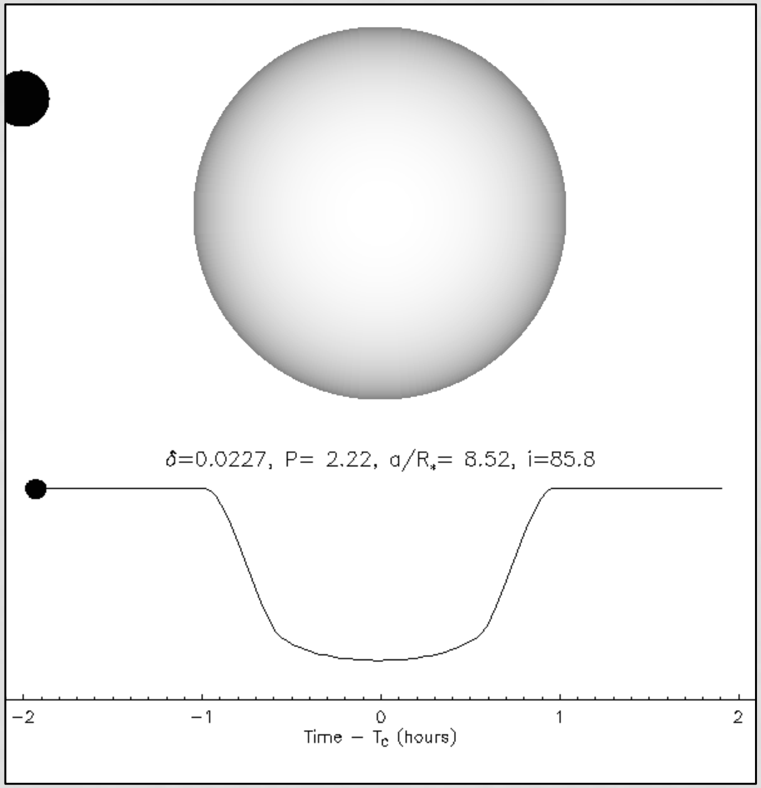
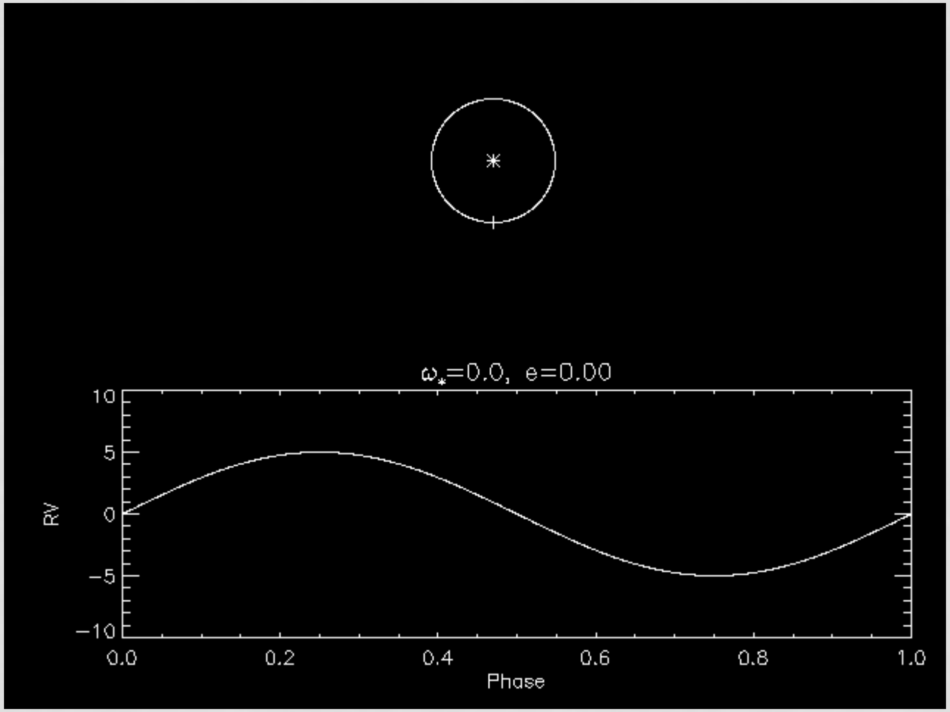
WORKING WITH MCMC CODES: EXOFAST



