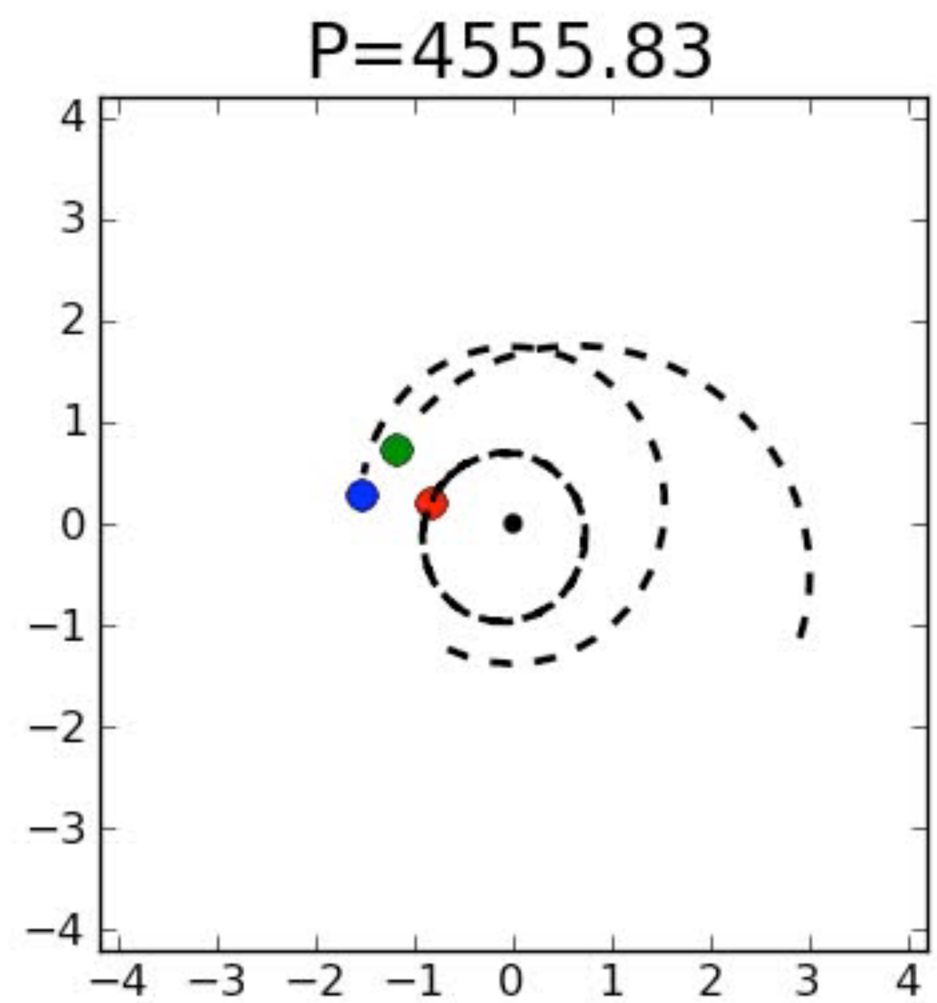


Multi-Planet Stability: >3 Close Planets

- No more "clean" analytic rules for stability:
  - average instability timescale (Chambers et al 1996, *fit Youdin, Kratter, & Kenyon 2012*)

$$\log(\mathrm{t_c}/\mathrm{P_1}) = -9.11 + 4.39\Delta'\mu^{1/12} - 1.07\log(\mu)$$

  - Above 5, number of planets makes little difference to instability timescale
  - Two and three body resonances important (see e.g. Quillen 2011)



$$\Delta \sim 3.5R'_H, m_* = 1.0,$$

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Peale 1976

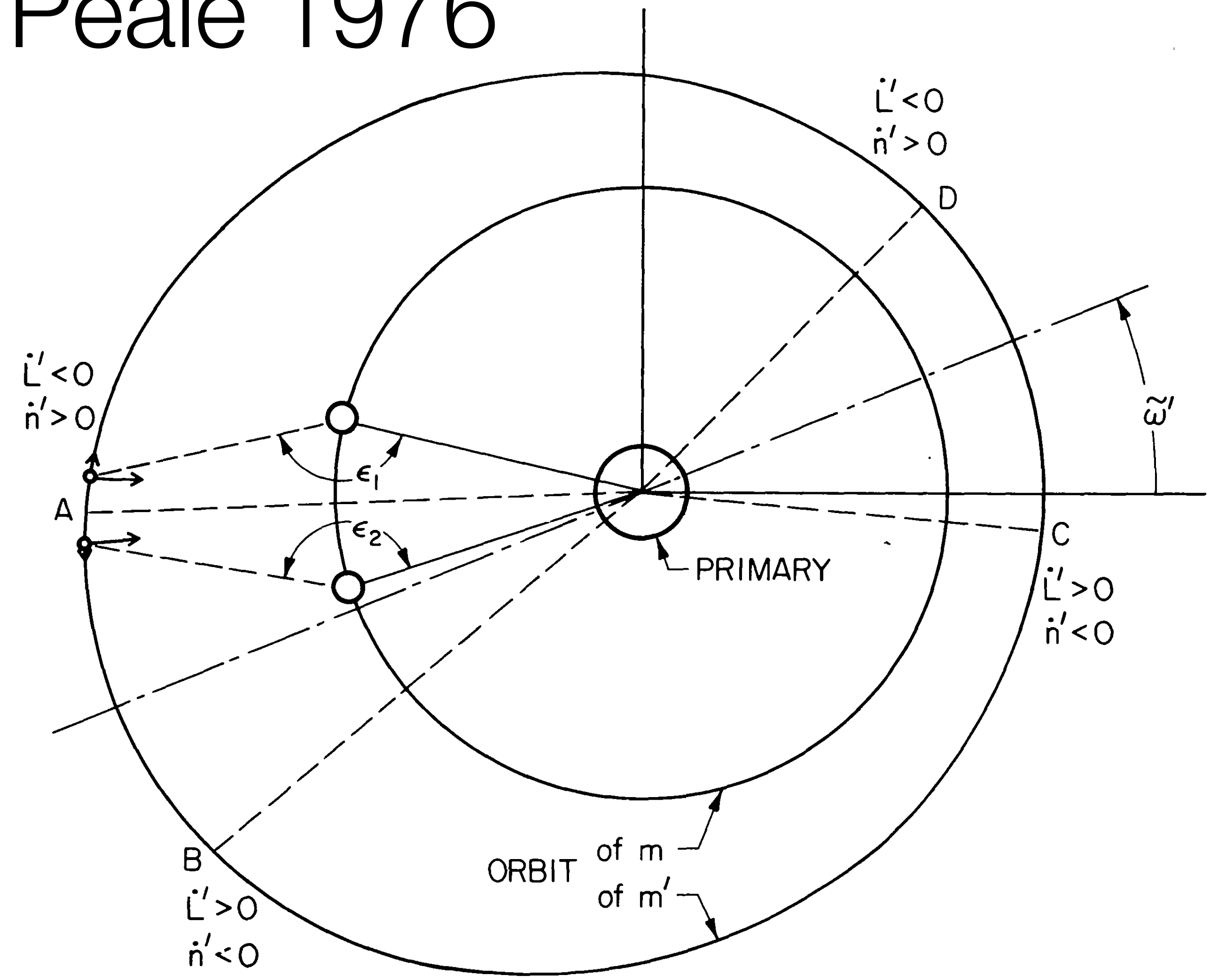
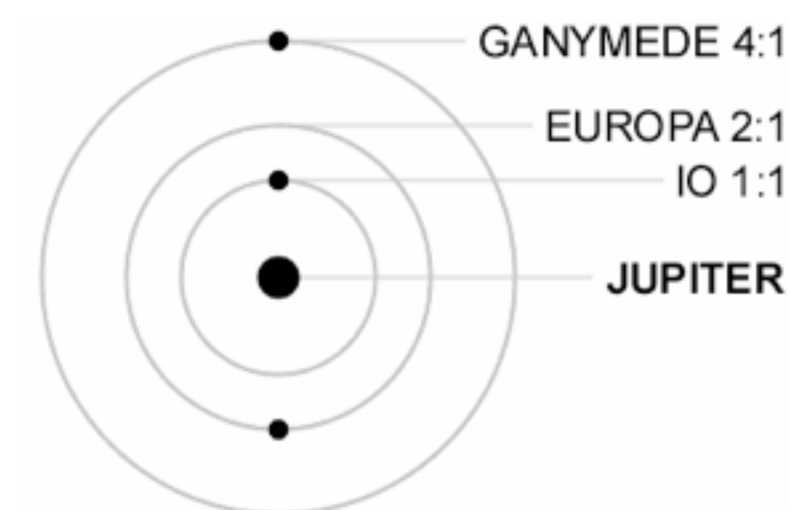


Figure 1 Large-eccentricity stability mechanism. Arbitrary positions of repetitive conjunctions are at points *A*, *B*, *C*, *D*. *L* and *n* are angular momentum and mean motions, respectively.

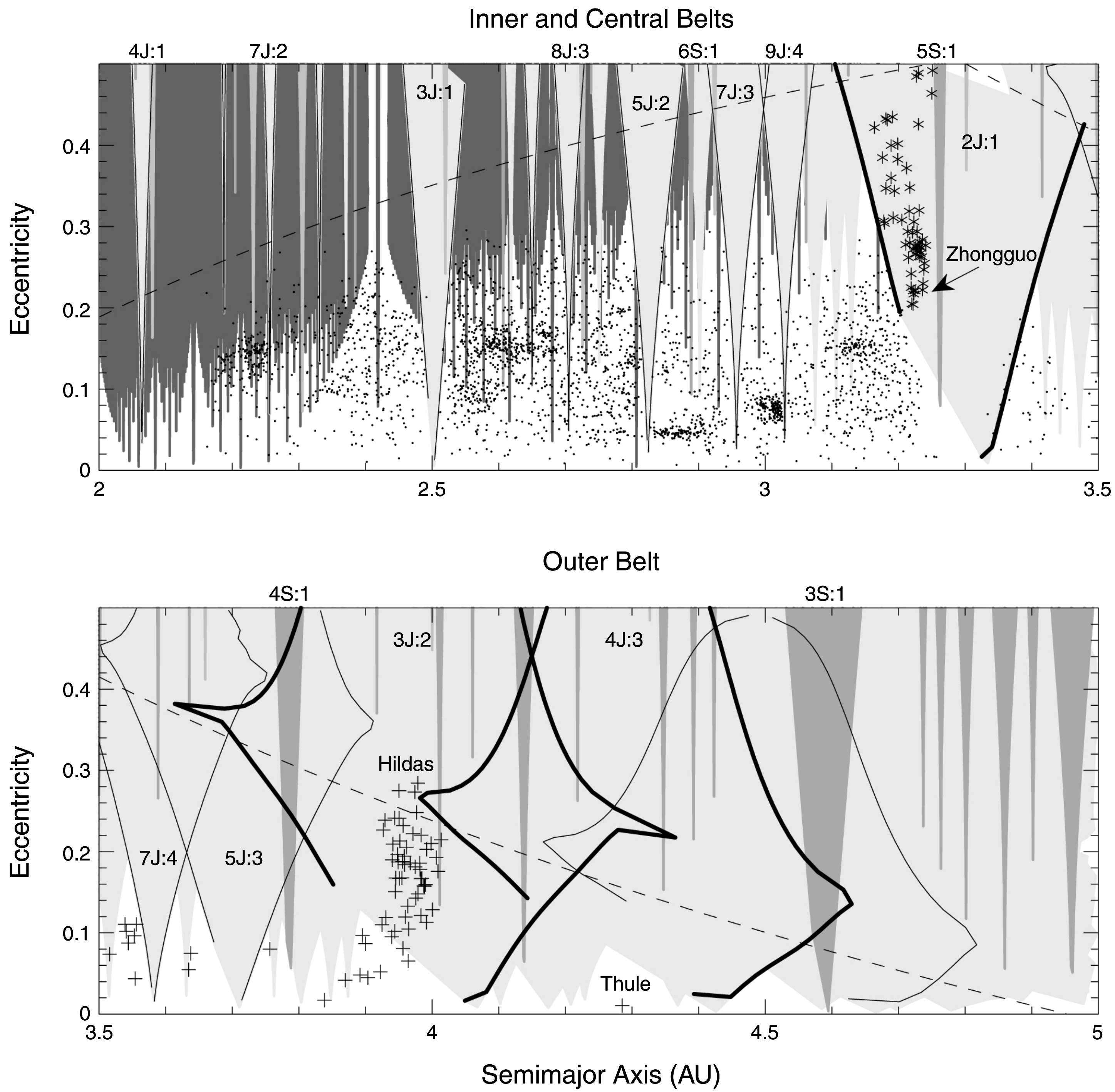
Resonances

- Definition: Resonances are precise numerical relationships between frequencies or periods (Murray & Dermott 1999)
  - Spin-Orbit (Earth, Moon)
  - **Orbit-Orbit / mean motion (Neptune-Pluto)**
  - Secular Resonance (e.g. Kozai)
- Resonances can stabilize orbits
  - torques always return you to the resonance
  - Less time averaging at higher *e*



Example:
Laplace Resonance

Resonance: Stable or Unstable?



Nesvorný et al 2002

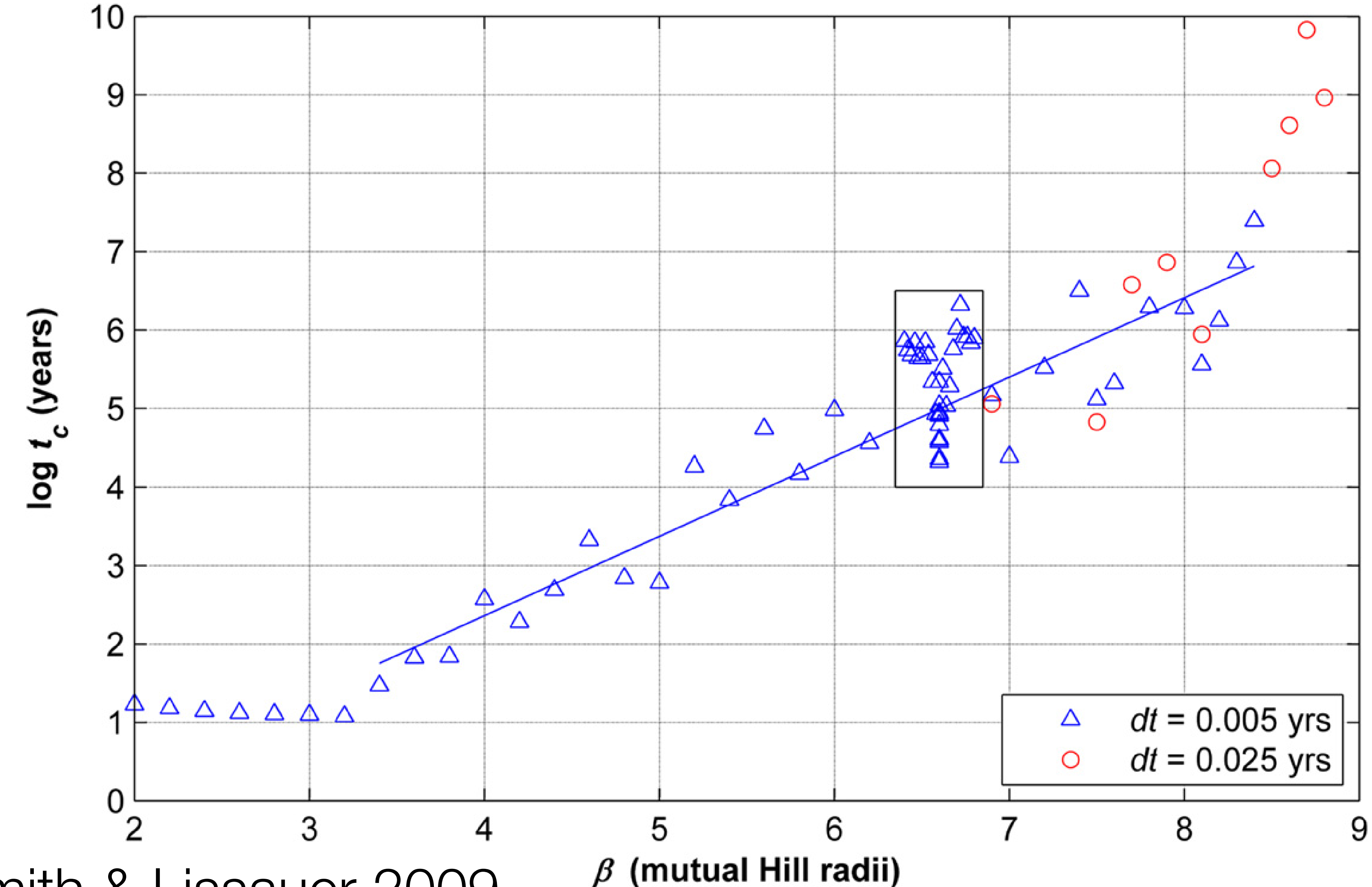
- Stabilize by preventing close interactions (Neptune, Pluto)
- Destabilize due to resonance overlap: two (or more) resonances occur in the same phase space

see e.g. Wisdom 1980

Multi-Planet Stability: >3 Close Planets

- Good Scaling (for equal mass, equal spacing)

- Real systems?



Smith & Lissauer 2009

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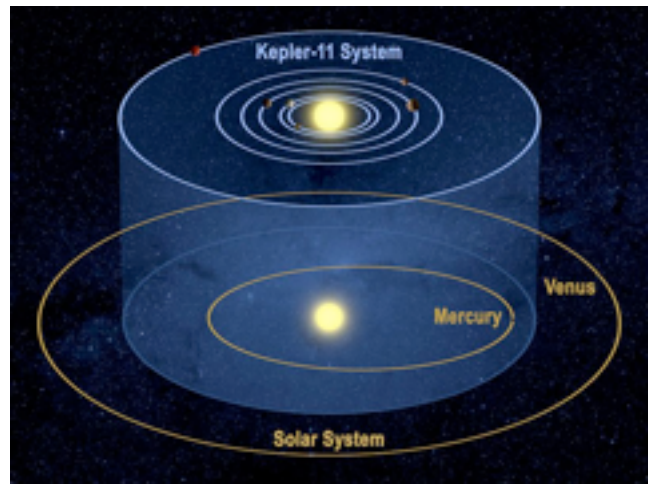
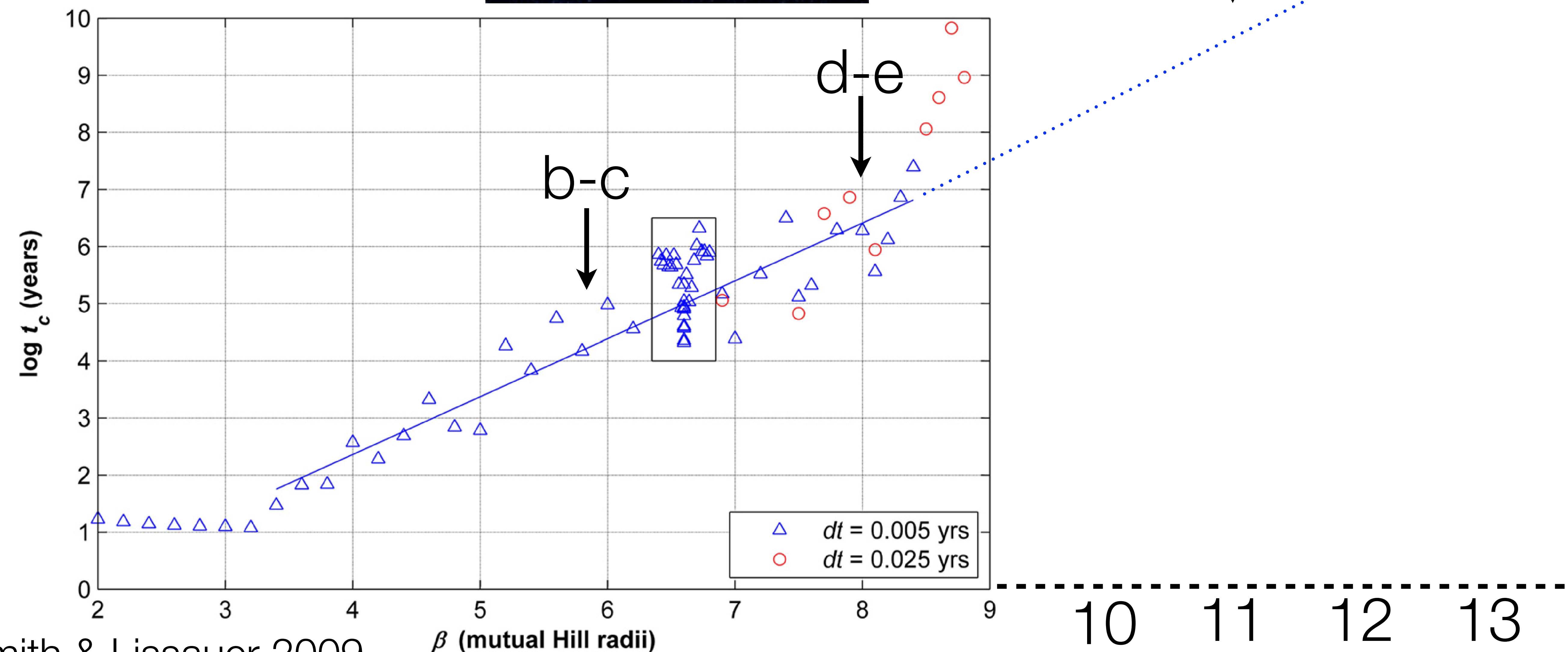
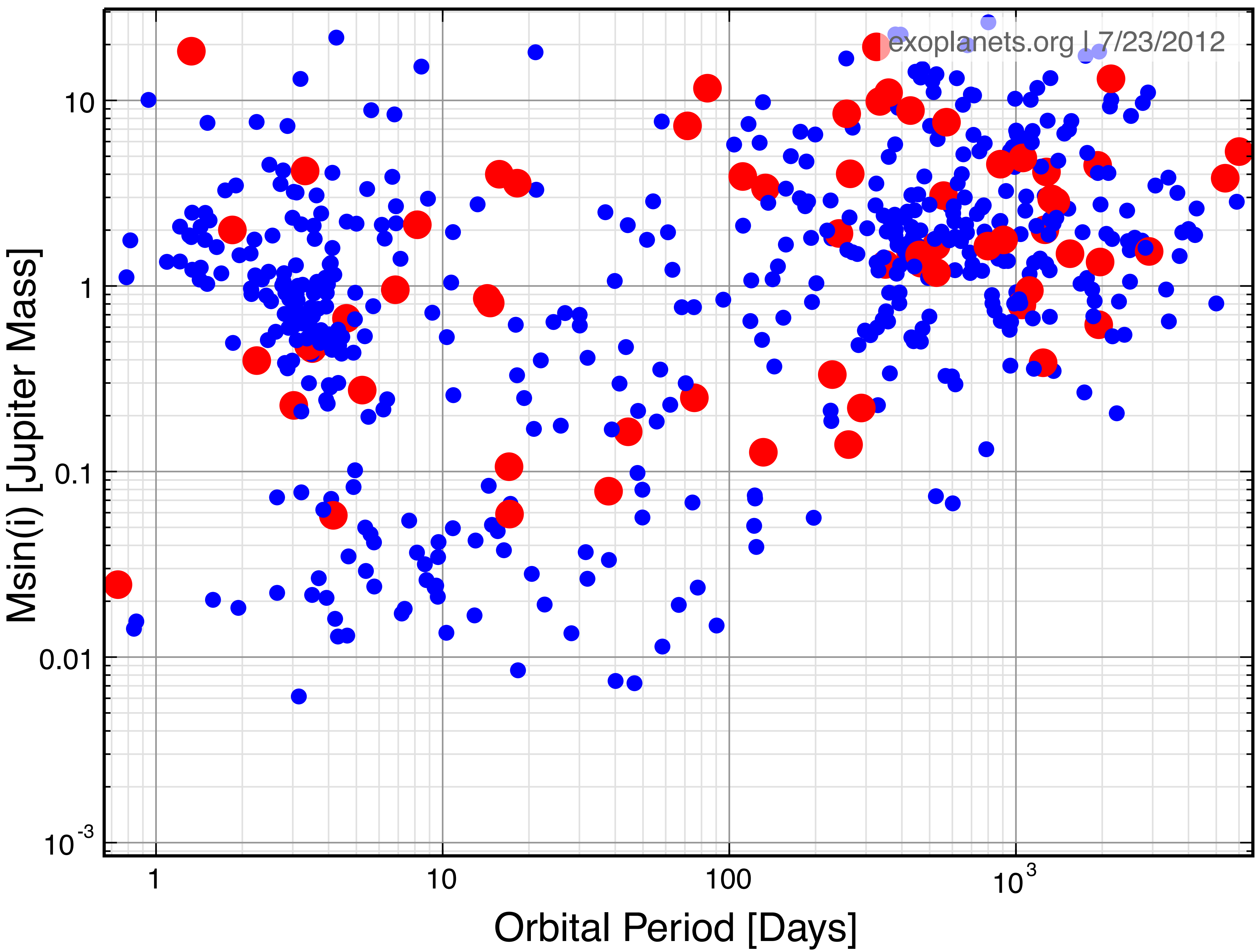