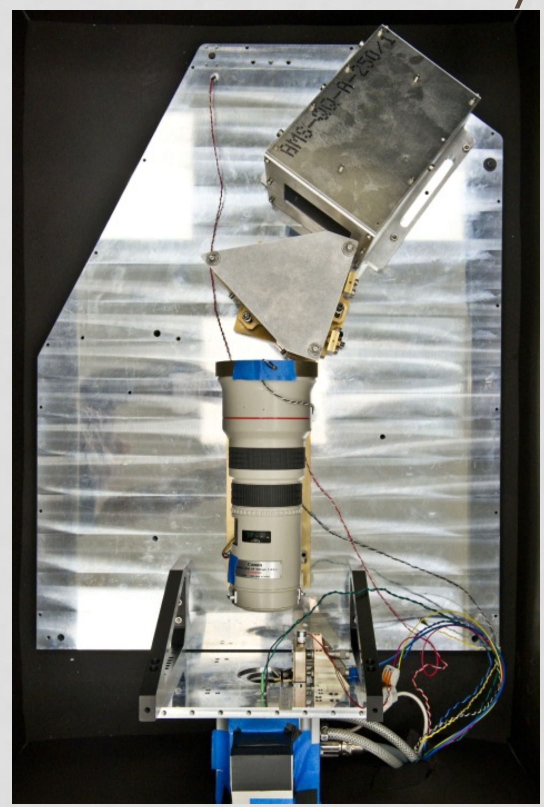


Jason Eastman
LCOGT/UCSB
With B. Scott Gaudi and Eric Agol

Summer Sagan Workshop
2012.07.23

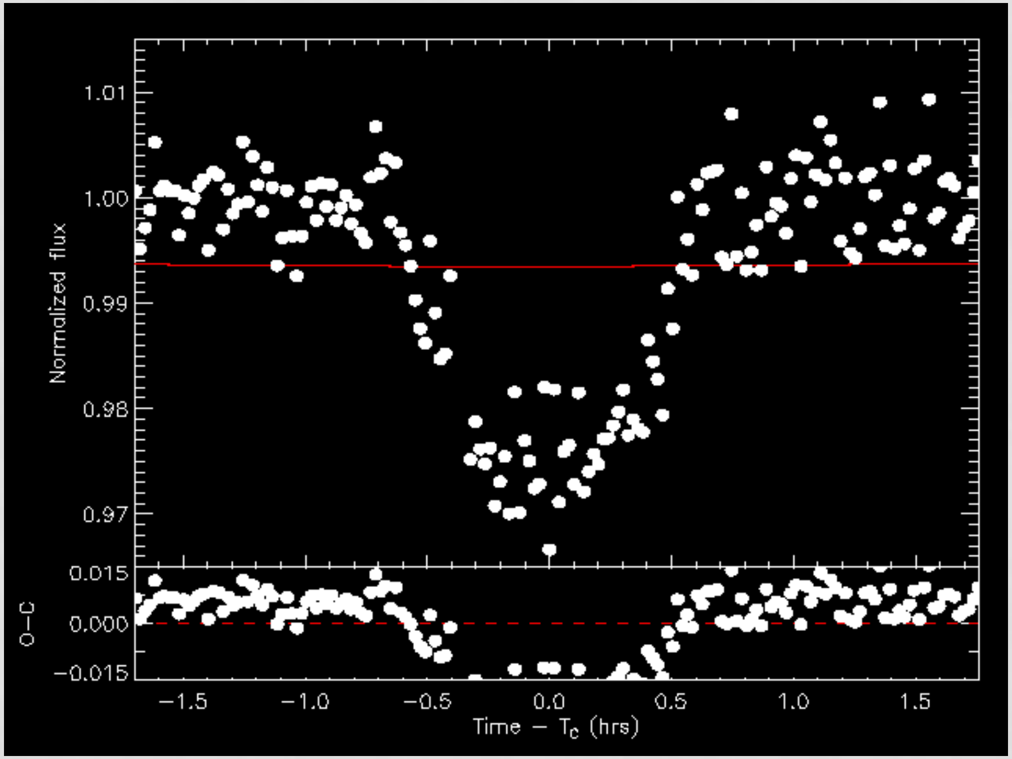
GOALS

- We have photometry and (usually) RV of exoplanets
- Many overlapping constraints -- best determine planetary properties if we fit them simultaneously



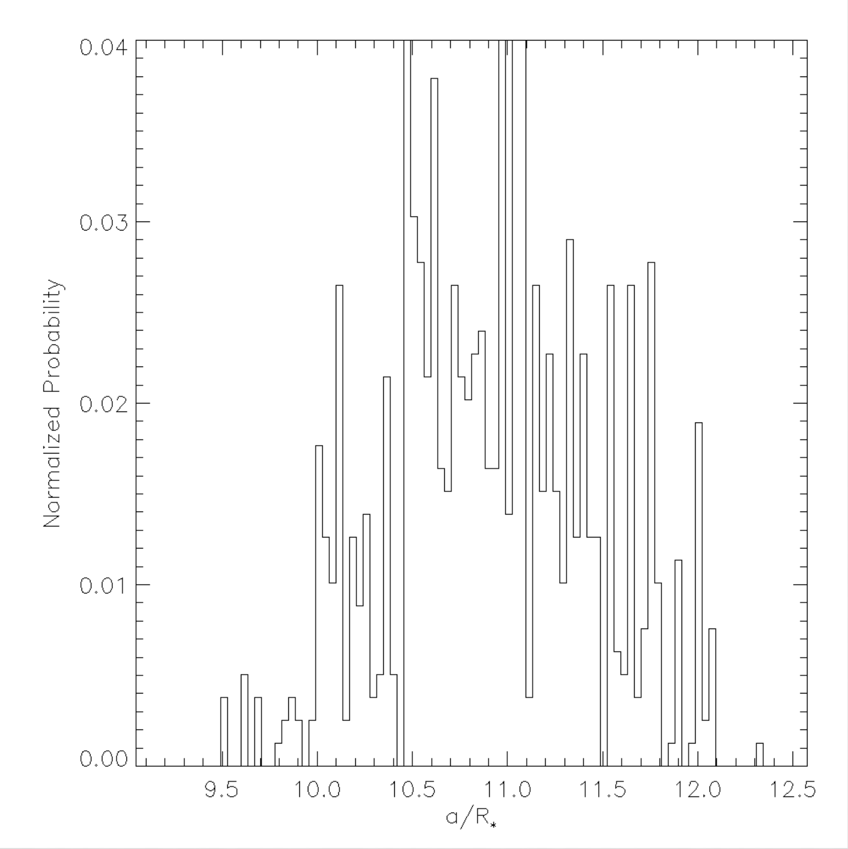
BEST-FIT

- Global Fits
  - BLS
  - Lomb-Scargle Periodogram
- Local Fits
  - Amoeba
  - Levenberg-Marquardt
- Scale errors



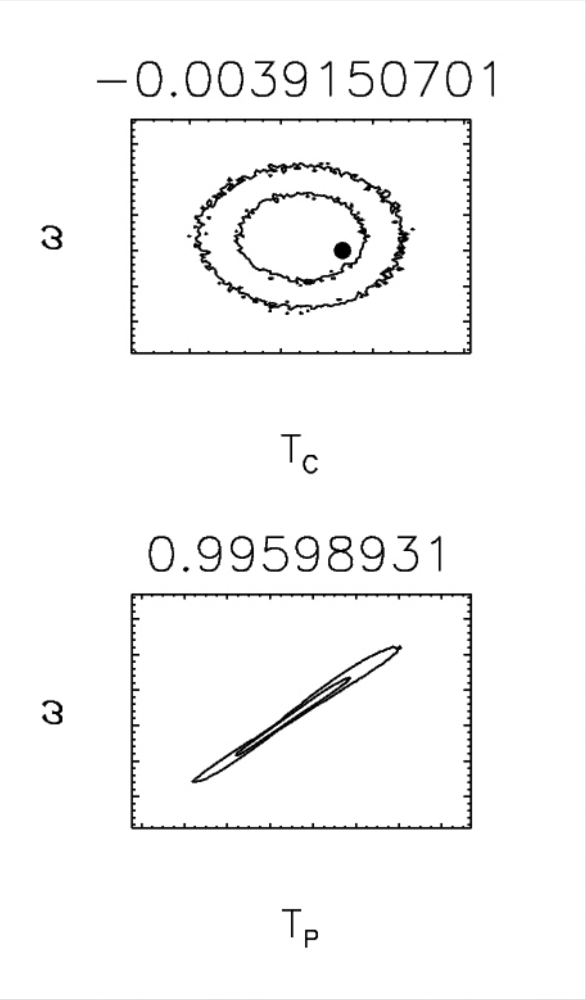
UNCERTAINTIES: MCMC

- Best fit by itself has very little meaning
- MCMC characterizes uncertainties
- $L \alpha e^{-\chi^2/2}$



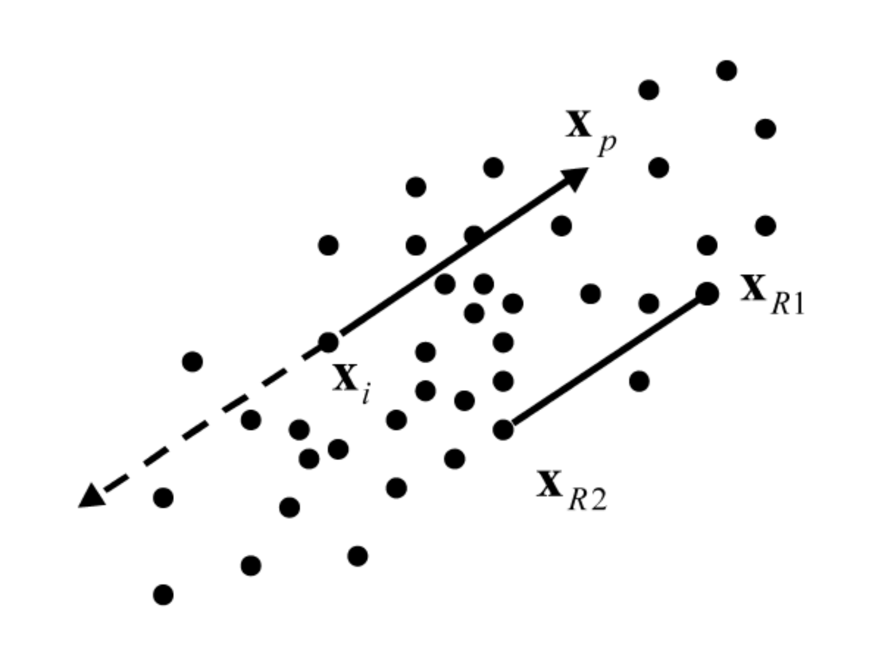
PARAMETERIZATION:
COVARIANCES VS PRIORS

- Trade off in parameterization
- Want minimal covariances
- Want physical priors
- Not always clear what those are



STEPPING SCALE: DIFFERENTIAL EVOLUTION MCMC

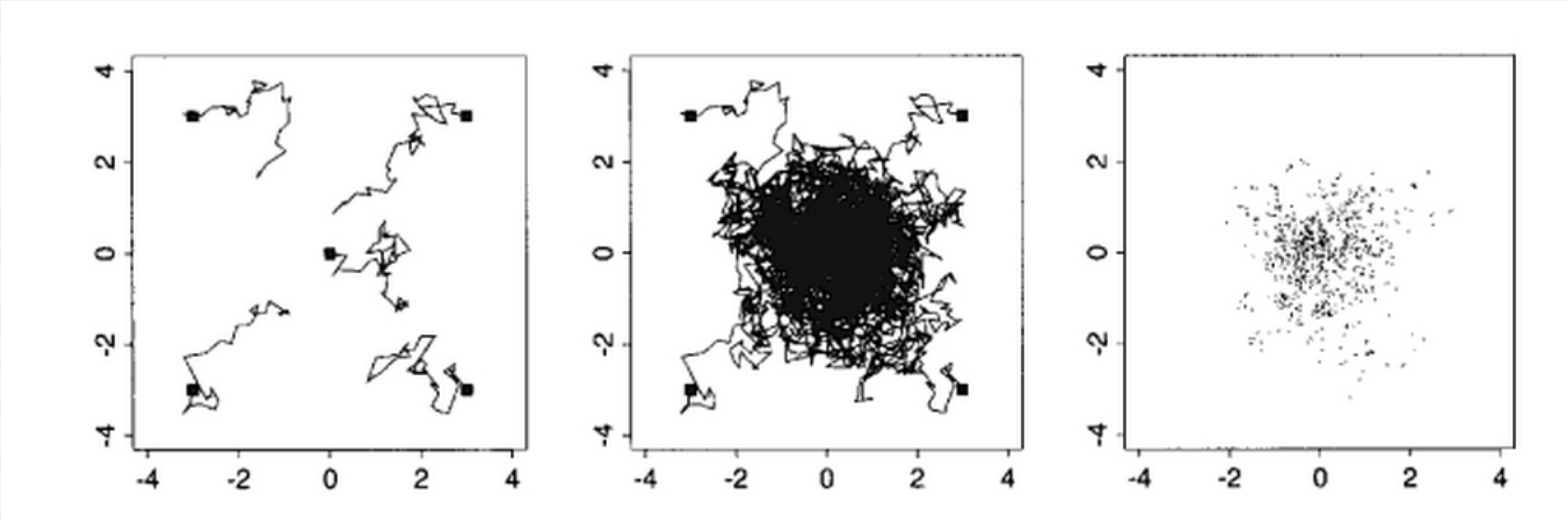
- How do you step?
  - Too big, too many steps get rejected
  - Too small, too many steps get accepted
  - Want ~20% acceptance
- Ideal step mirrors the covariance matrix
  - But that's what we're trying to figure out!