Lissauer et al 2011



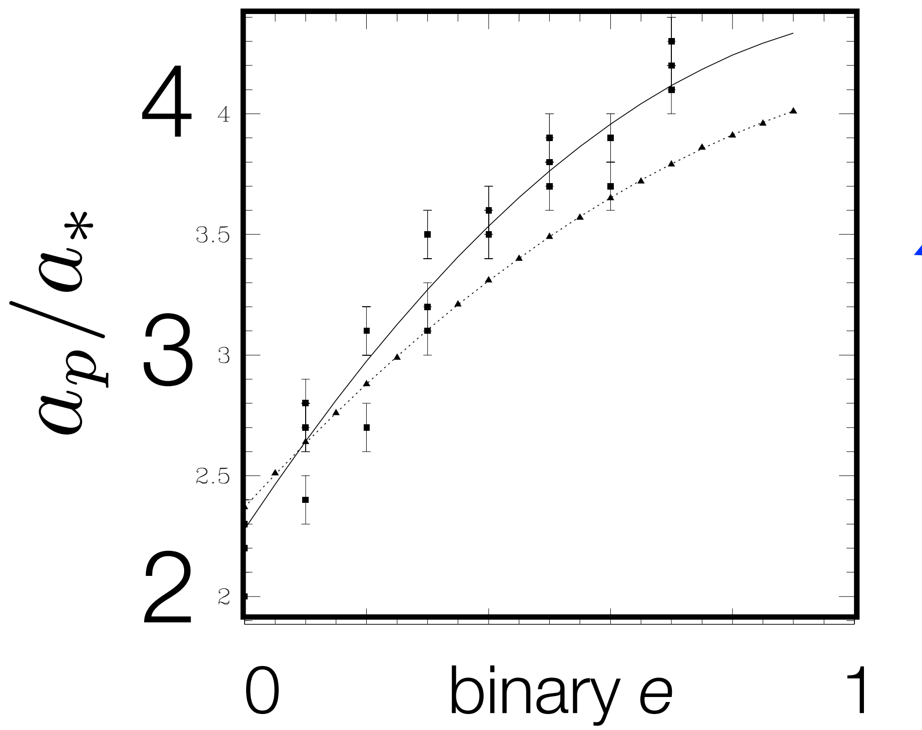
Smith & Lissauer 2009

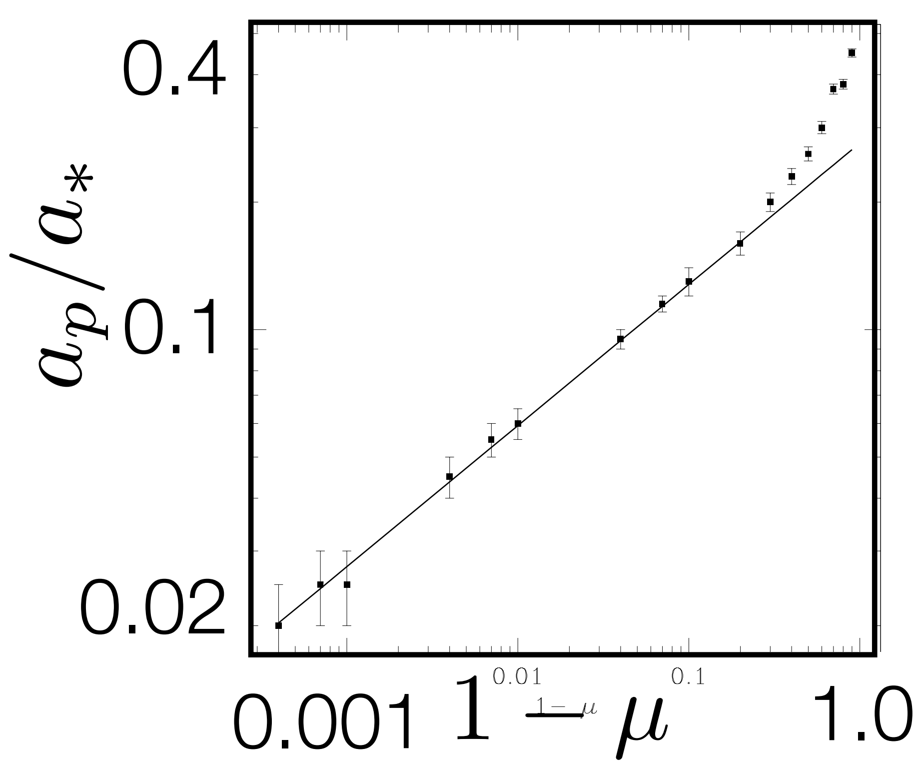
More complicated: Binary Planetary Systems



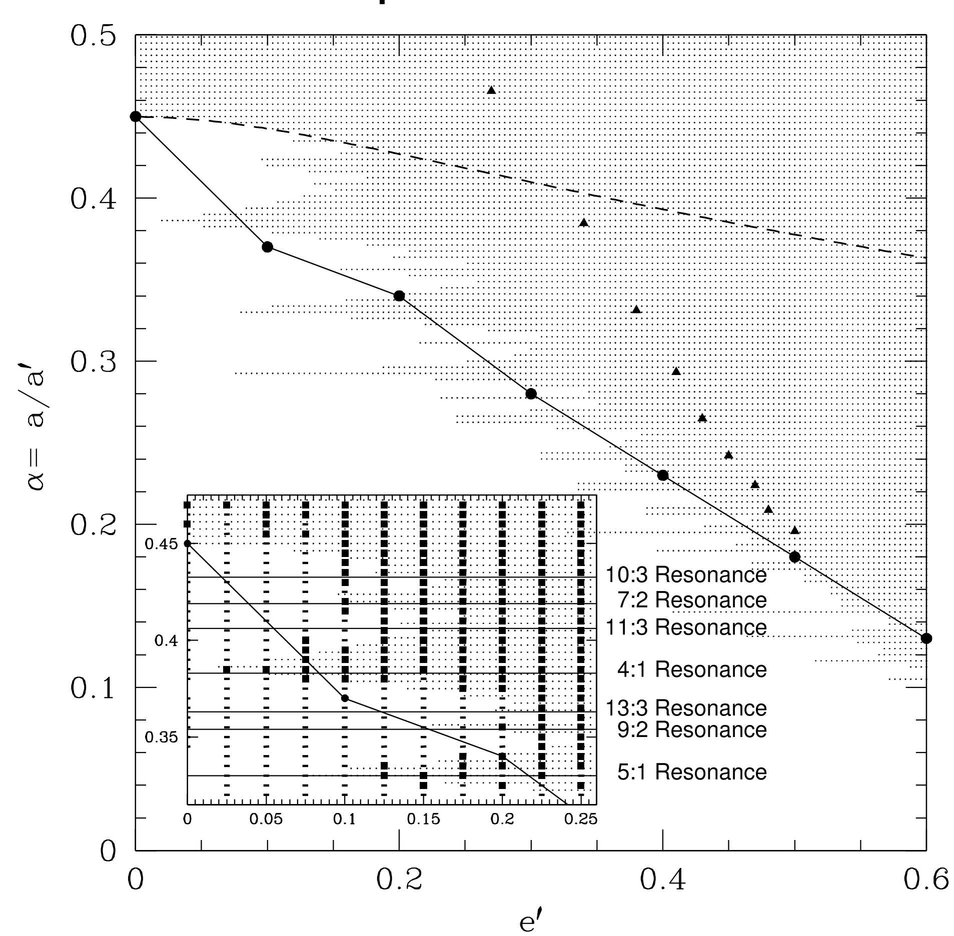
Stability of Planets around Binaries: R3BP

Holman & Wiegert 1999





- Two types of orbits
  - "P Type" orbits have planet outside of stellar binary
  - "S Type" orbits have planet around one star in the binary
- Mudryk & Wu 2006 show that the cause is resonance overlap



Applications of Dynamics

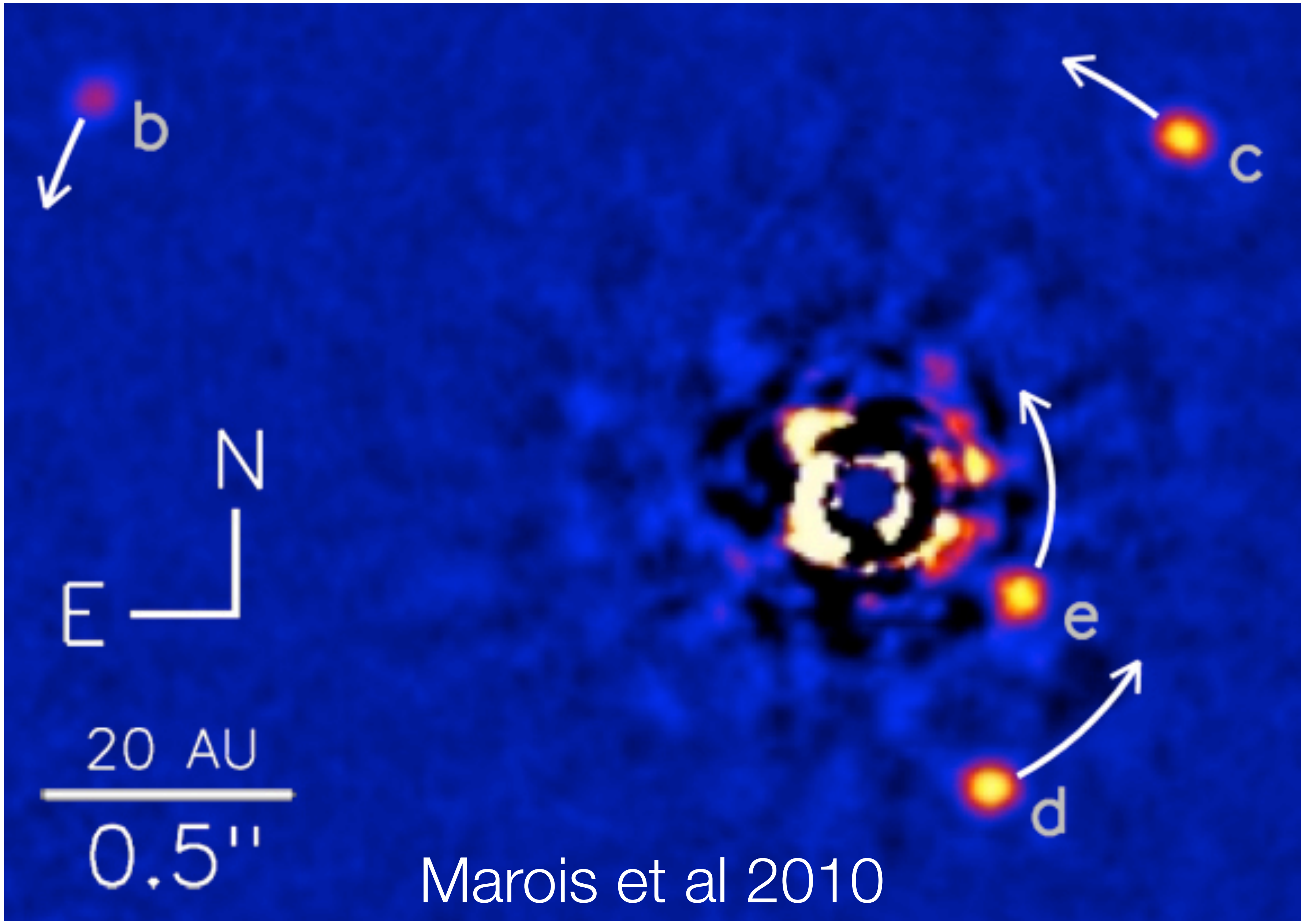
Q: What do we do in the absence of exquisite Kepler light curves, and great software packages?

Applications of Dynamics

Q: What do we do in the absence of exquisite Kepler light curves, and great software packages?

A: We learn a lot by modeling system dynamics alone

Example I: Masses and Formation of HR 8799



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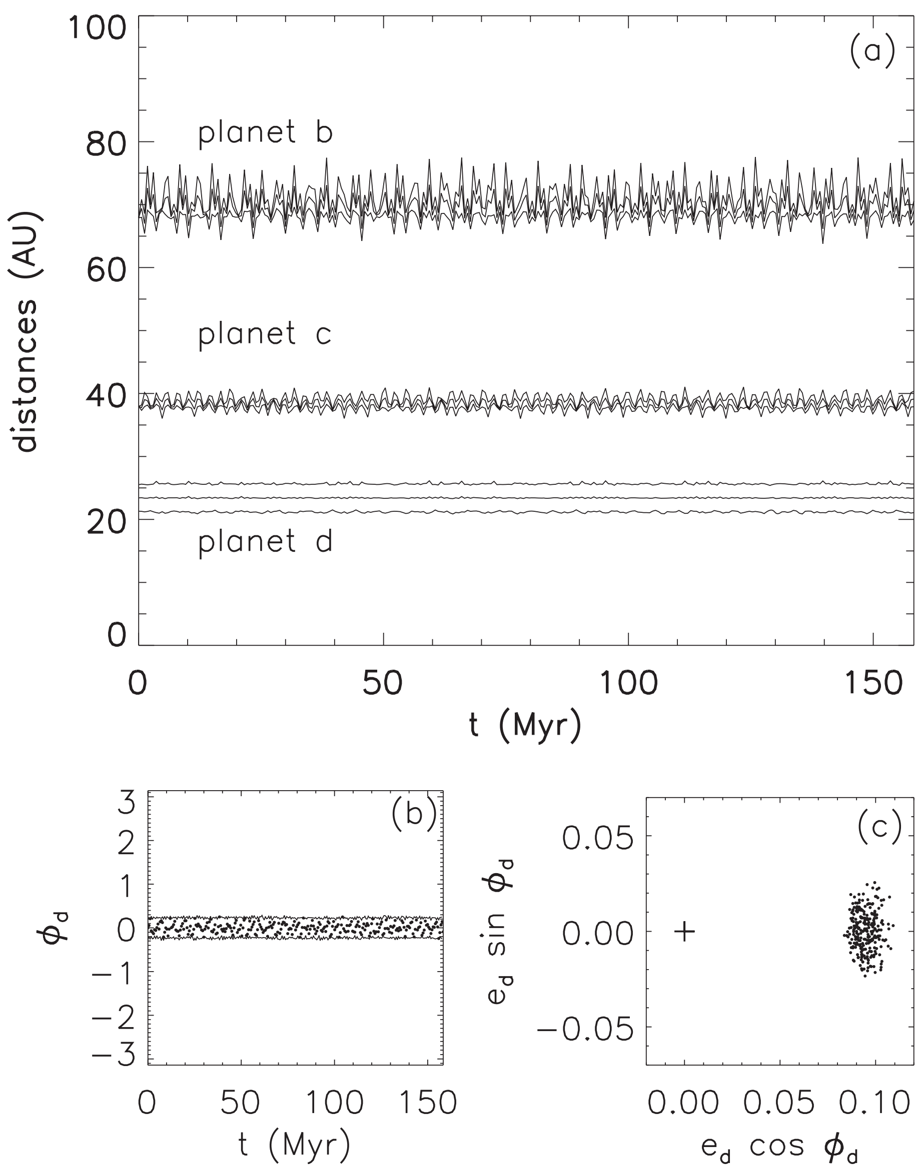

Fabrycky & Murray-Clay 2010

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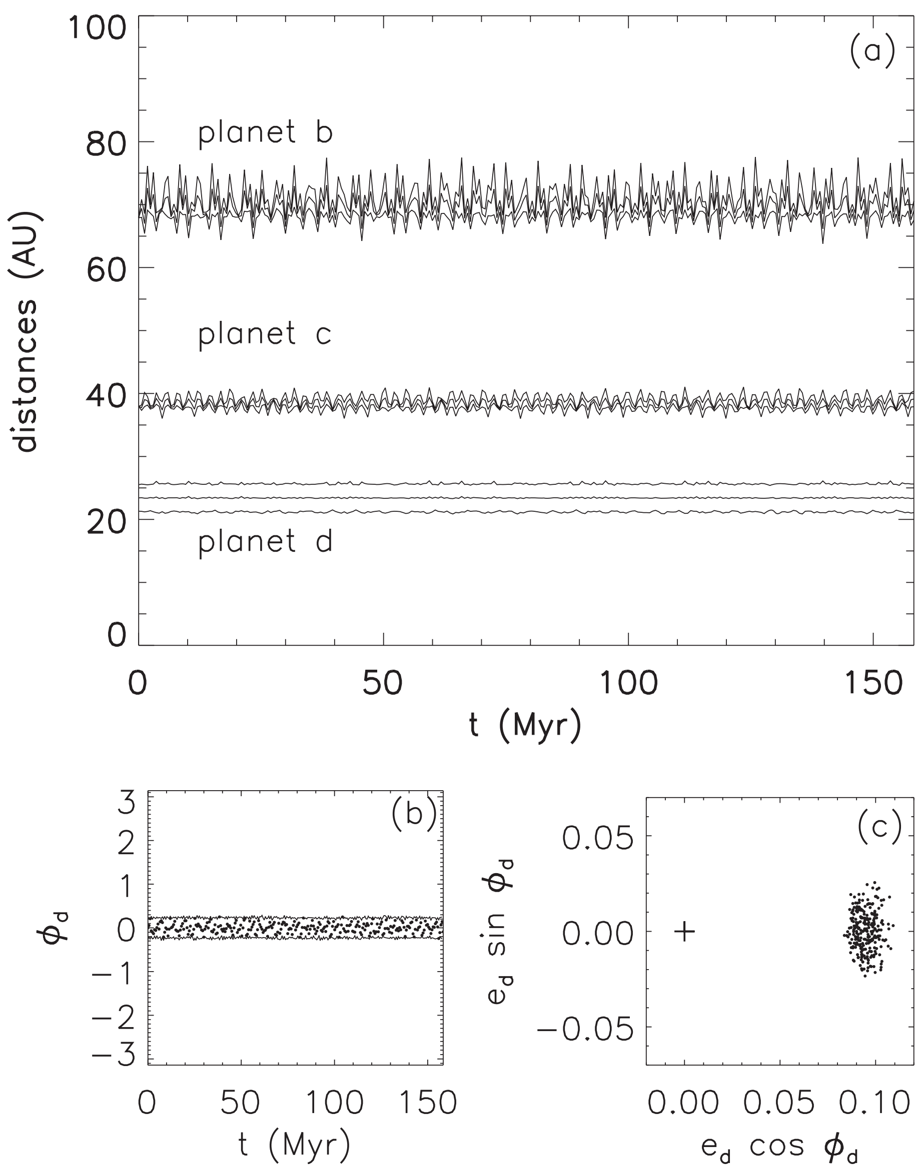