ter Braak, 2006

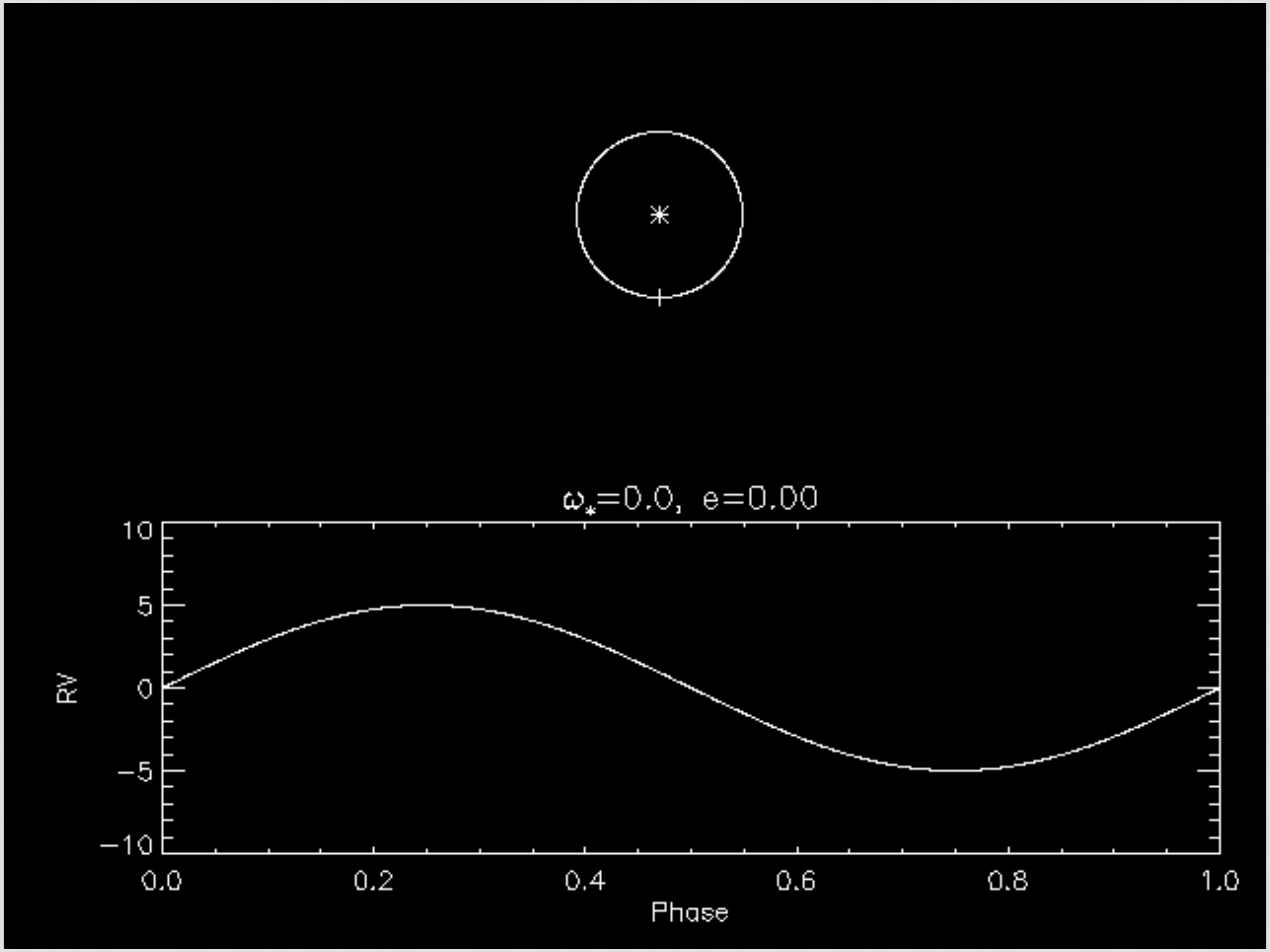
CONVERGENCE

- Each chain starts 5*stepsize*Gaussian Random away from best fit and run independently
- Stop when they all agree within some limit