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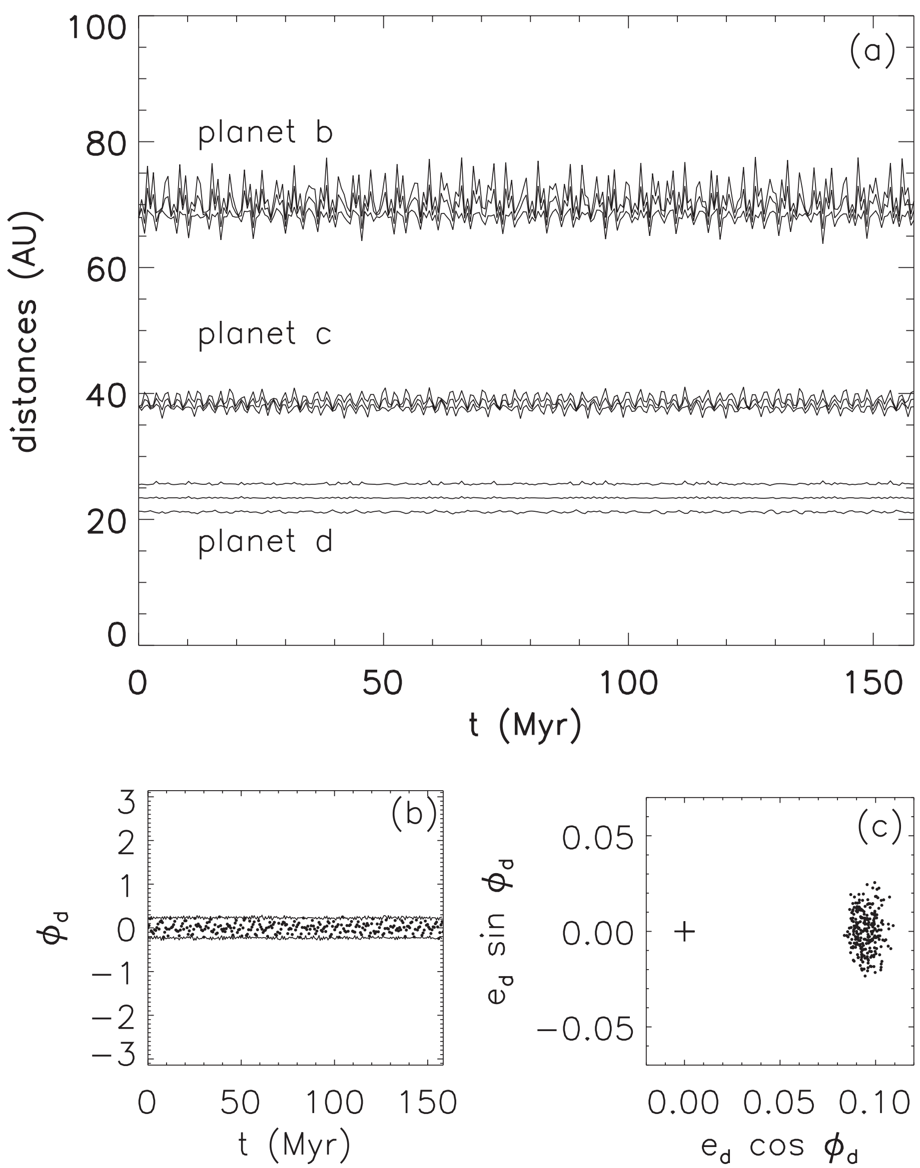

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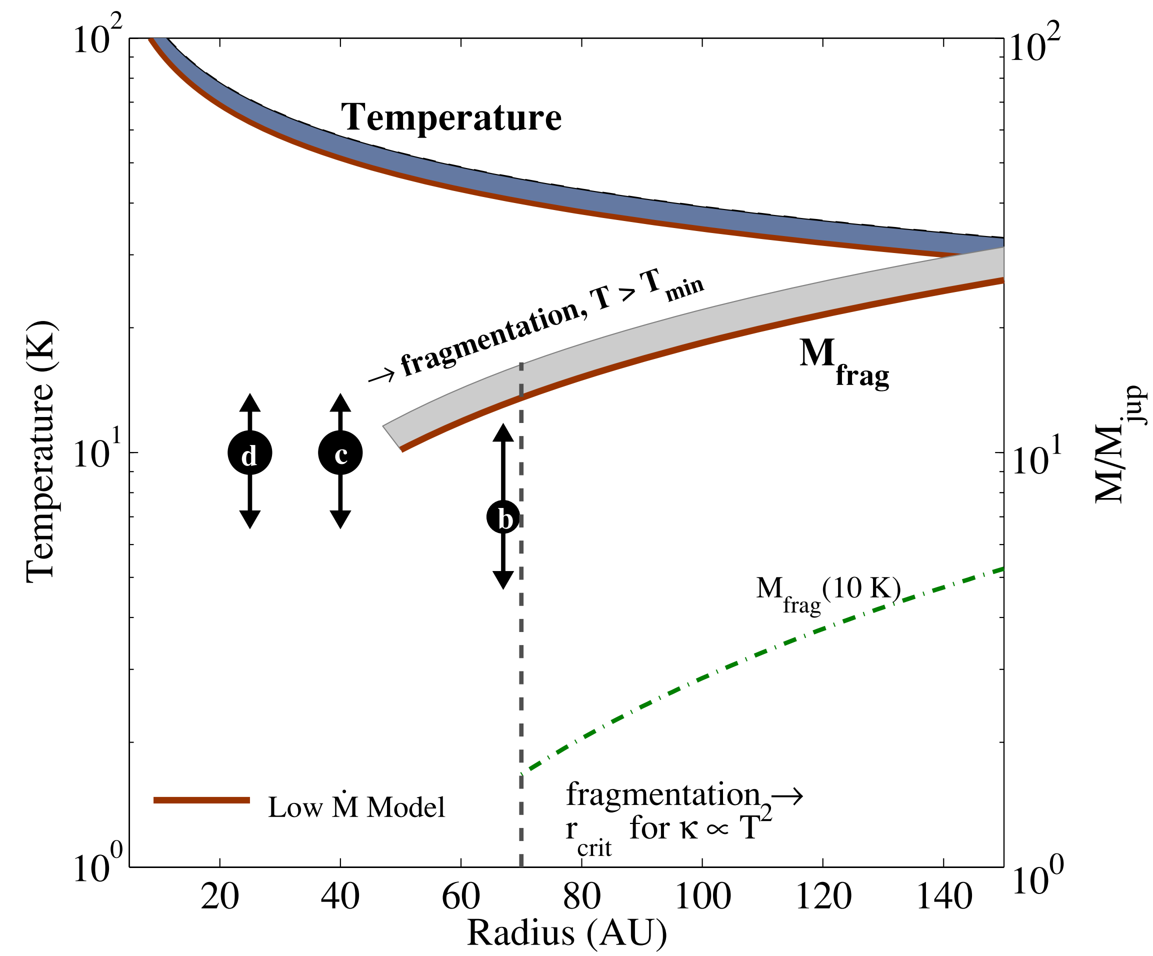
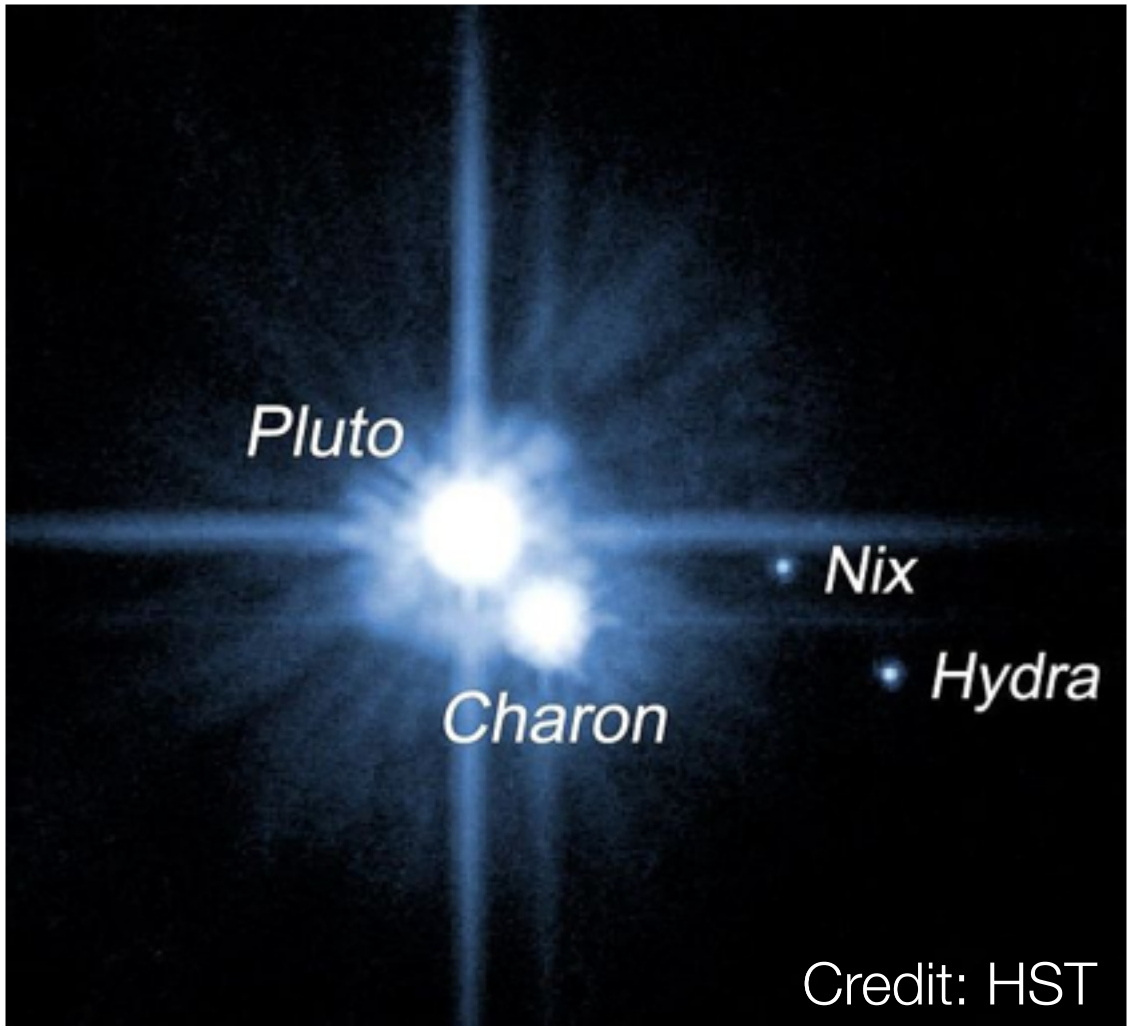
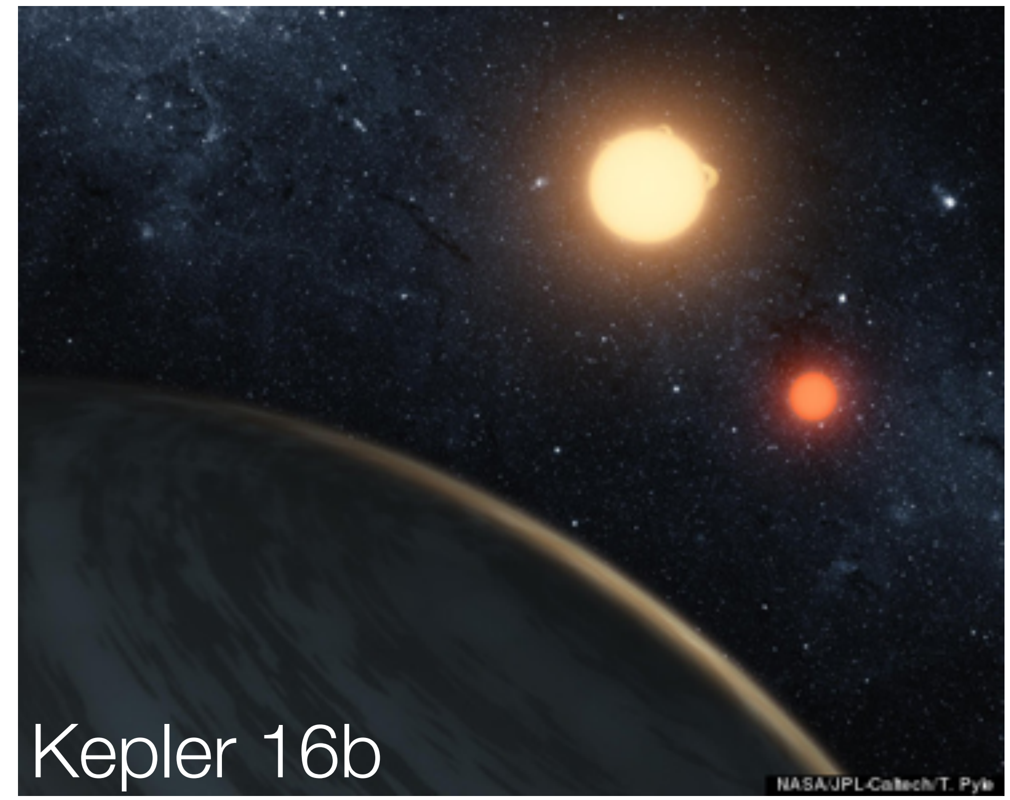


Kratter, Murray-Clay, Youdin 2010

- Dynamical modeling shows that low masses and resonant high mass configurations are stable
- Formation models prefer low masses or migration scenarios that don't favor resonance

Example II: The most famous circumbinary system





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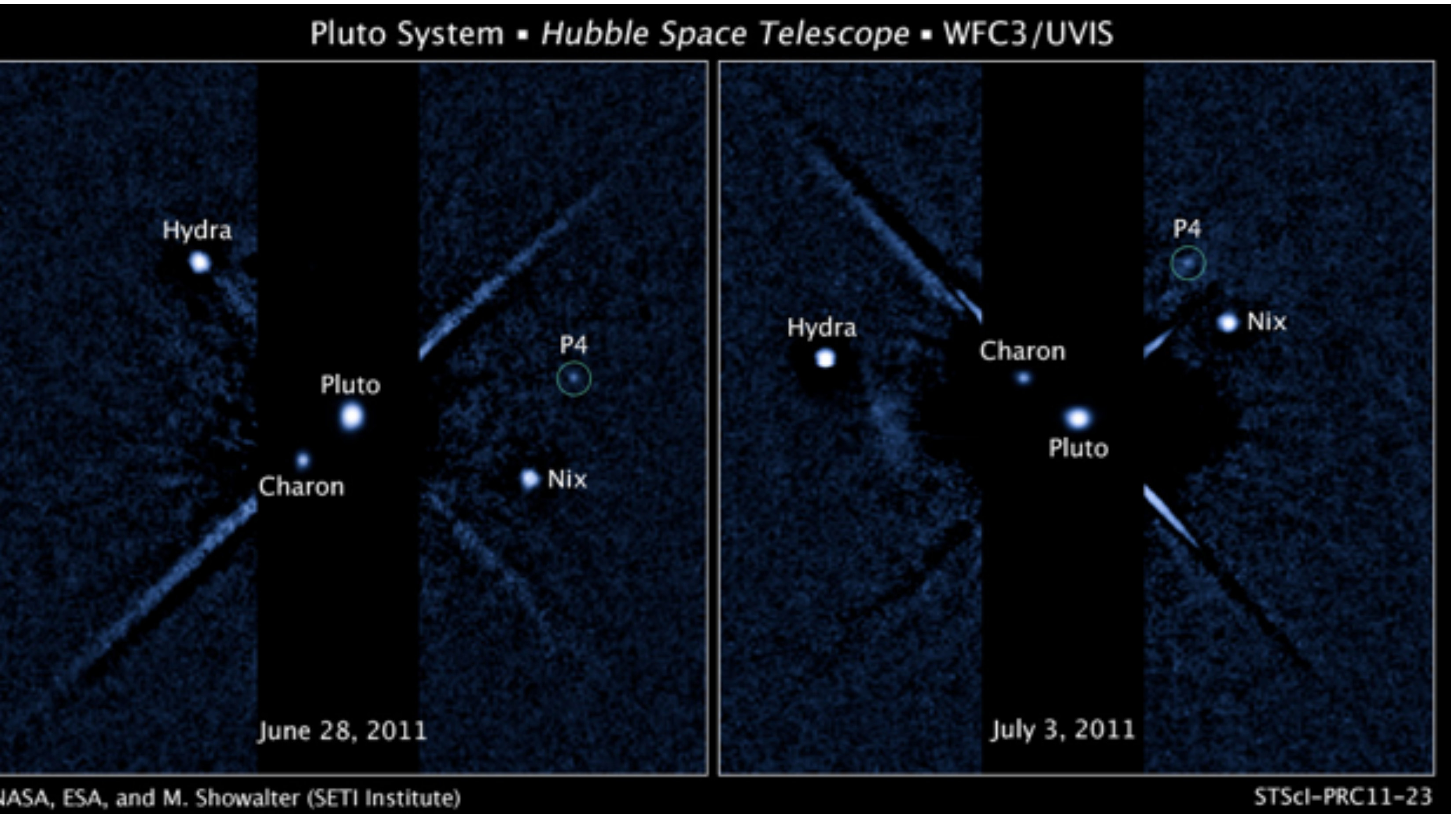
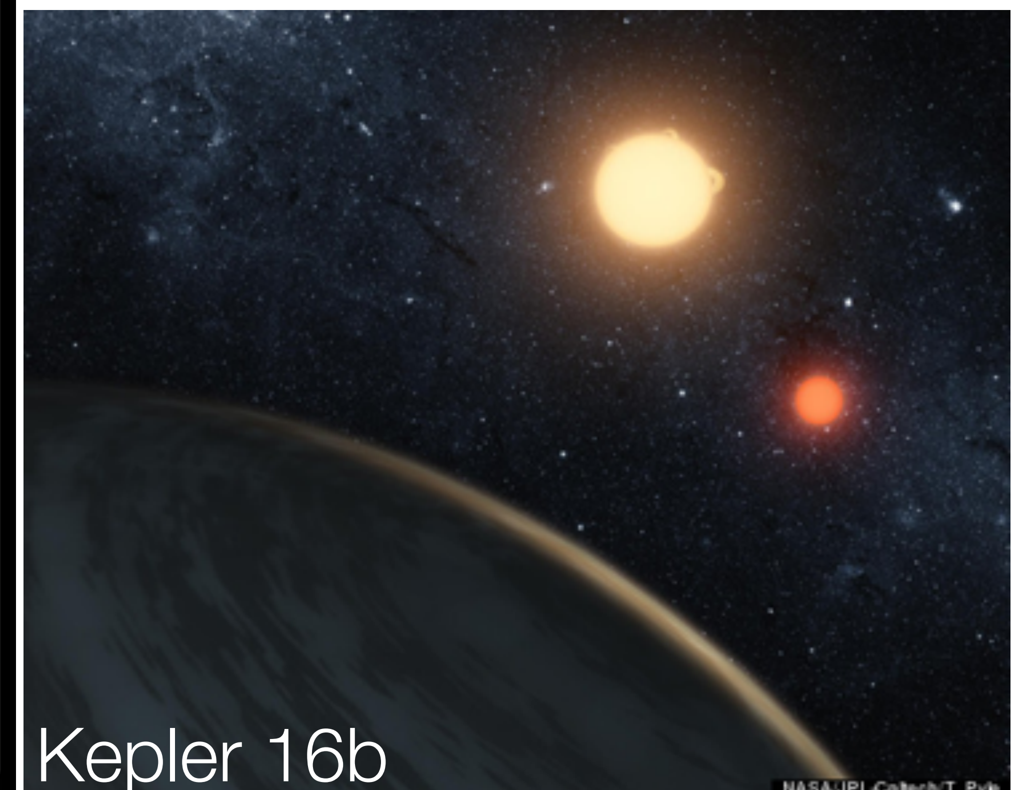


Credit: HST



Kepler 16b

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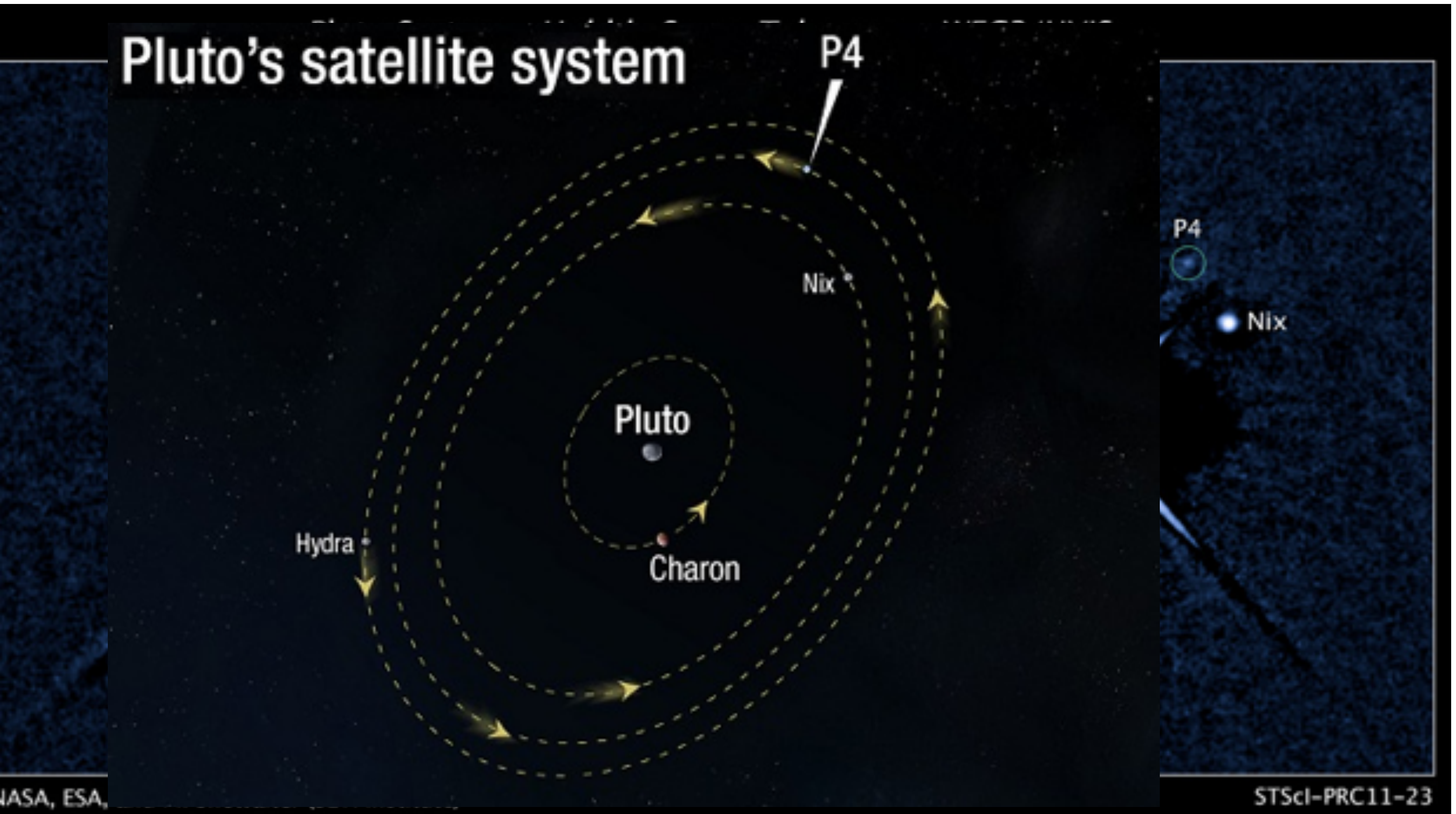

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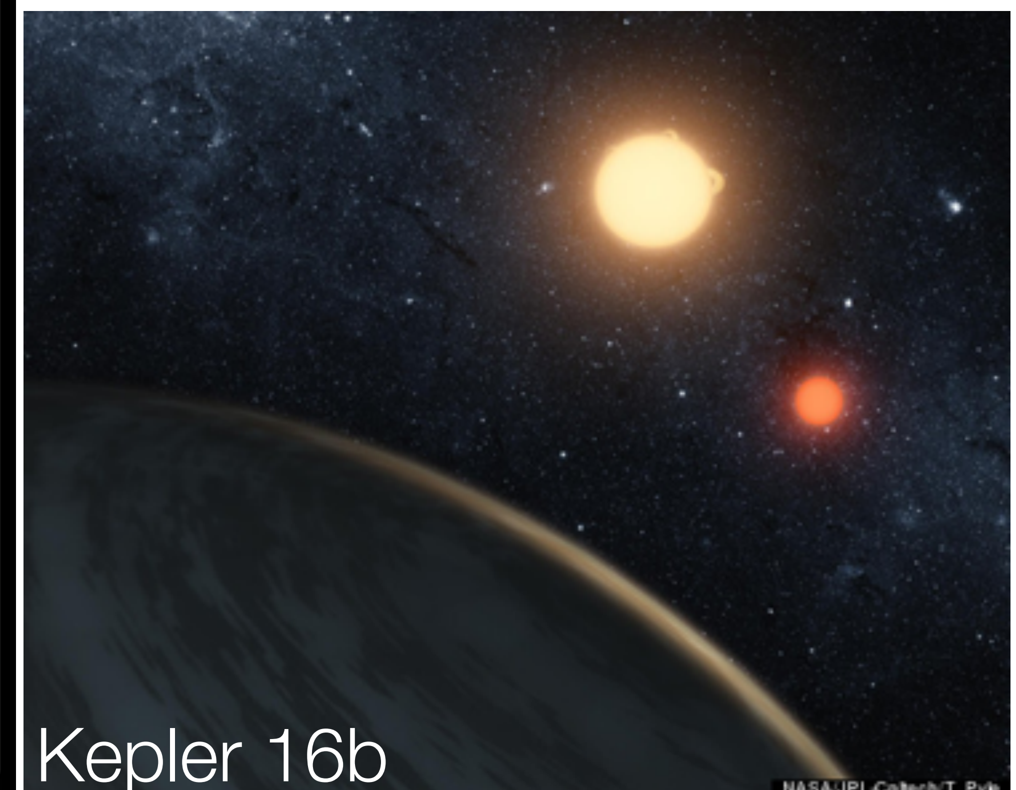

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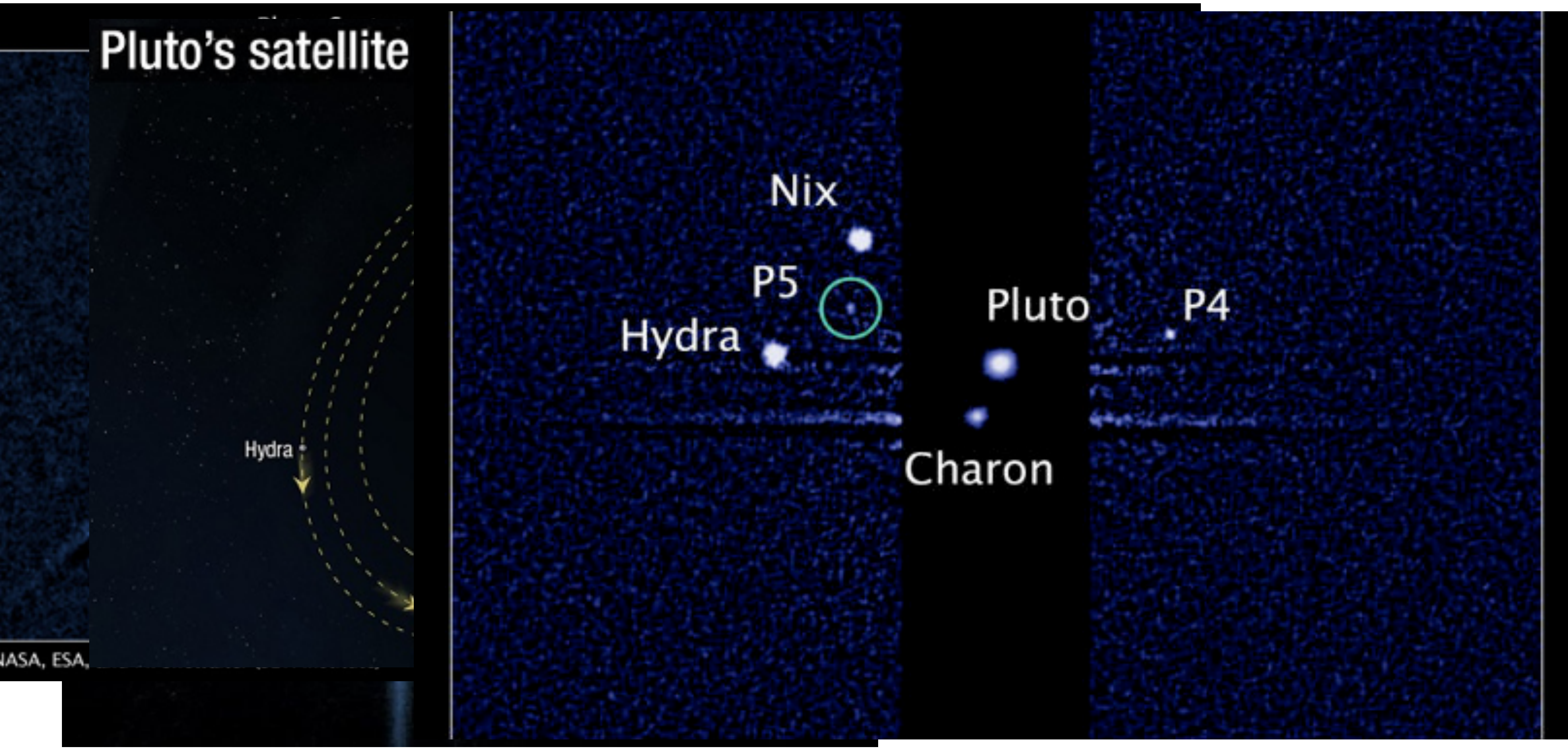
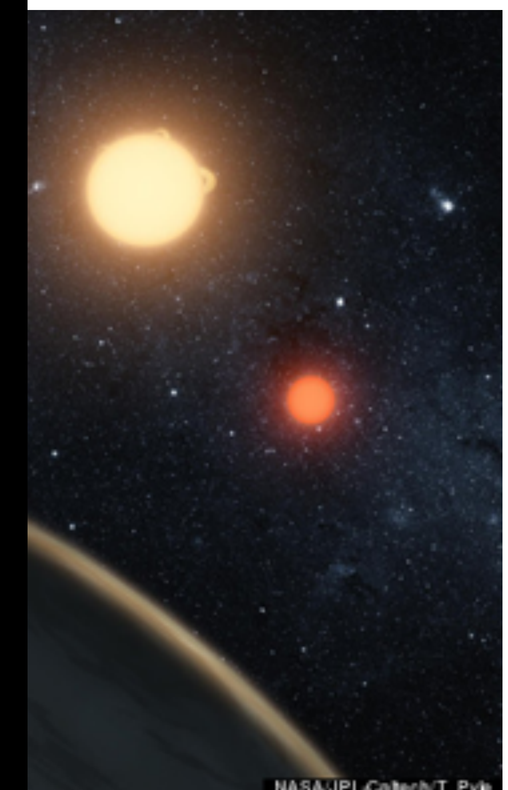
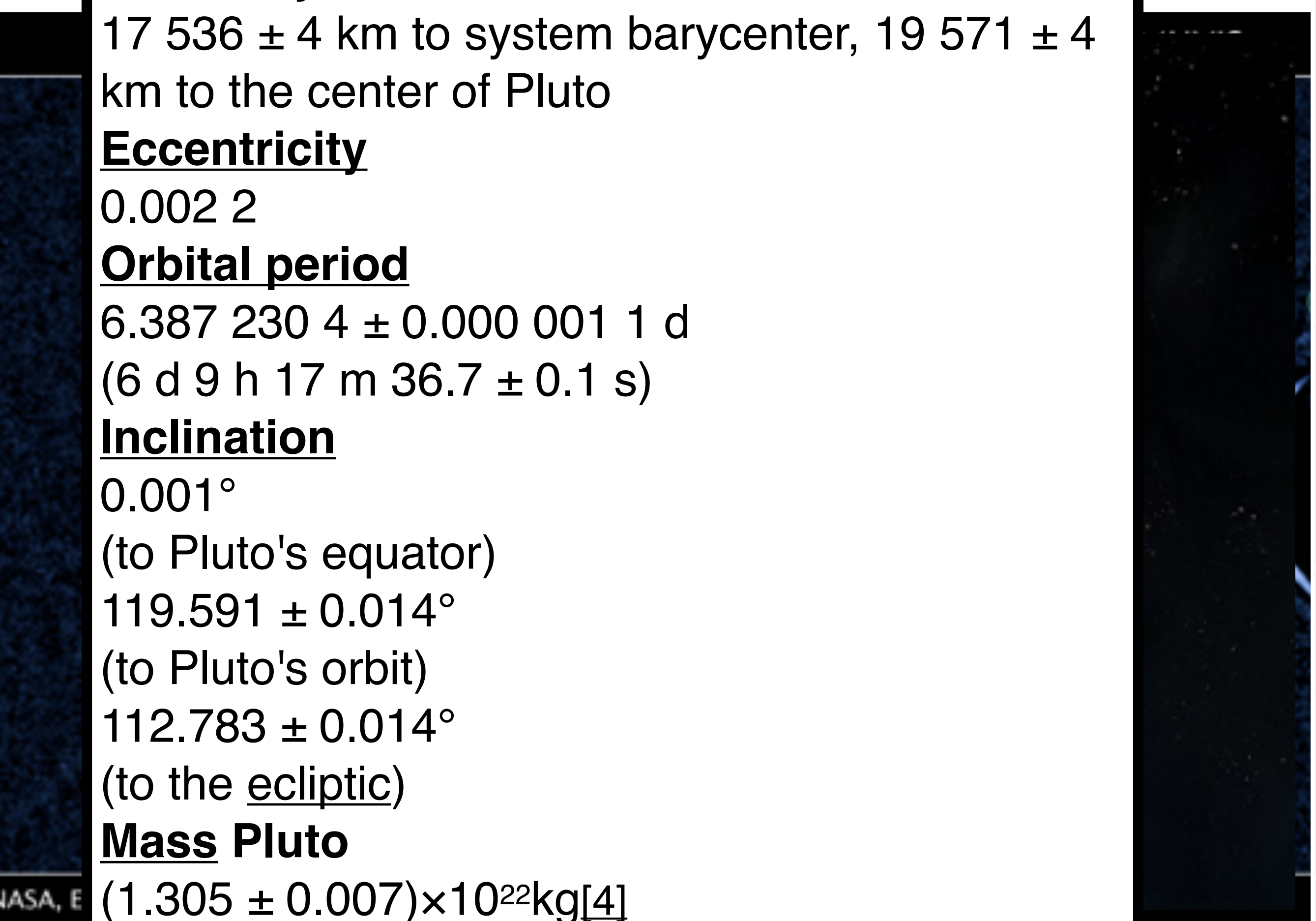
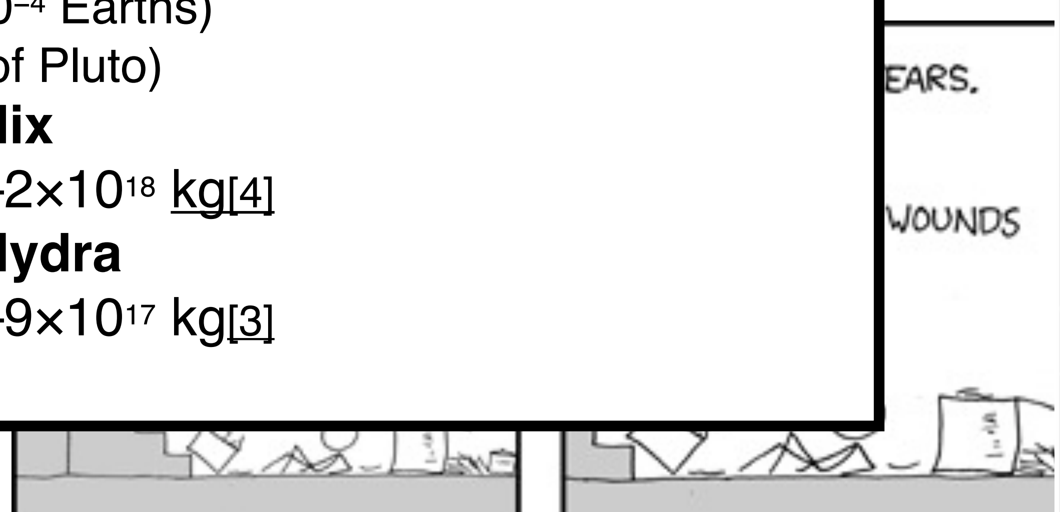




Pluto-Charon





Semi-major axis

17 536 ± 4 km to system barycenter, 19 571 ± 4 km to the center of Pluto

Eccentricity

0.002 2

Orbital period

6.387 230 4 ± 0.000 001 1 d
(6 d 9 h 17 m 36.7 ± 0.1 s)

Inclination

0.001°
(to Pluto's equator)
119.591 ± 0.014°
(to Pluto's orbit)
112.783 ± 0.014°
(to the ecliptic)

Mass Pluto

(1.305 ± 0.007)×10^{22}kg[4]

Mass Charon

(1.52 ± 0.06)×10^{21} kg[2]
(2.54×10^{-4} Earths)
(11.6% of Pluto)

Mass Nix

5×10^{16}–2×10^{18} kg[4]

Mass Hydra

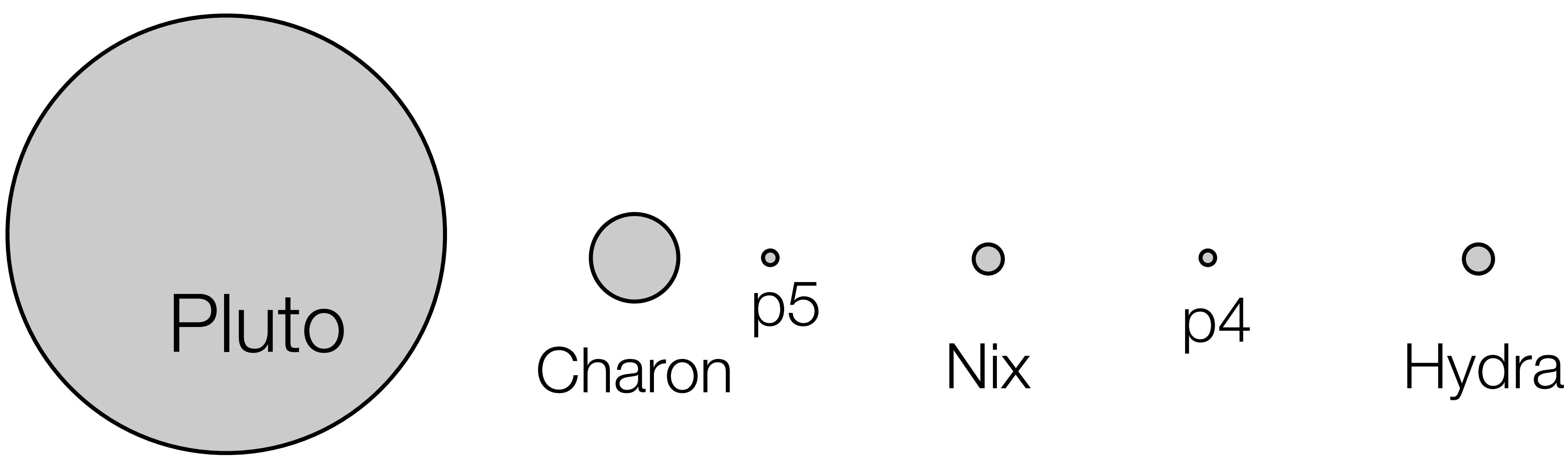
1×10^{17}–9×10^{17} kg[3]

Kepler 16b

Parameter	Value and Uncertainty
Star A	
Mass, M_A ($M_\odot$)	$0.6897^{+0.0035}_{-0.0034}$
Radius, R_A ($R_\odot$)	$0.6489^{+0.0013}_{-0.0013}$
Mean Density, ρ_A (g cm^{-3})	$3.563^{+0.017}_{-0.016}$
Surface Gravity, $\log g_A$ (cgs)	$4.6527^{+0.0017}_{-0.0016}$
Effective Temperature, $T_{\rm eff}$ (K)	4450 ± 150
Metallicity, [m/H]	-0.3 ± 0.2
Star B	
Mass, M_B ($M_\odot$)	$0.20255^{+0.00066}_{-0.00065}$
Radius, R_B ($R_\odot$)	$0.22623^{+0.00059}_{-0.00053}$
Mean Density, ρ_B (g cm^{-3})	$24.69^{+0.13}_{-0.15}$
Surface Gravity, $\log g_B$ (cgs)	$5.0358^{+0.0014}_{-0.0017}$
Planet b	
Mass, M_b ($M_{\rm Jupiter}$)	$0.333^{+0.016}_{-0.016}$
Radius, R_b ($R_{\rm Jupiter}$)	$0.7538^{+0.0026}_{-0.0023}$
Mean Density, ρ_b (g cm^{-3})	$0.964^{+0.047}_{-0.046}$
Surface Gravity, g_b (m s^{-2})	$14.52^{+0.70}_{-0.69}$
Binary star orbit	
Period, P_1 (day)	$41.079220^{+0.000078}_{-0.000077}$
Semi-major axis length, a_1 (AU)	$0.22431^{+0.00035}_{-0.00034}$
Eccentricity, e_1	$0.15944^{+0.00061}_{-0.00062}$
Argument of Periapse, ω_1 (deg)	$263.464^{+0.026}_{-0.027}$
Mean Longitude, λ_1 (deg)	$92.3520^{+0.0011}_{-0.0011}$
Inclination, i_1 (deg)	$90.3401^{+0.0016}_{-0.0019}$
Longitude of Nodes, Ω_1 (deg)	$\equiv 0$ (by definition)
Circumbinary planet orbit	
Period, P_2 (day)	$228.776^{+0.020}_{-0.037}$
Semi-major axis length, a_2 (AU)	$0.7048^{+0.0011}_{-0.0011}$
Eccentricity, e_2	$0.0069^{+0.0010}_{-0.0015}$