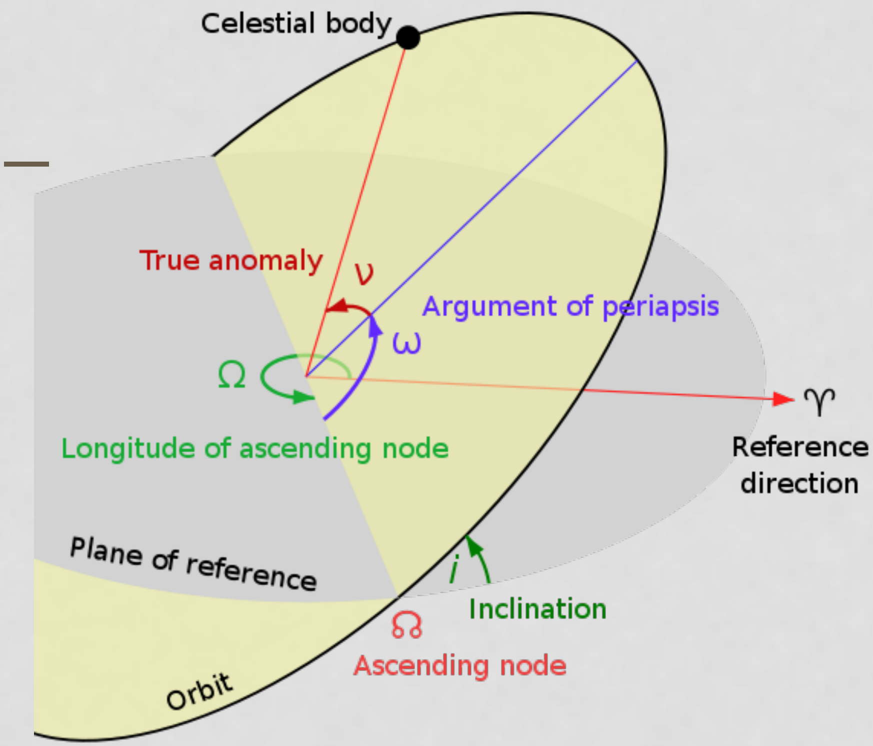
Gelman, et. al., 2003

RV MODEL

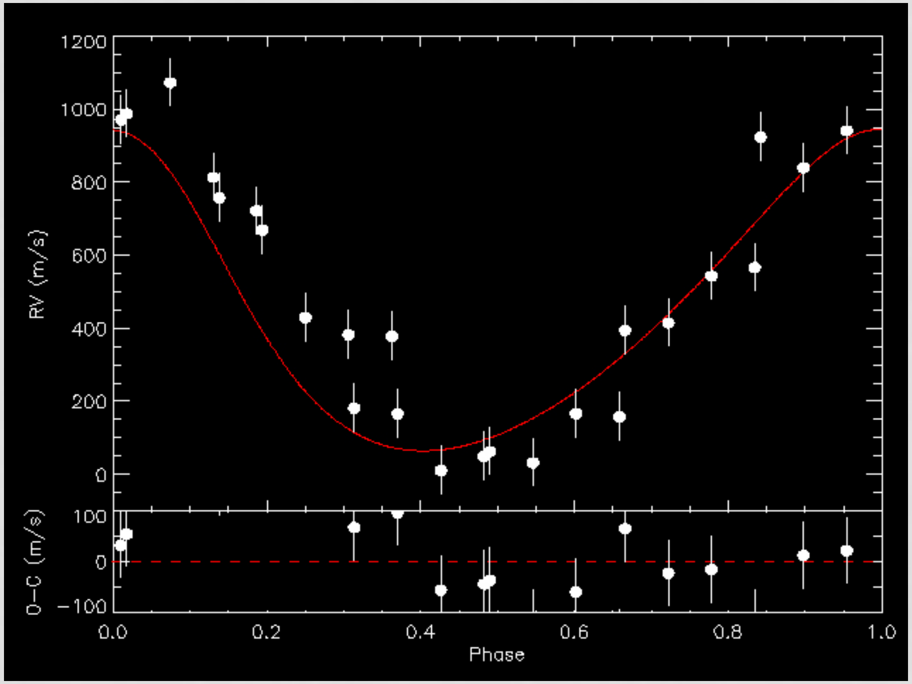


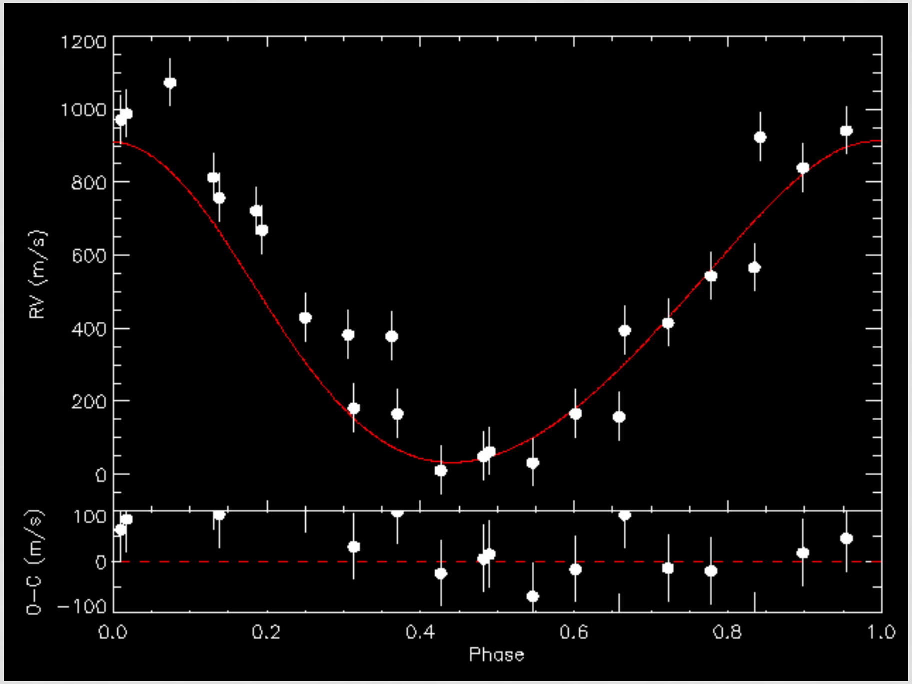
RV PARAMETERIZATION

- $\log P, T_C, \sqrt{e}\cos\omega_*, \sqrt{e}\sin\omega_*, \log K, \gamma$
- $K = \left(\frac{2\pi G}{P (M_* + M_P)^2} \right)^{1/3} \frac{M_P \sin i}{\sqrt{1-e^2}}$
- T_P?
- $e\cos\omega_*, e\sin\omega_*$?
- e, ω_*?