Decomposing the Pluto and Charon System

- Understanding **stability in a multi-planet, binary system** not possible through any 3 body arguments
  - Stable as test particles about a binary
  - Any two satellites are stable about P-C barycenter
  - Three satellites about P-C barycenter are stable for some masses
  - Full System (numerically)



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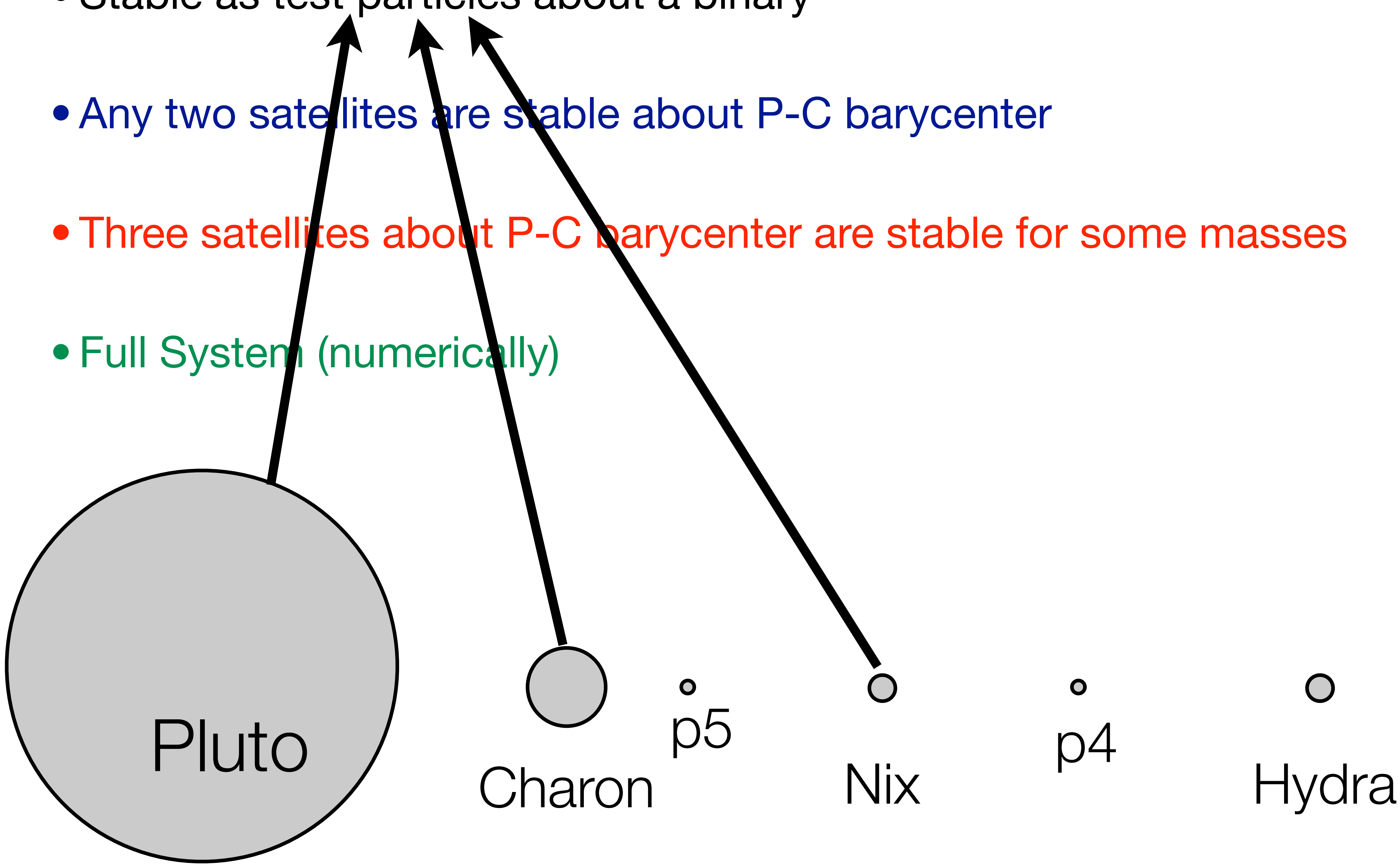

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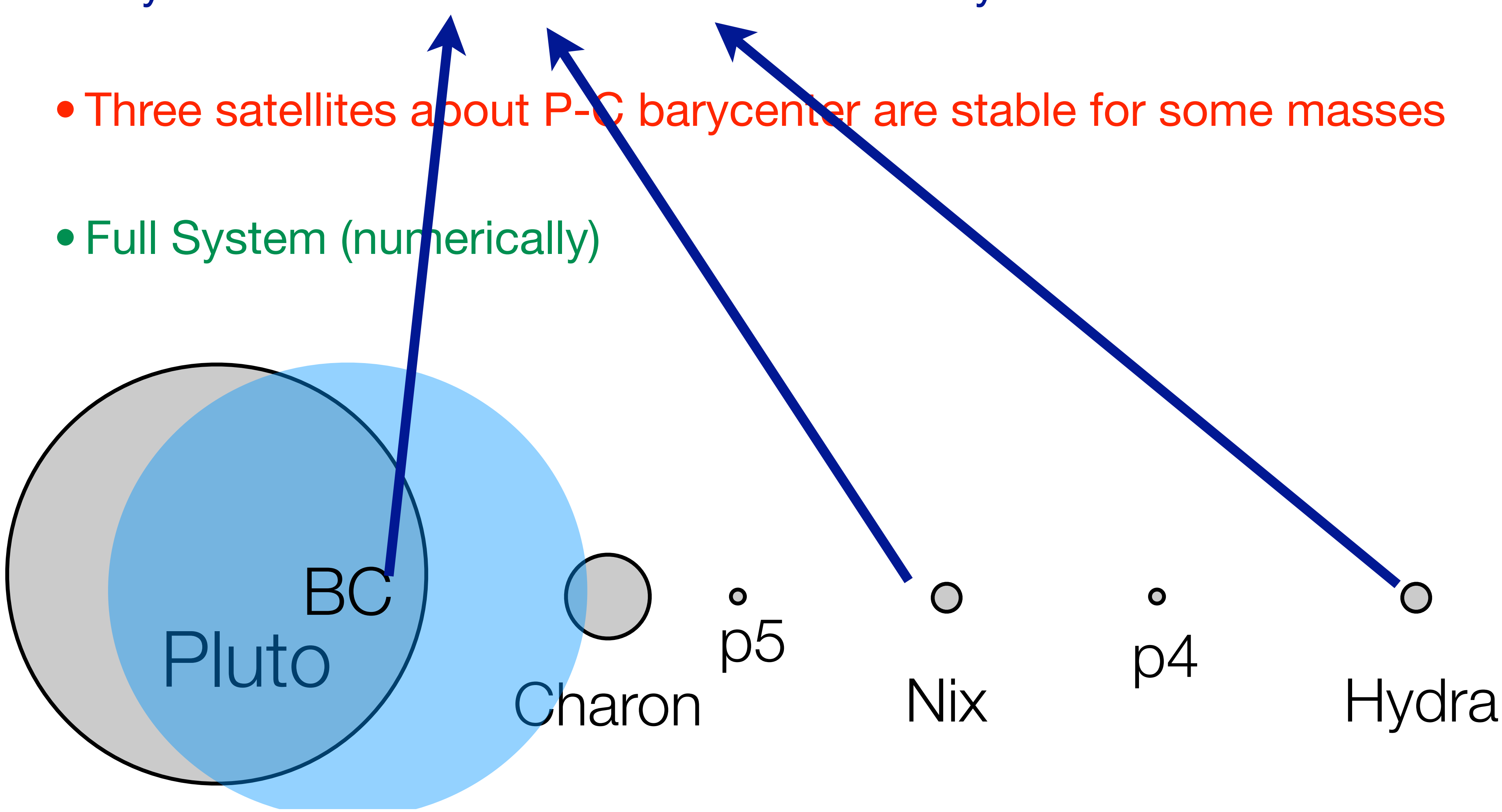

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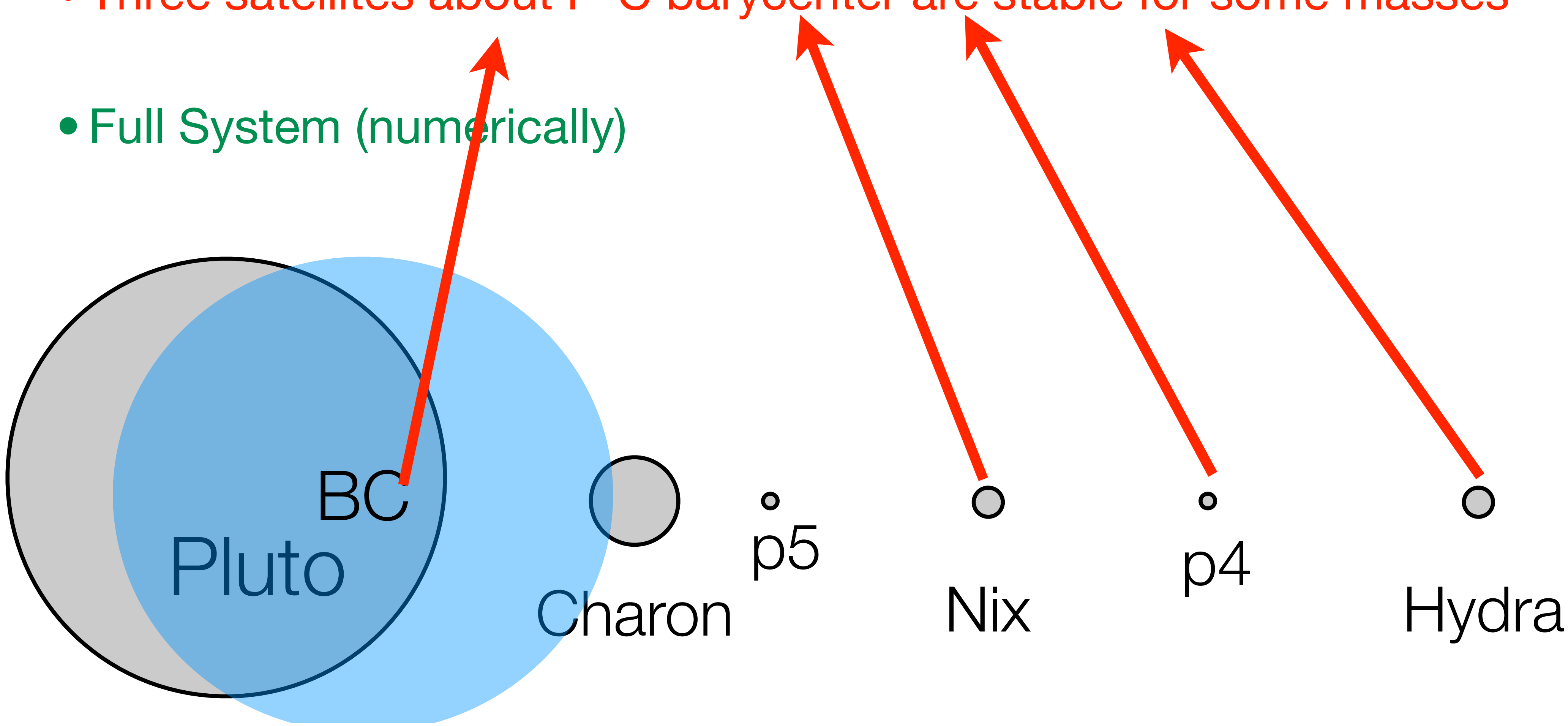

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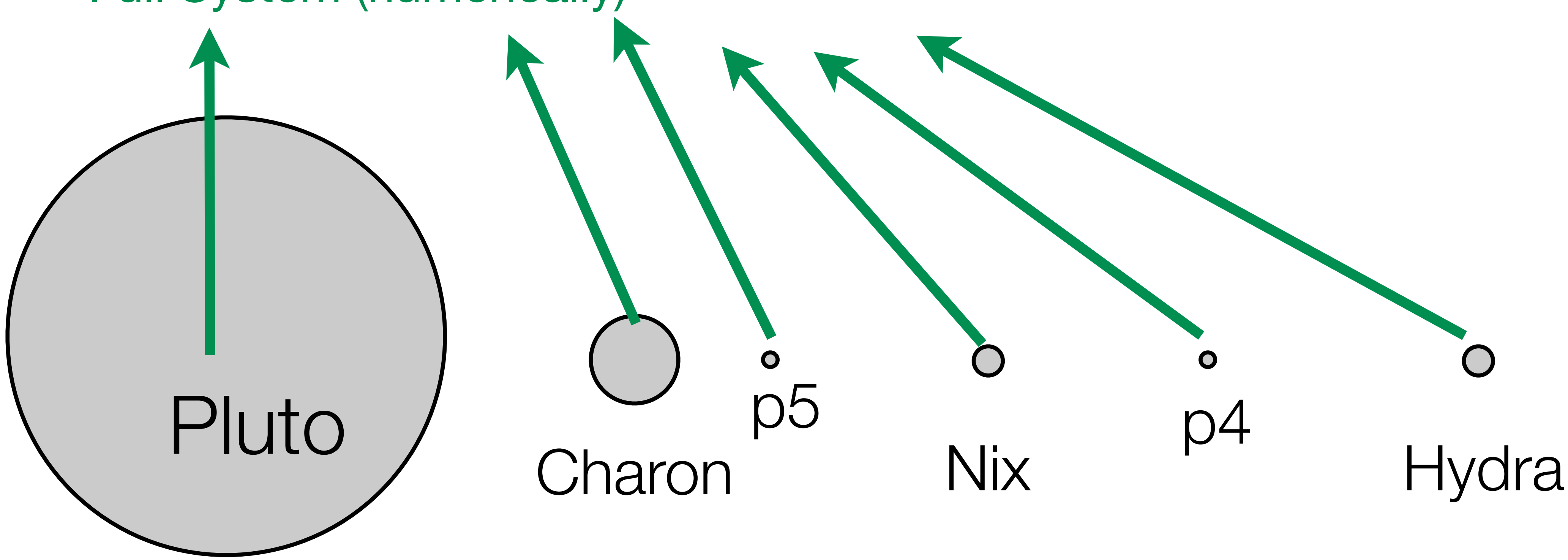

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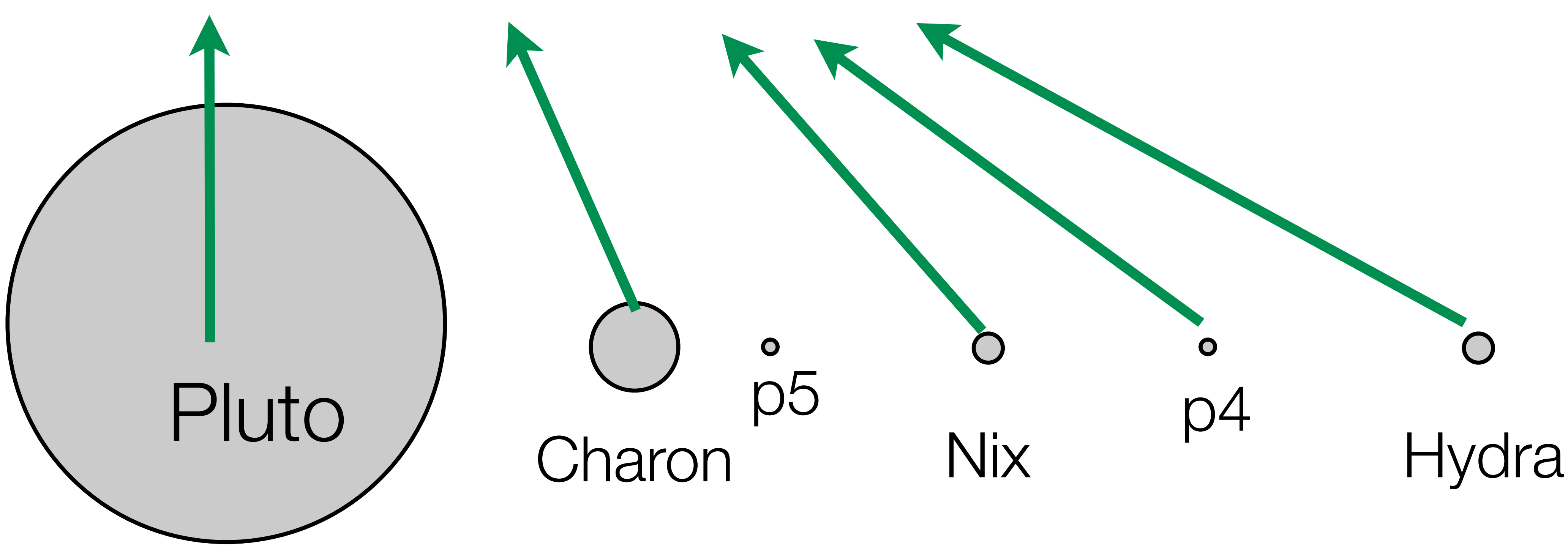

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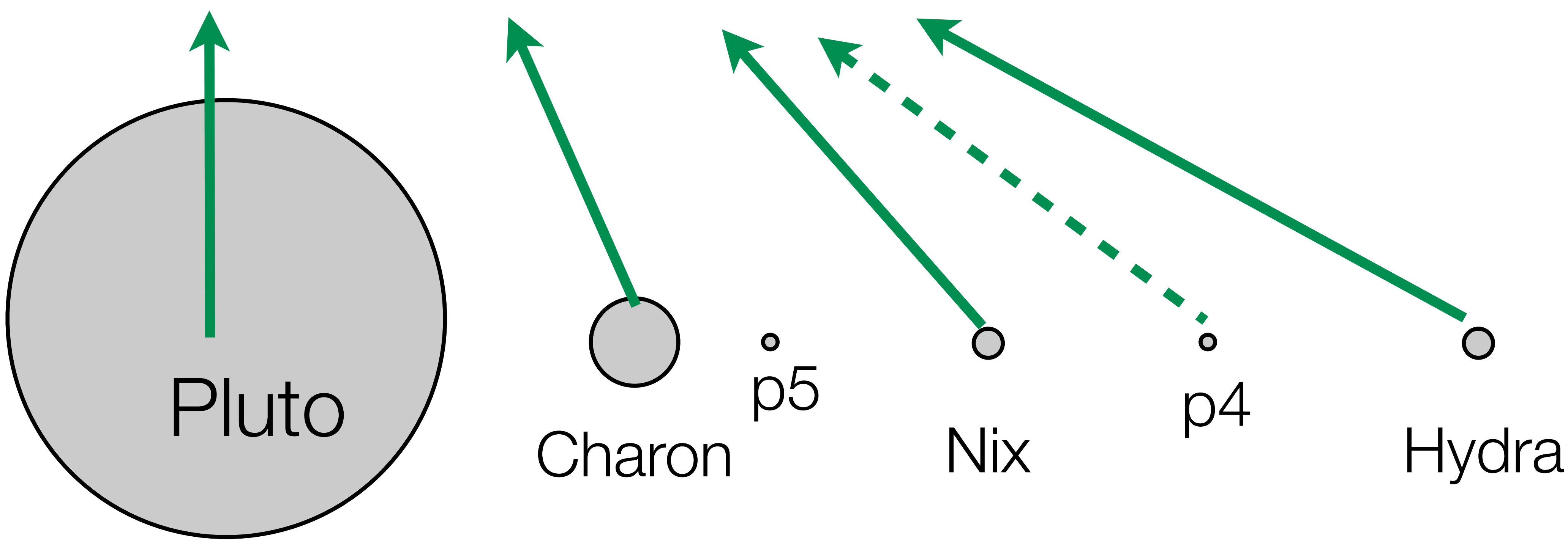
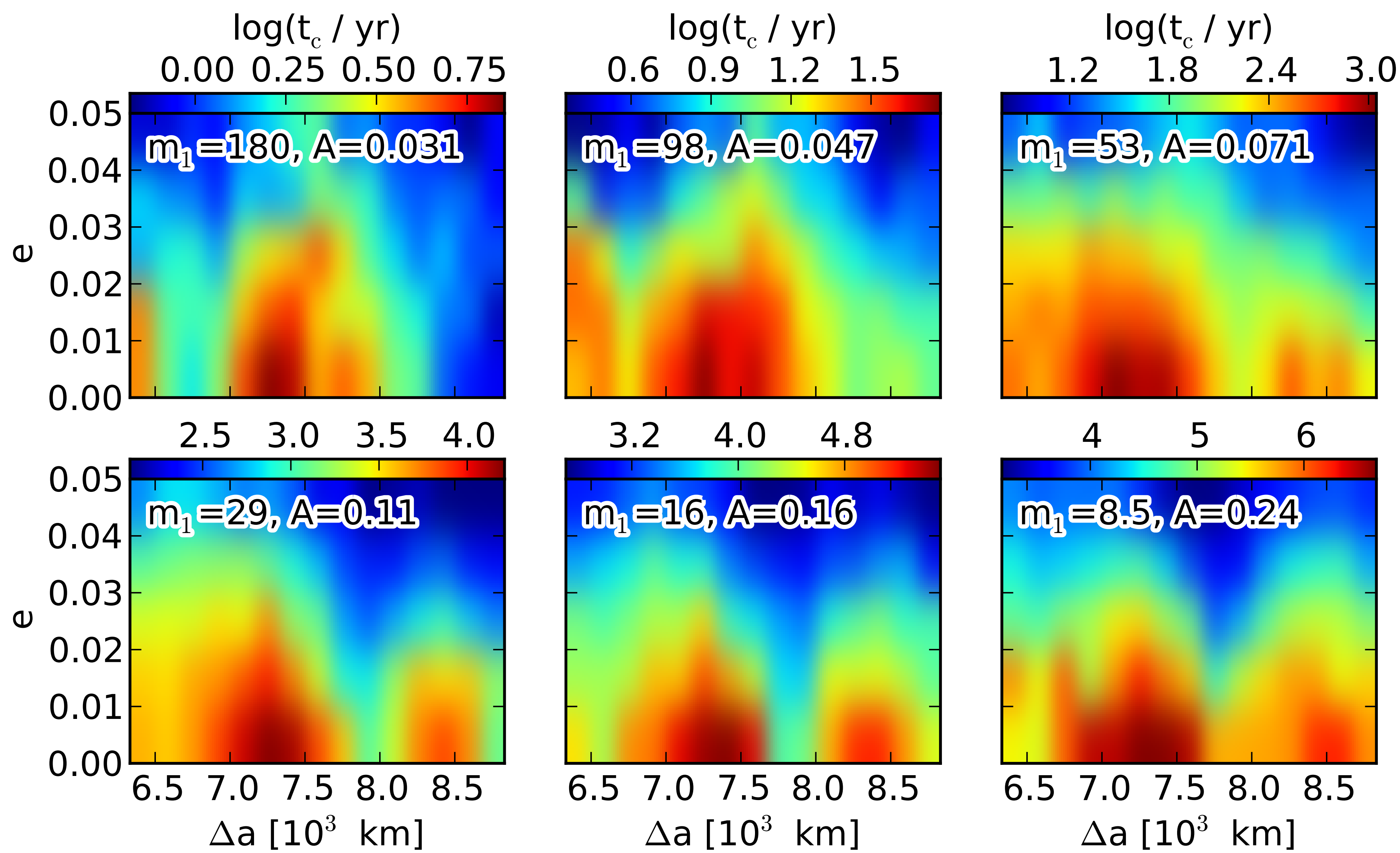