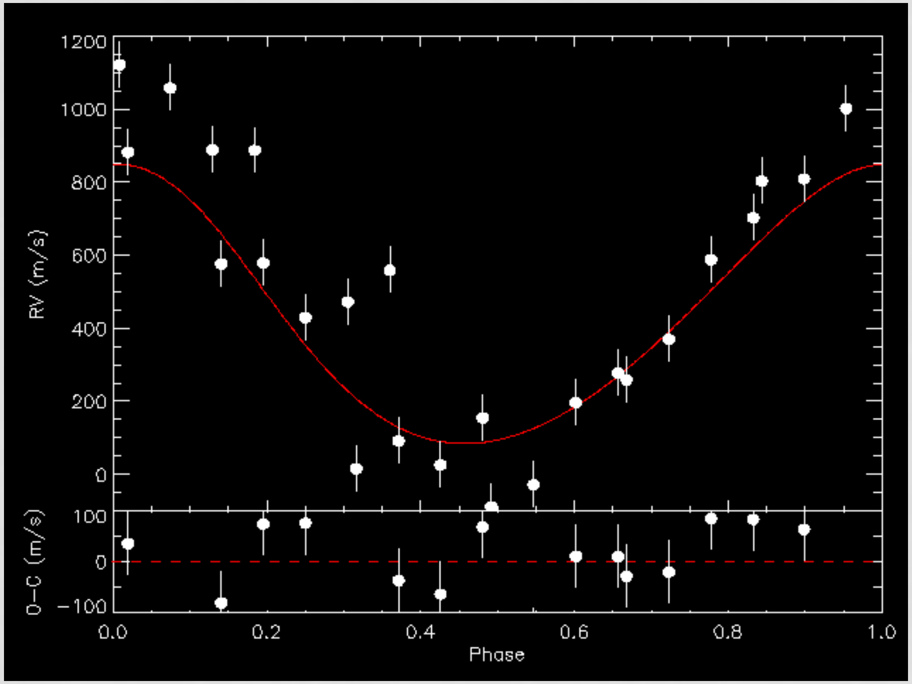


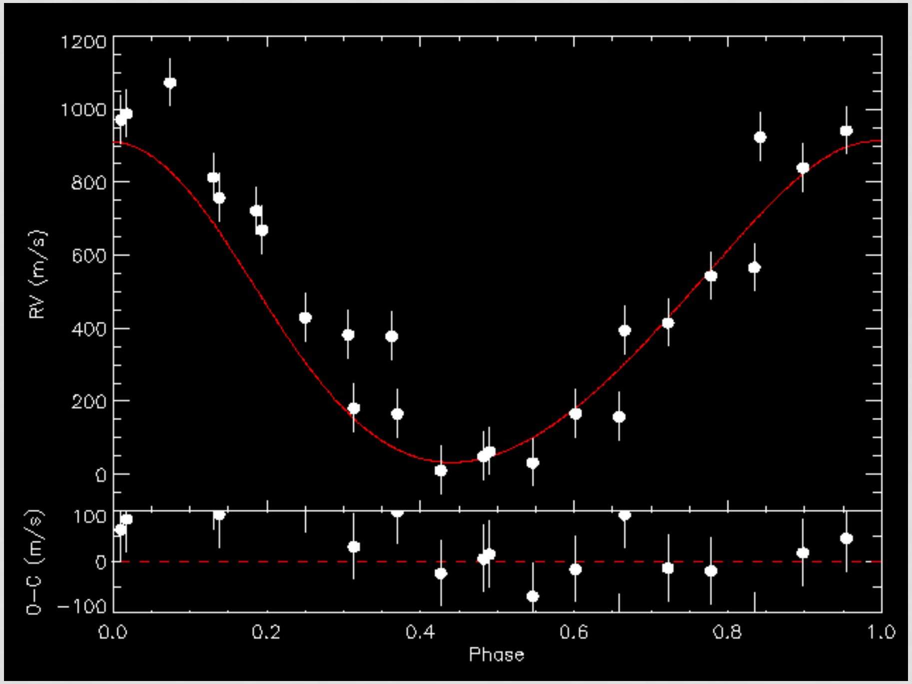
T_P VS T_C





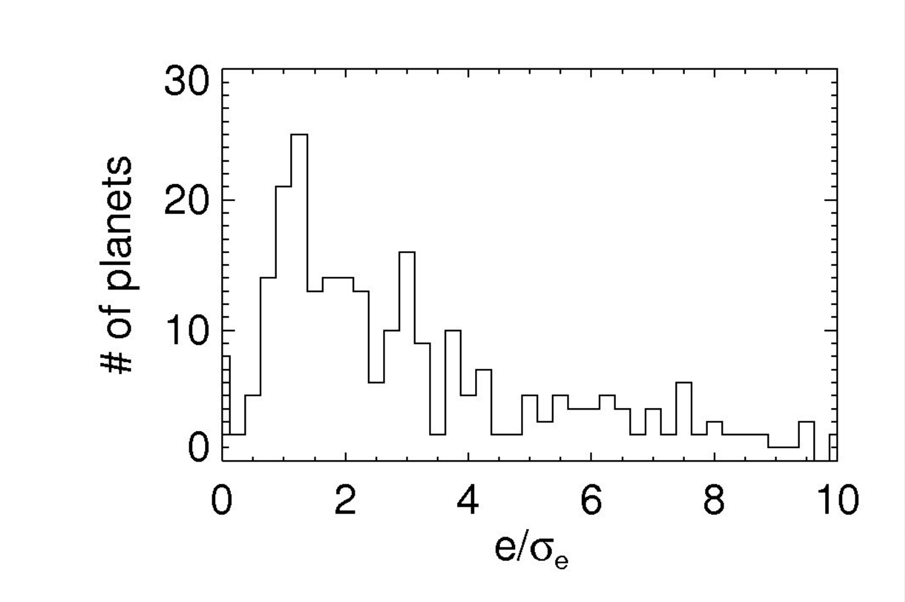
T_P VS T_C


1200
1000
800
600
400
200
0
RV (m/s)
100
0
−100
O−C (m/s)
0.0
0.2
0.4
0.6
0.8
1.0
Phase


1200
1000
800
600
400
200
0
RV (m/s)
100
0
−100
O−C (m/s)
0.0
0.2
0.4
0.6
0.8
1.0
Phase

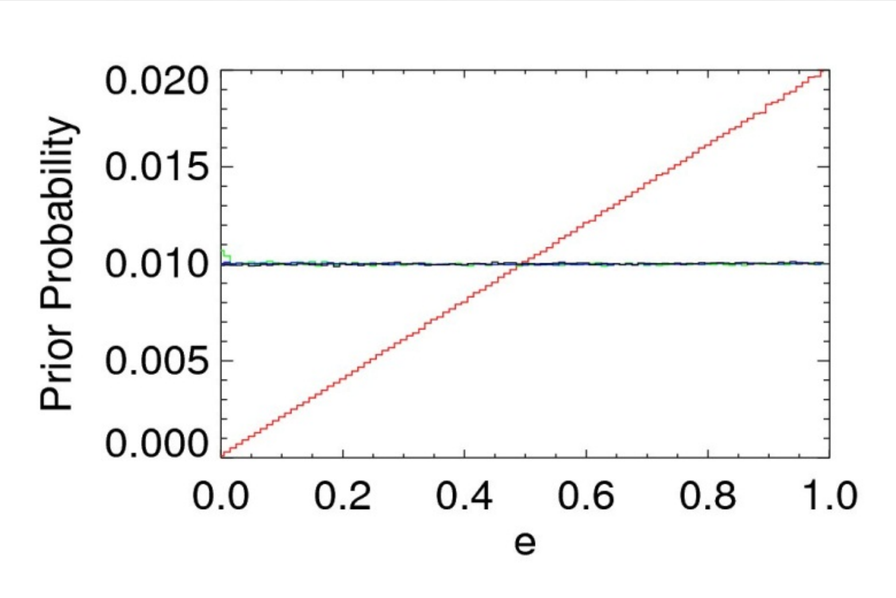
ECCENTRICITY

- Expect tidally circularized
- Small eccentricities significant
- Other bodies?
- Tidal Q?



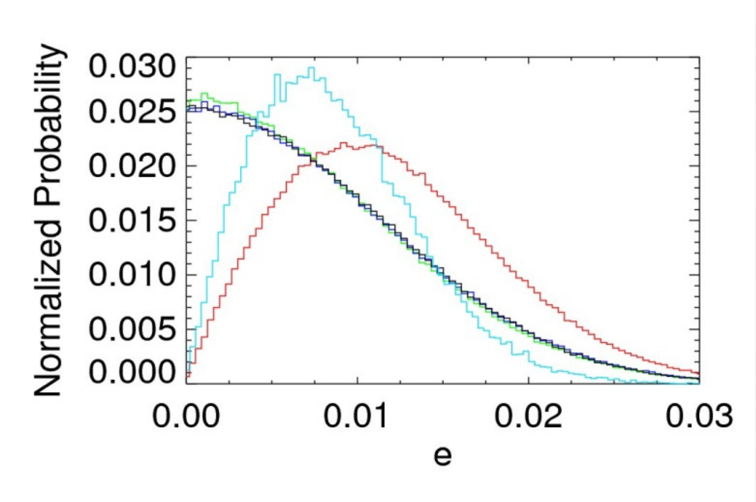
ECCENTRICITY BIAS
PRIOR BIAS

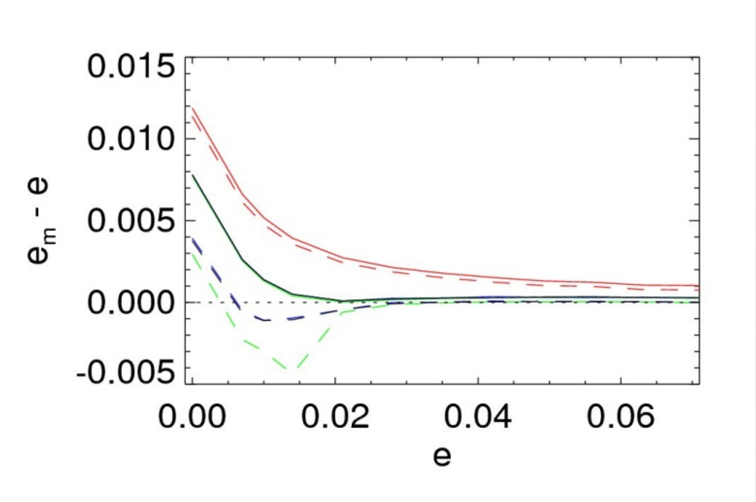
- Without correction, priors are uniform in stepping parameters
- $e\cos\omega_*$, $e\sin\omega_*$ imply linear prior in e
- Must correct or use a parameterization intrinsically uniform