Circumbinary Multi-planet stability

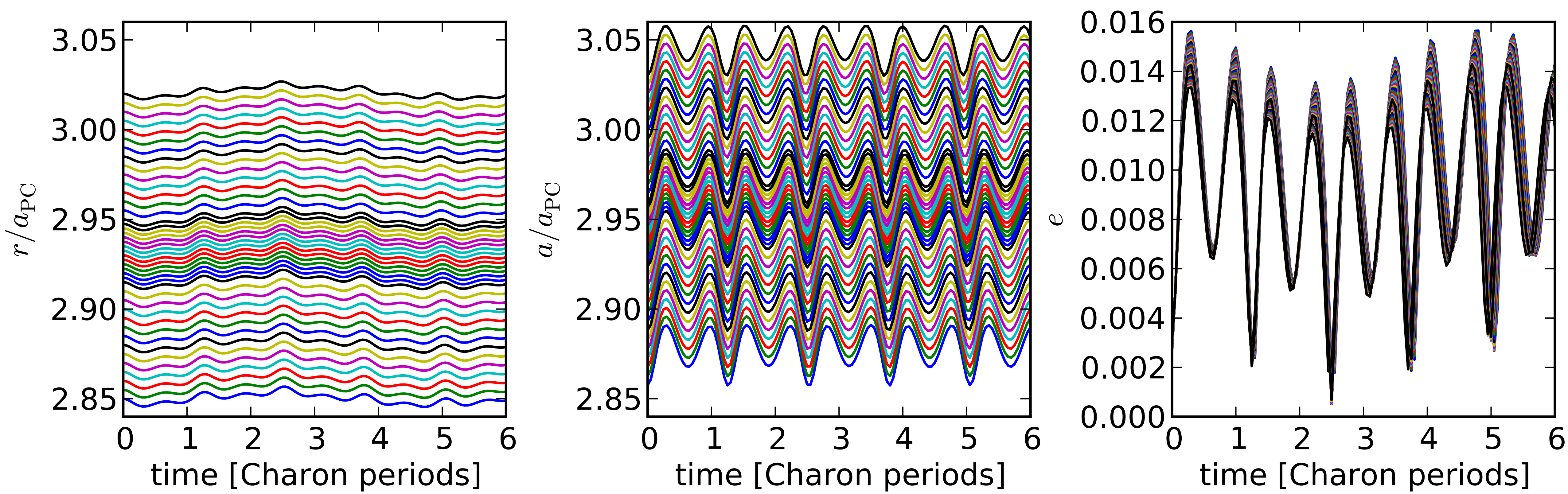
- Locations fixed by observations, so vary masses of Nix & Hydra
- populate with potential orbits of new satellite (test particle) and run for 1 billion Pluto-Charon orbits
- Examine lifetime as a function of semi-major axis and eccentricity



Youdin, Kratter, & Kenyon, 2012

Most Circular Orbit: Beware non-Keplerian orbits

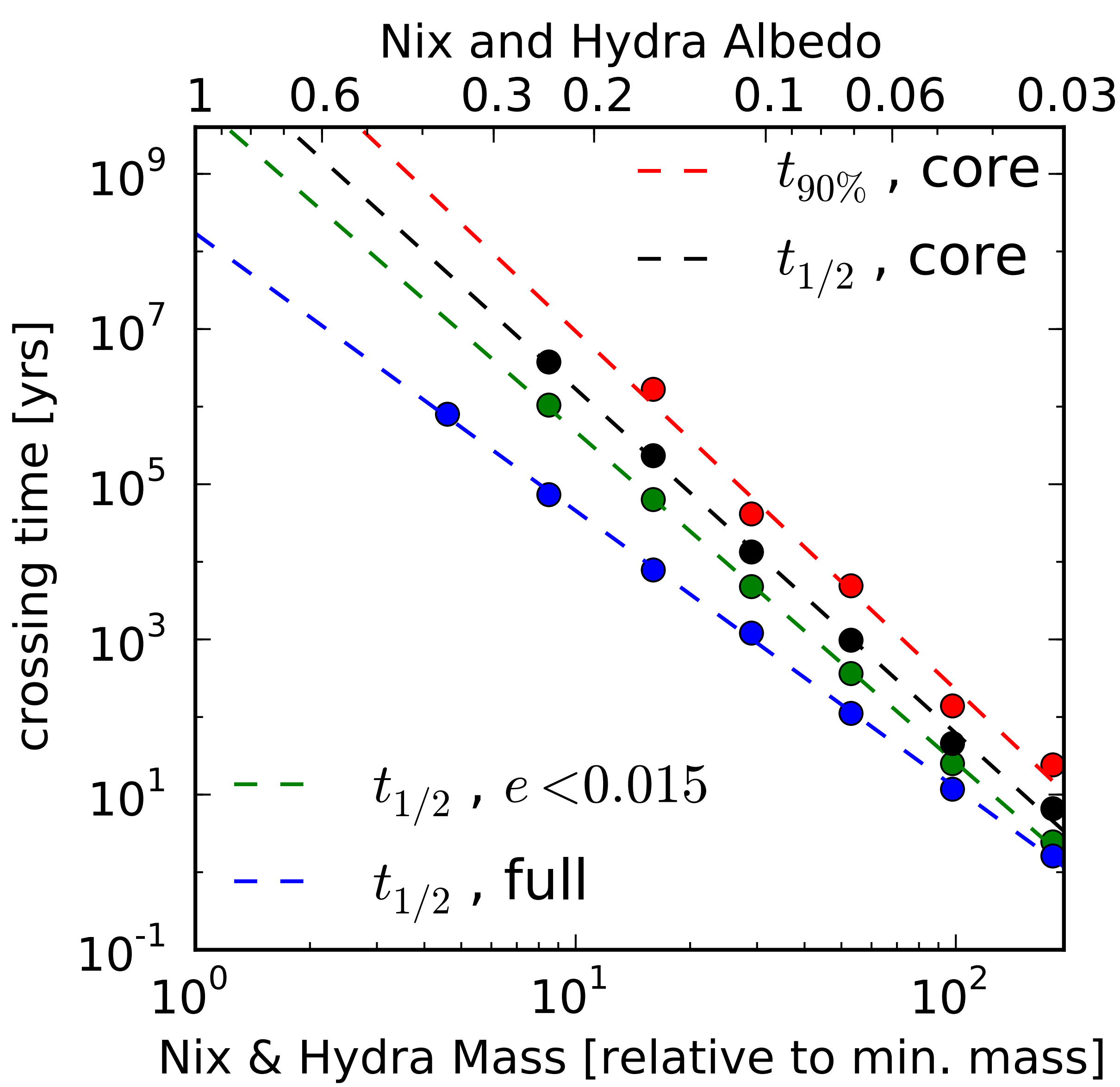
- Orbits about binary are significantly non-Keplerian *(Lee & Peale 2006)*
- To get “circular” orbits, cannot simply set e=0
- Are cold orbits more stable?



Youdin, Kratter, & Kenyon, 2012

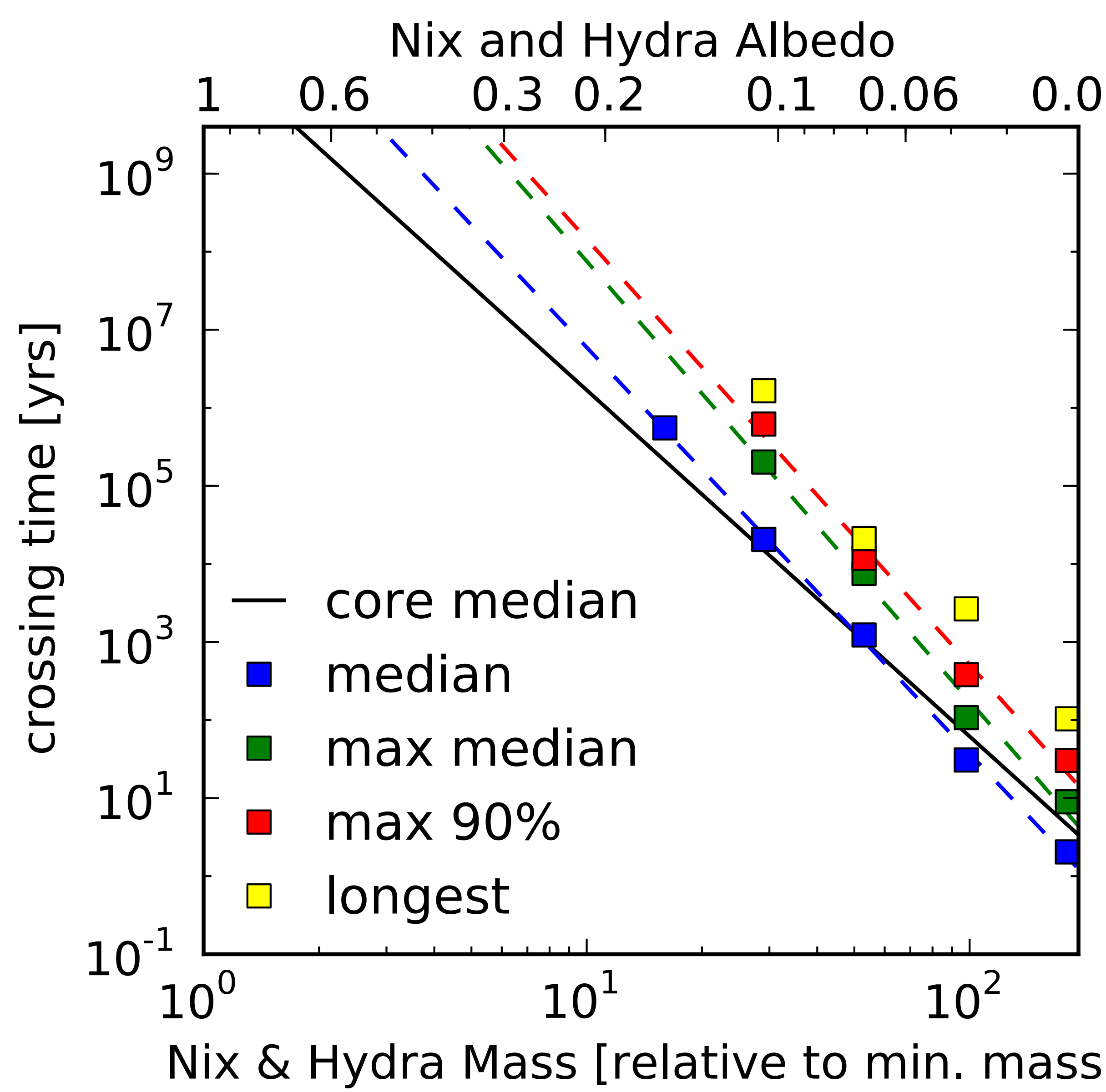
Dynamics tell us that Nix & Hydra are bright

"Keplerian"



$$\mu_{\rm Nix} \approx 6.4\times10^{-7}\rho_1 A^{-3/2}$$
$$\mu_{\rm Hyd} \approx 1.1\times10^{-6}\rho_1 A^{-3/2}$$
$$\mu_{\rm P4} \approx 2.0\times10^{-8}\rho_1 A^{-3/2}$$

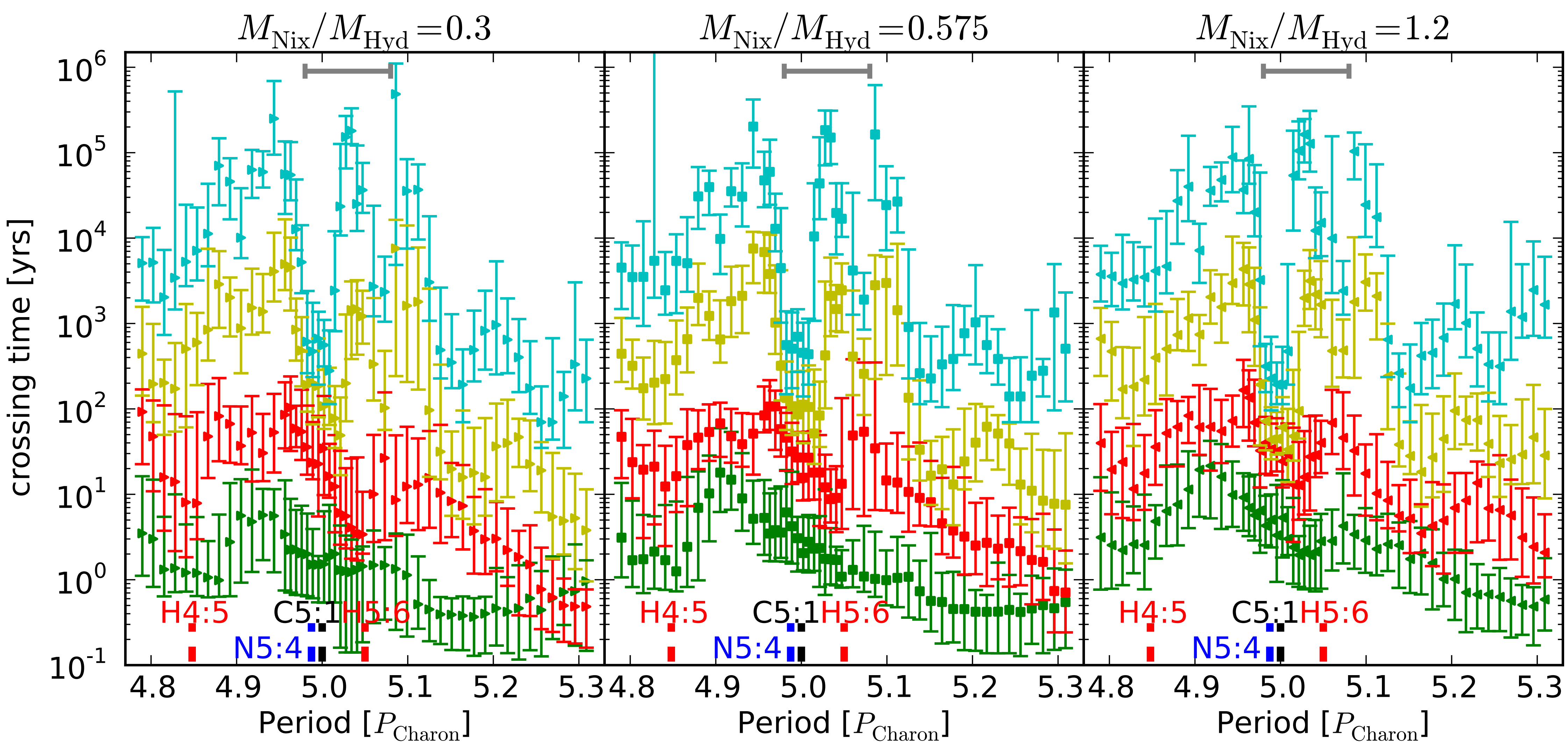
"Circular"



Youdin, Kratter, & Kenyon, 2012

Resonance destabilization

- 5:1 appears to be unstable for a range of parameters
- Eccentricity of Pluto-Charon is a relatively weak effect



Credit: Alex Parker

Youdin, Kratter, & Kenyon, 2012

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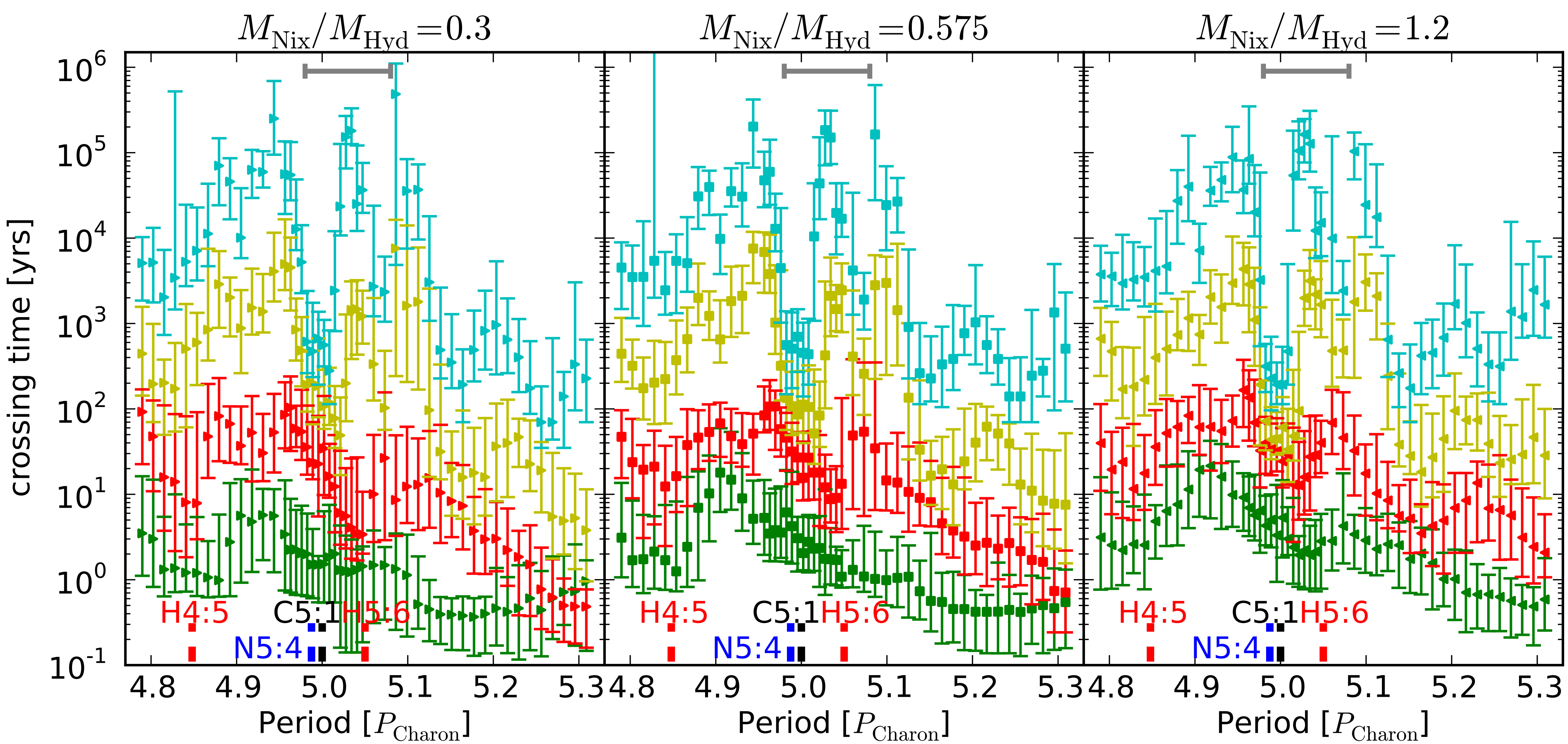
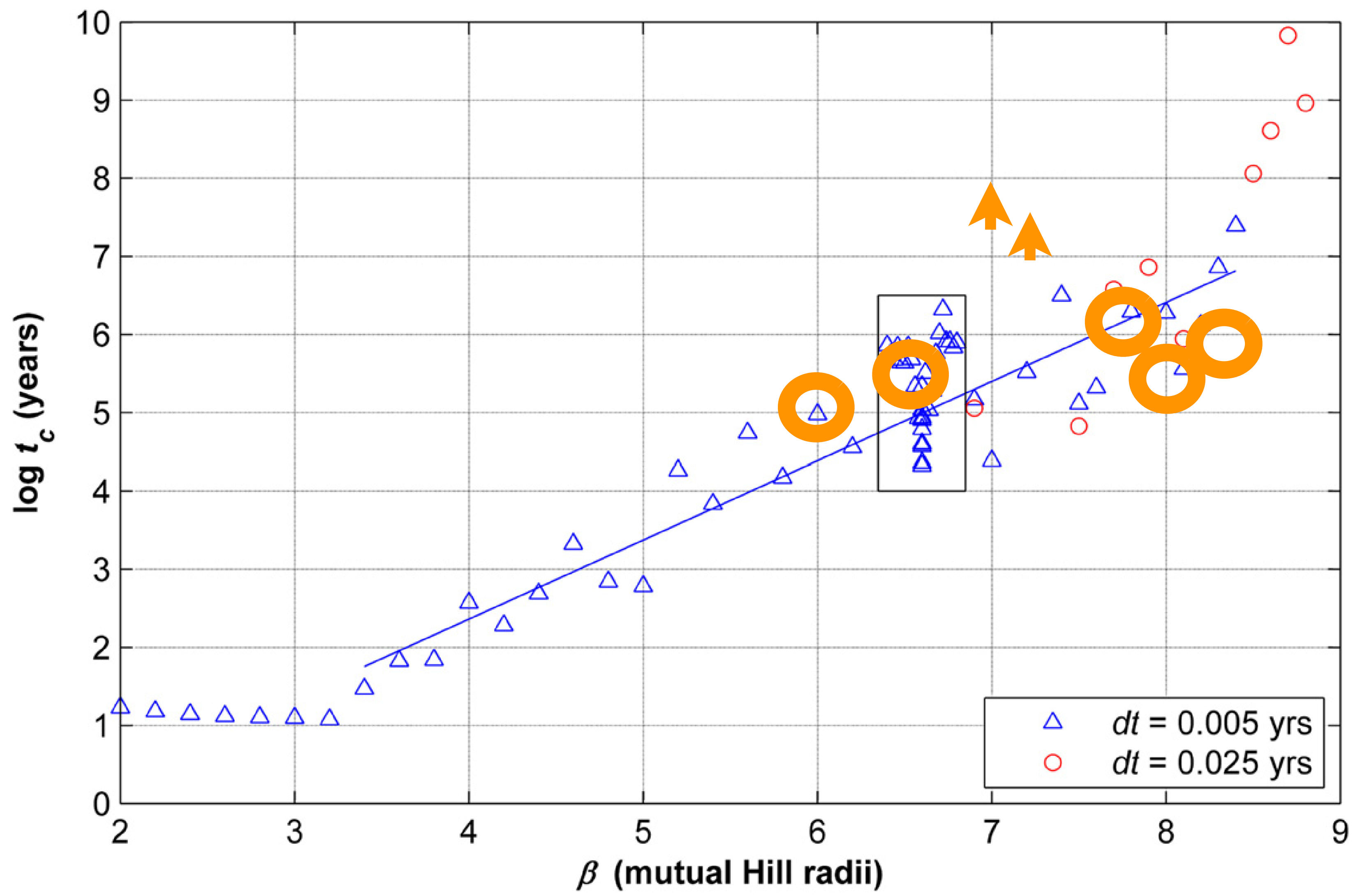


Credit: Alex Parker

Youdin, Kratter, & Kenyon, 2012

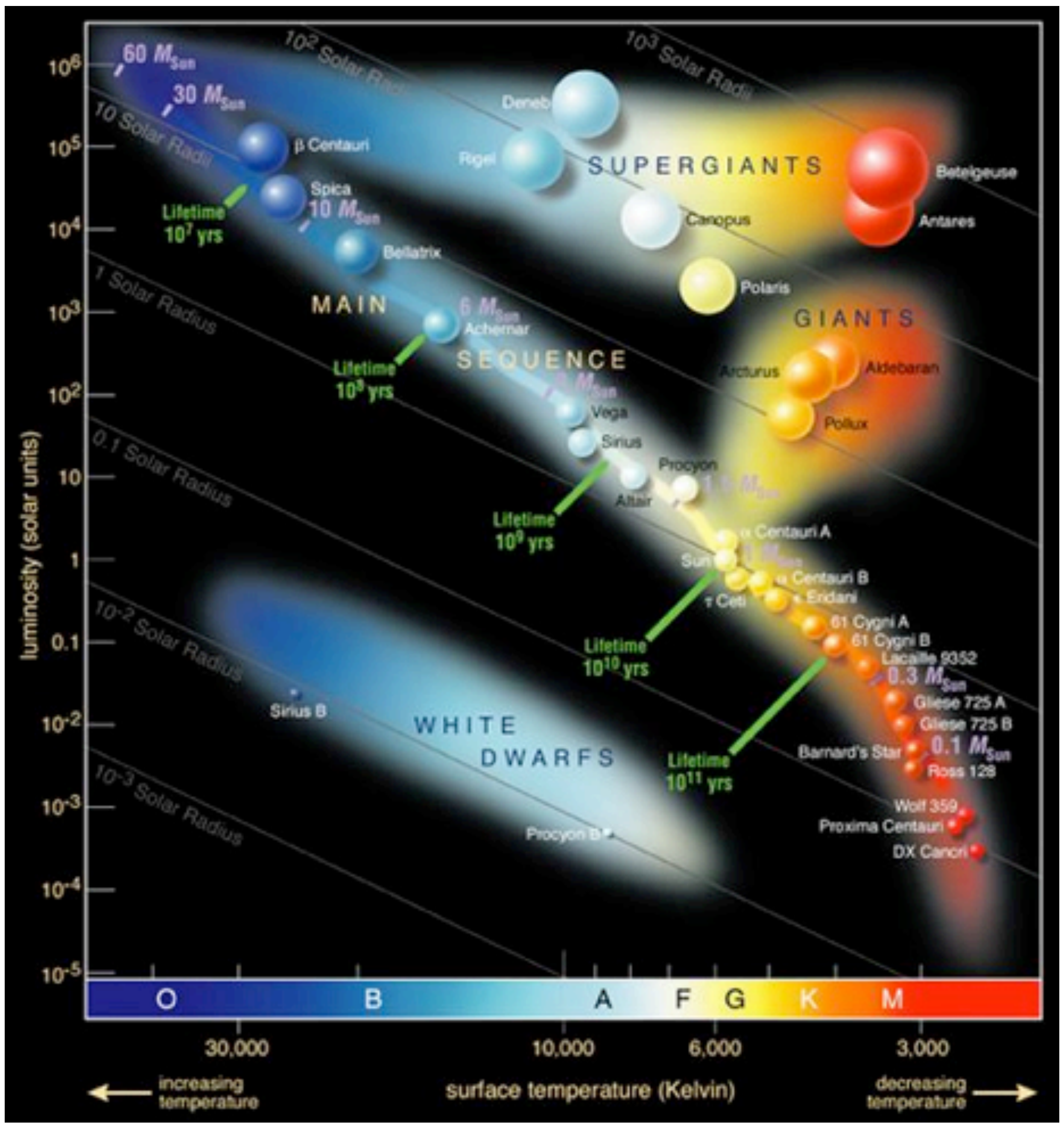
Circumbinary vs Single multi-planet stability

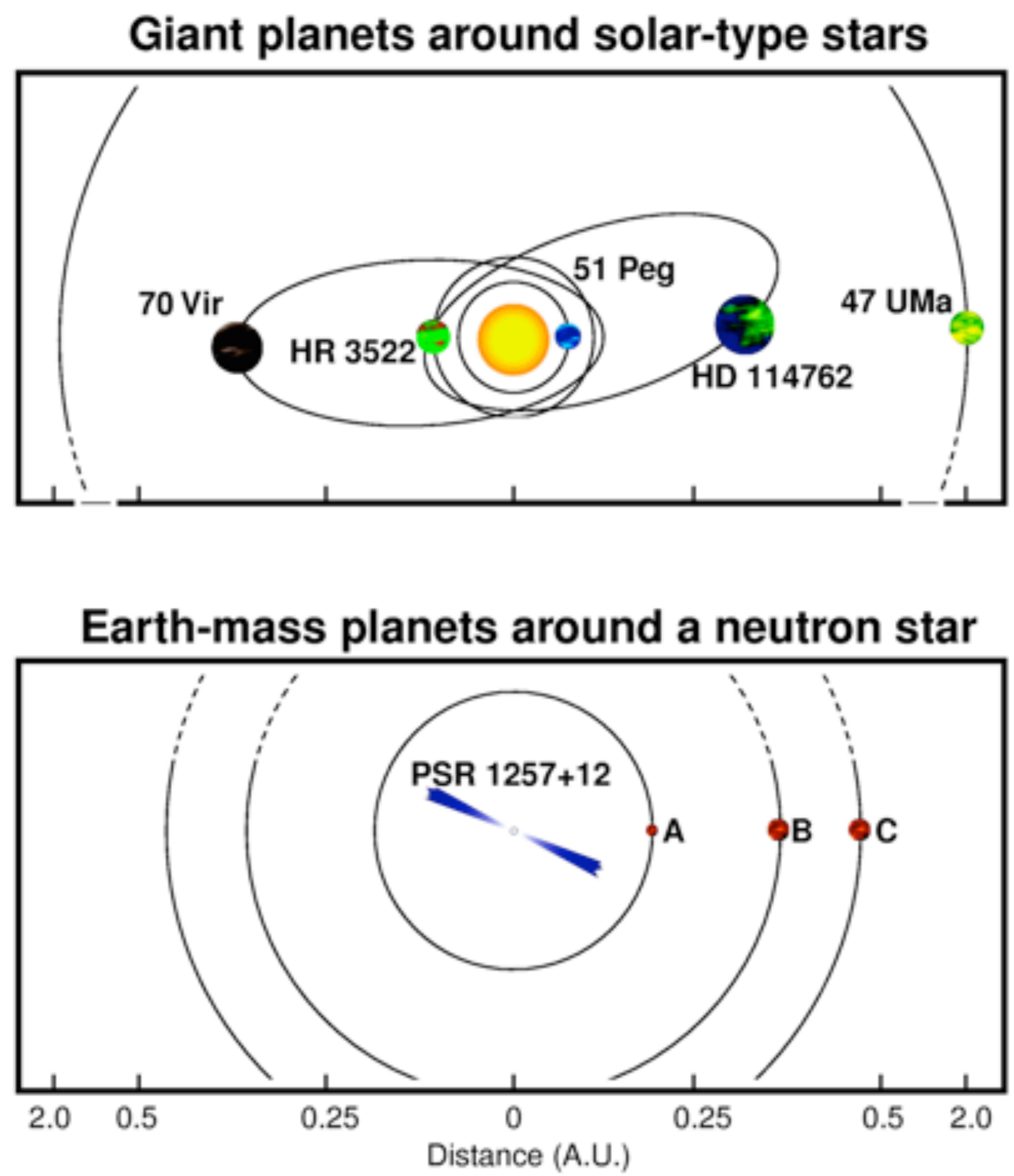
- Evidence for the role of resonances in controlling long term stability for Kepler 16 multi analog



Smith & Lissauer 2009, Kratter, Shannon, & Youdin in prep

Part III: When Stars Die



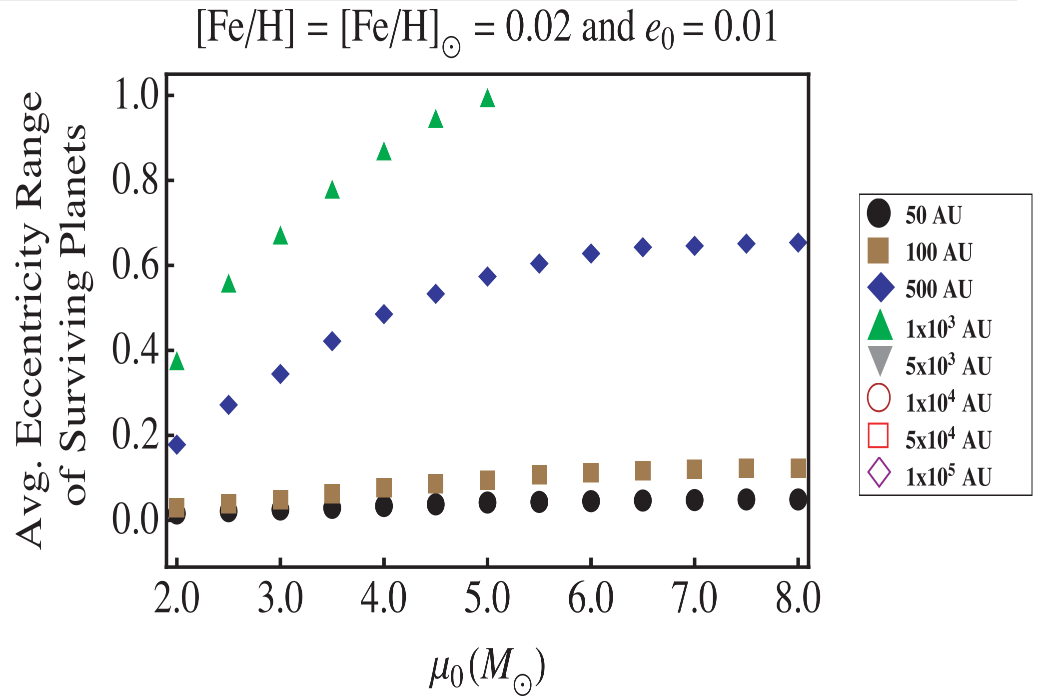


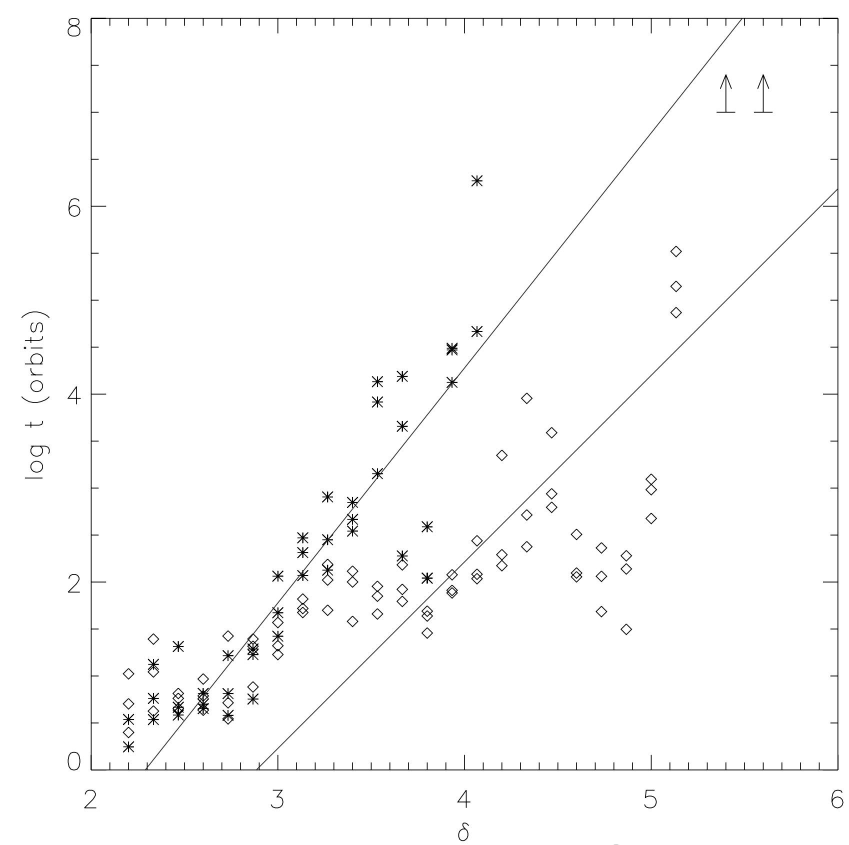
A. Wolszczan

Dynamics of Stellar Death

- Slow adiabatic mass loss conserves eccentricity, but semi-major axis grows
- Change relative spacings in multi-planet systems
- Non-constant mass loss can excite eccentricity and lead to planetary loss
- Engulfment

Veras et al 2011

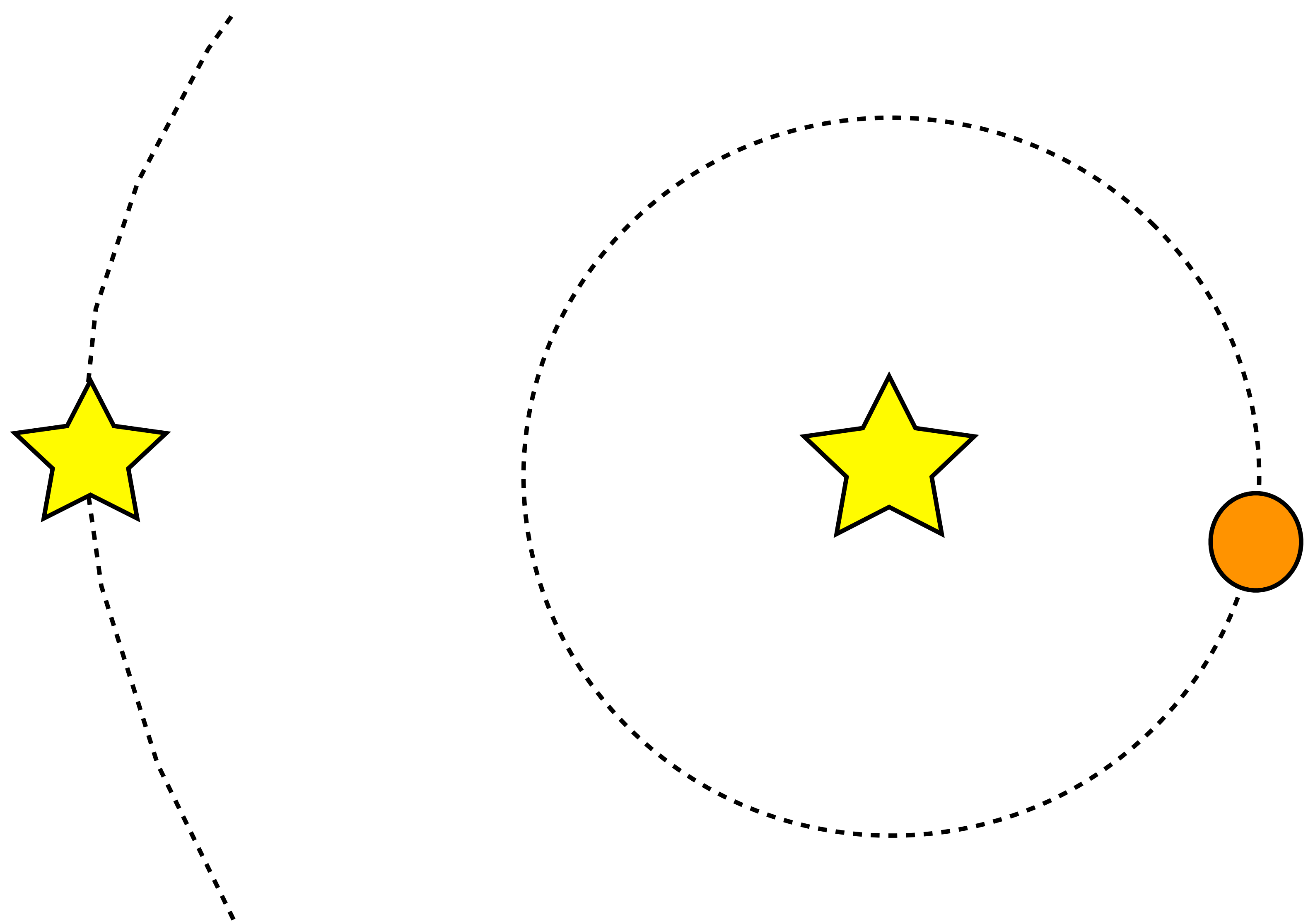




Debes & Sigurdsson 2002

Extreme Example: Mass loss in binary planetary systems

- Differential mass loss causes the planet to expand much more than the binary companion



Kratter & Perets, 2012

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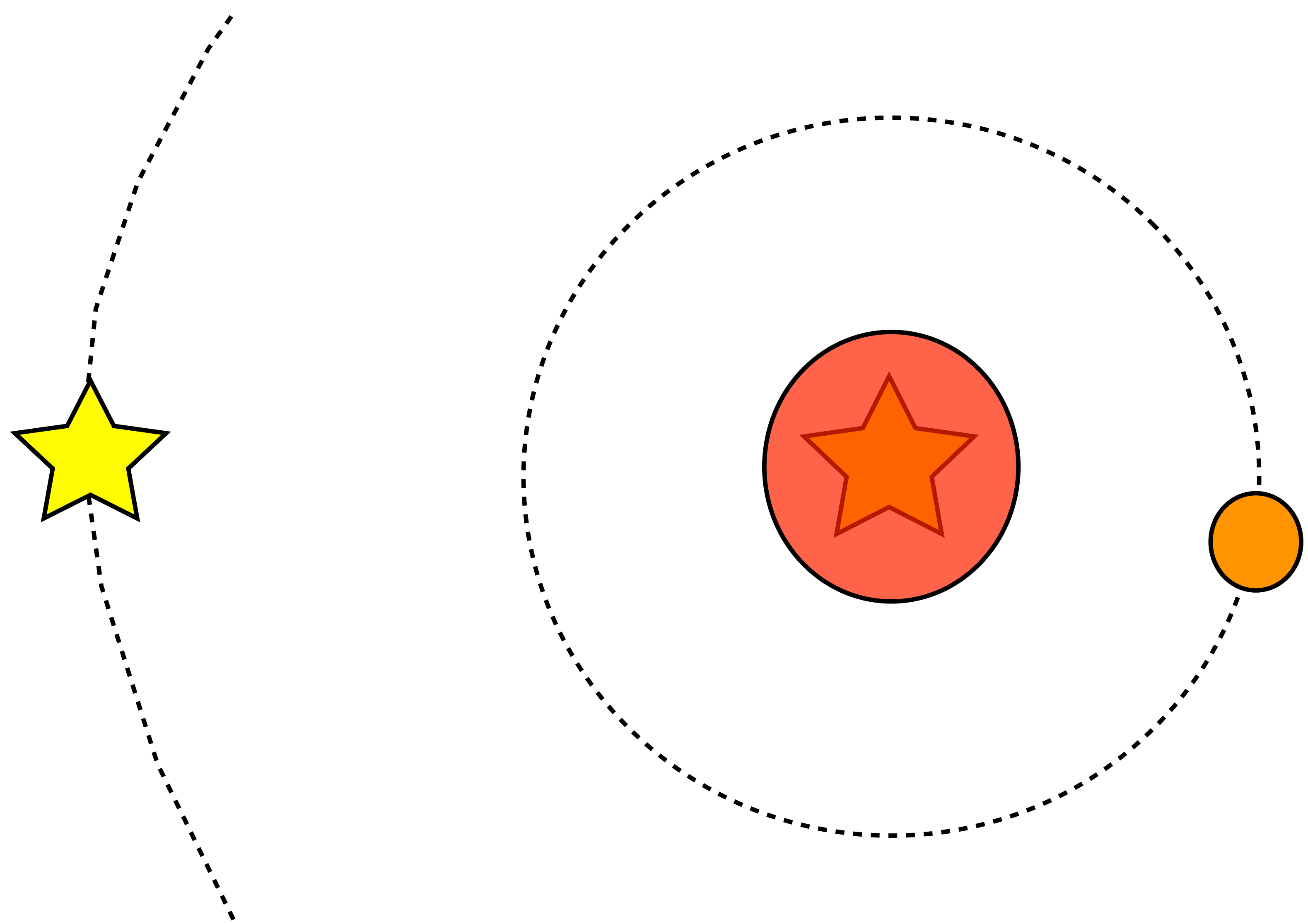

Kratter & Perets, 2012

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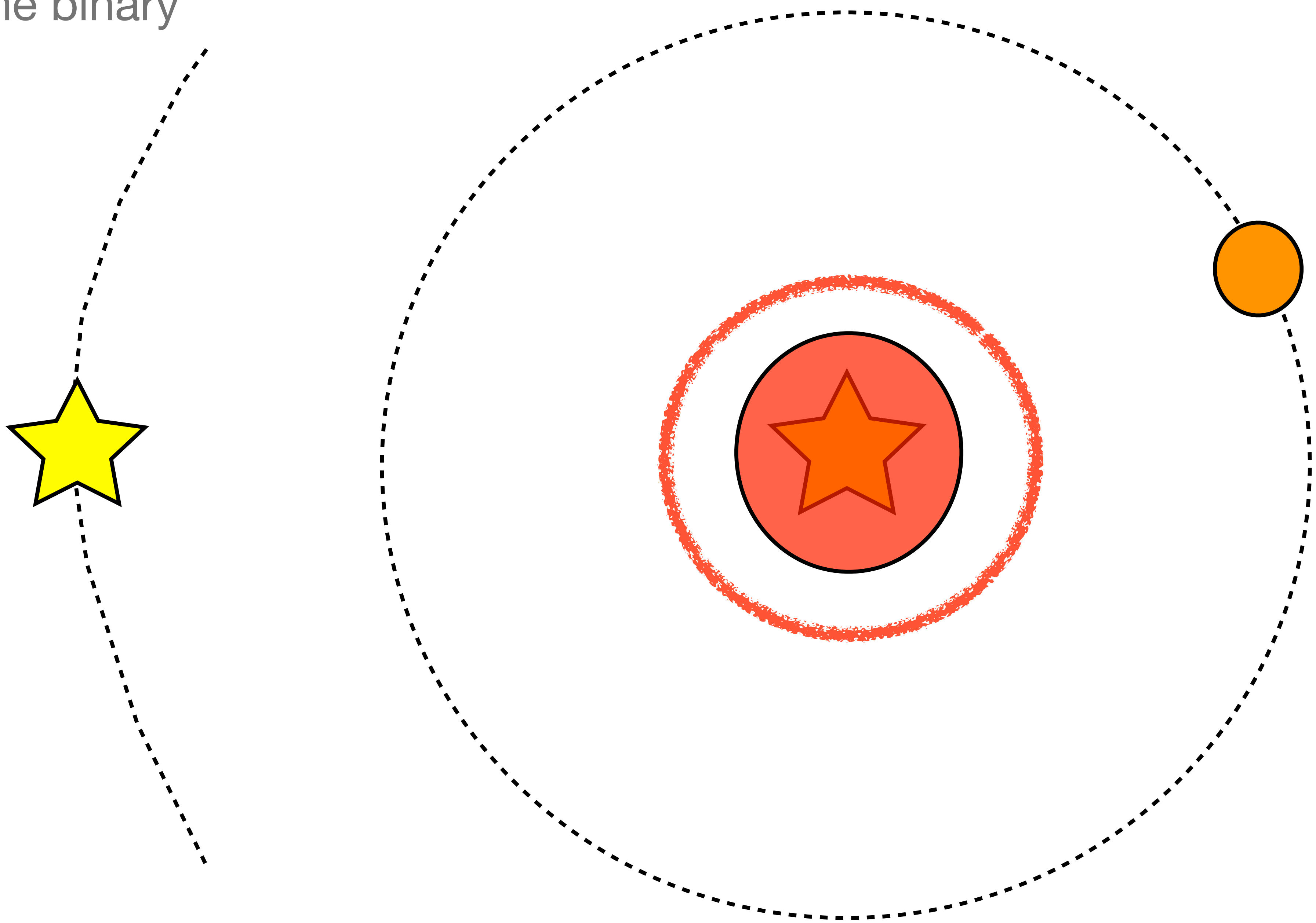

Kratter & Perets, 2012

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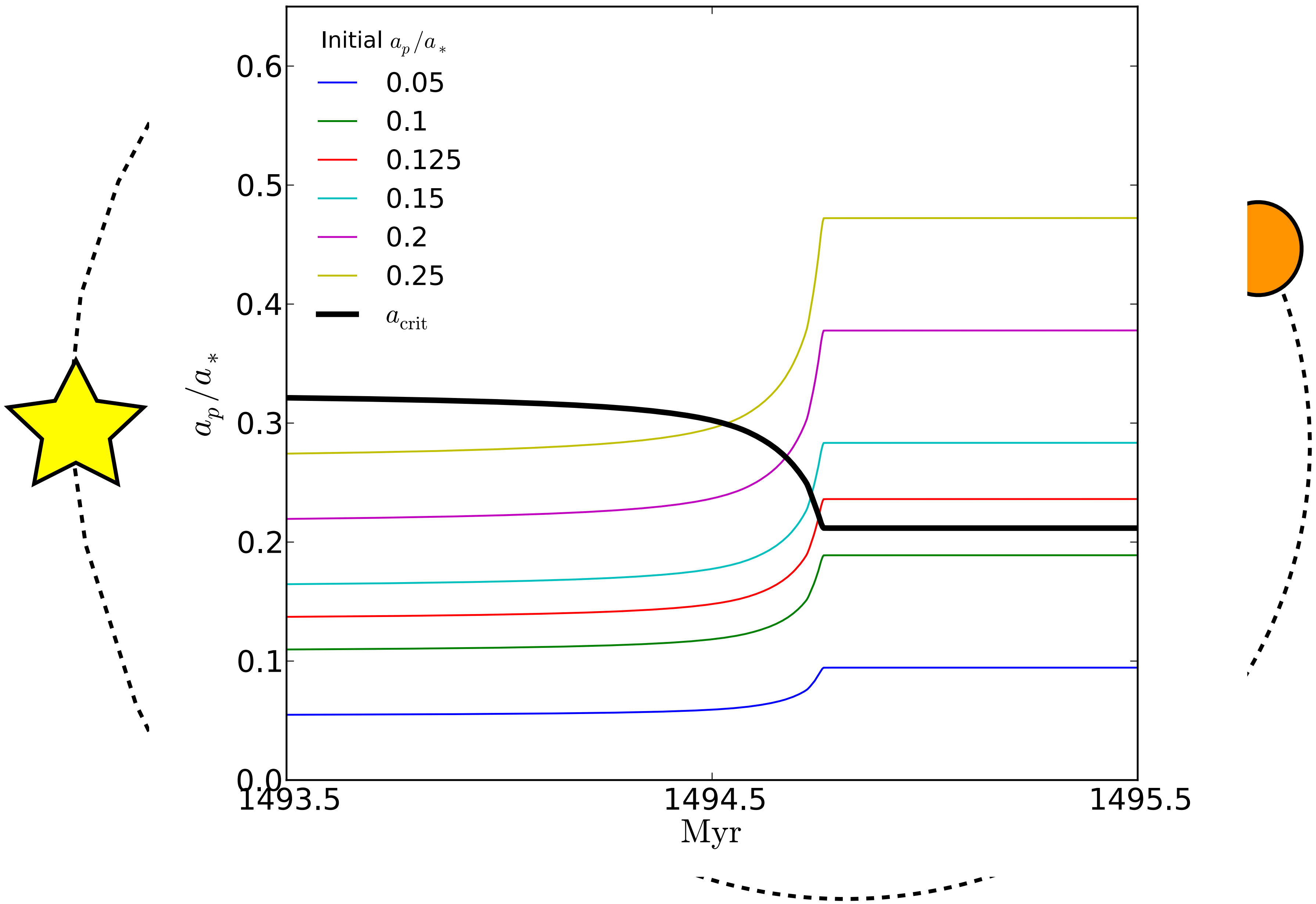

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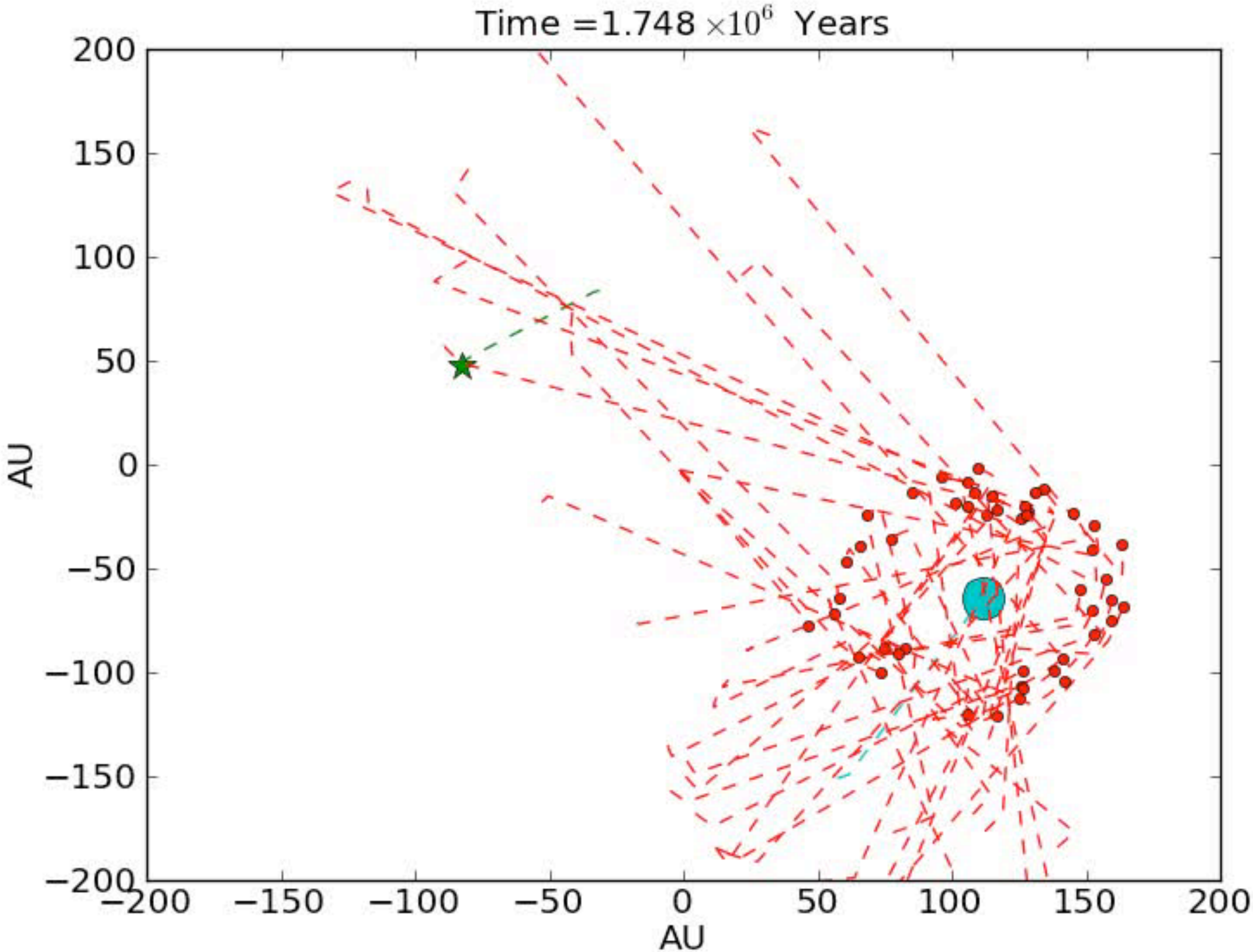
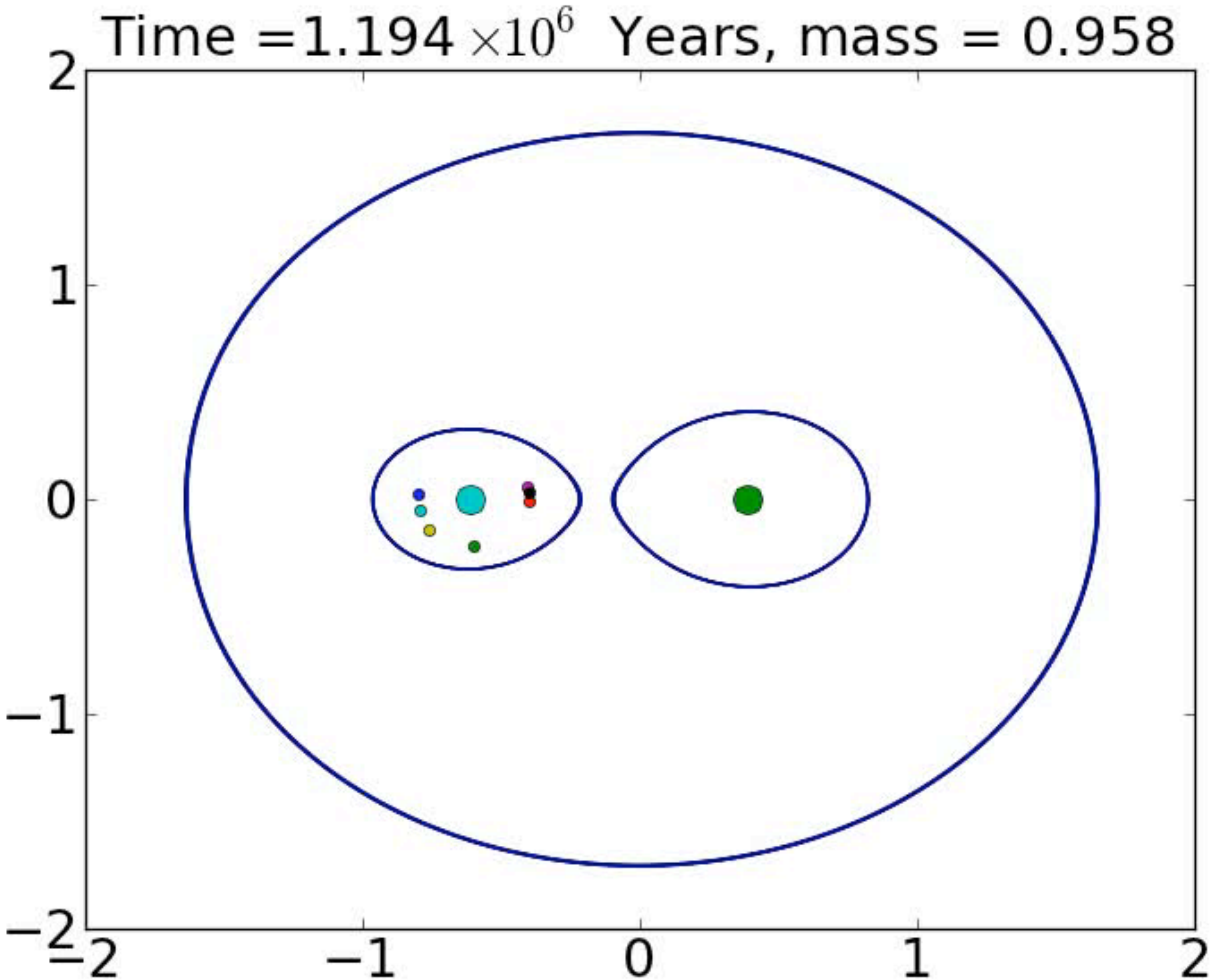
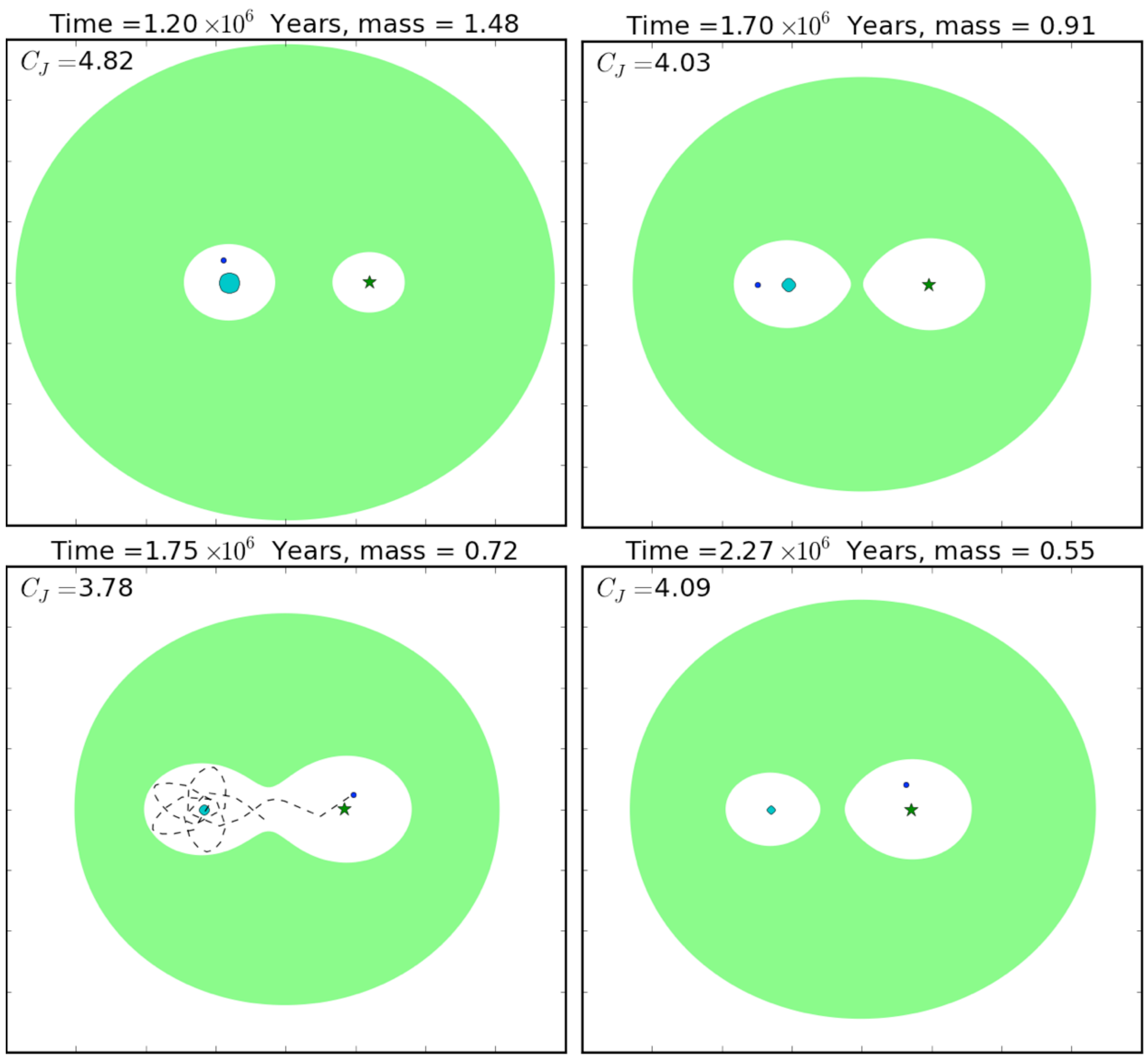

Time =1.748 ×10^6 Years
AU
AU
200
150
100
50
0
−50
−100
−150
−200
−200
−150
−100
−50
0
50
100
150
200

In the rotating reference frame...(CR3BP)



Capture Mechanism

- Zero velocity curves show the bounds of an object's orbit for fixed Jacobi constant



Kratter & Perets 2012

- Mass loss opens and closes the bottleneck (at L1) through which destabilized planets travel

- Long term (>100Myr) stability not guaranteed

Heppenheimer & Porco 1977 Vieira Neto et al 2006

Conclusions

1) Provide an overview of role of dynamics in formation and evolution of the systems you will observe

- Dynamics controls the birth, evolution, and death of planetary systems

2) Provide basic tools to help analyze and validate new systems

- Kepler light curves are fantastic. Dynamical modeling makes it even more powerful
- Light Curves + Dynamics sheds light on physics, formation, and fate
- Simple estimate can help (in)validate detections

2) Keep theory in mind in choosing project and writing papers