ECCENTRICITY BIAS
LUCY-SWEENEY BIAS

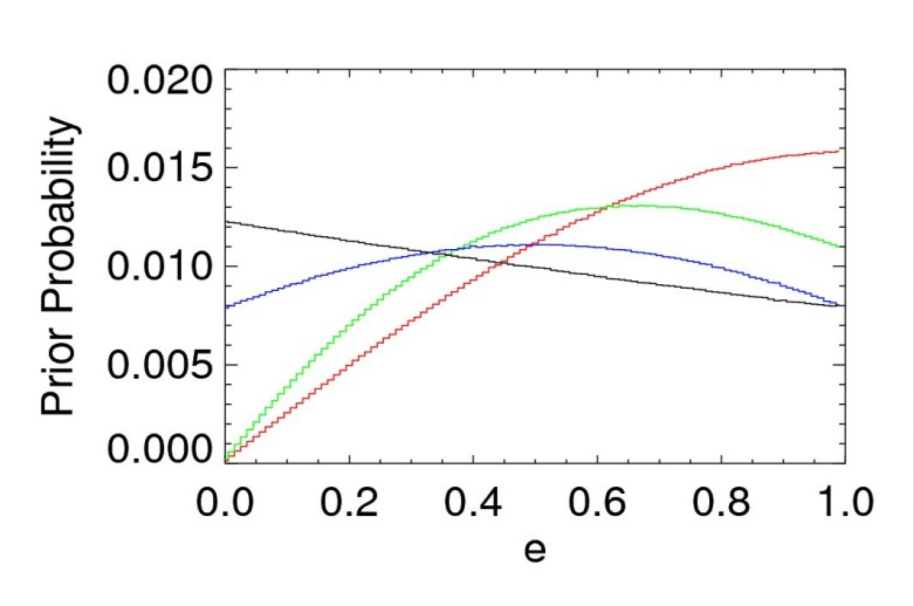
- Problem for all positive-definite parameters
- Zero phase space at exactly zero
- Any error (systematic or otherwise) necessarily skews result positive



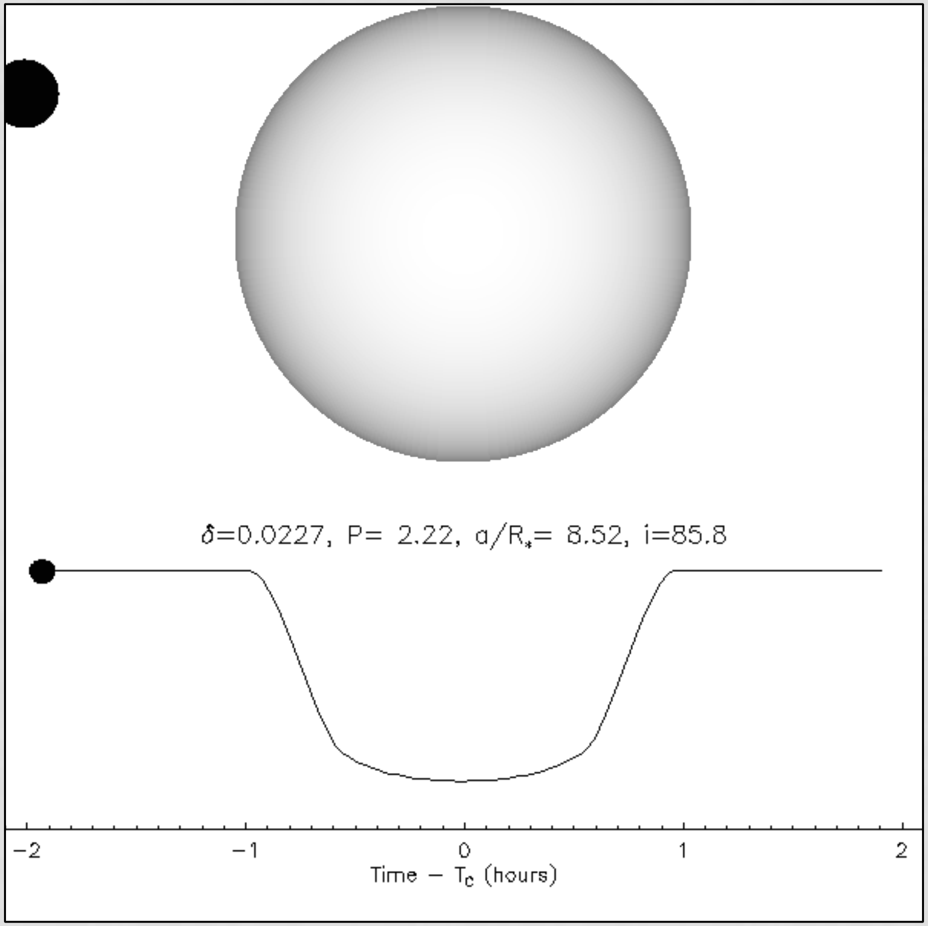


ECCENTRICITY BIAS
METROPOLIS-HASTINGS ALGORITHM

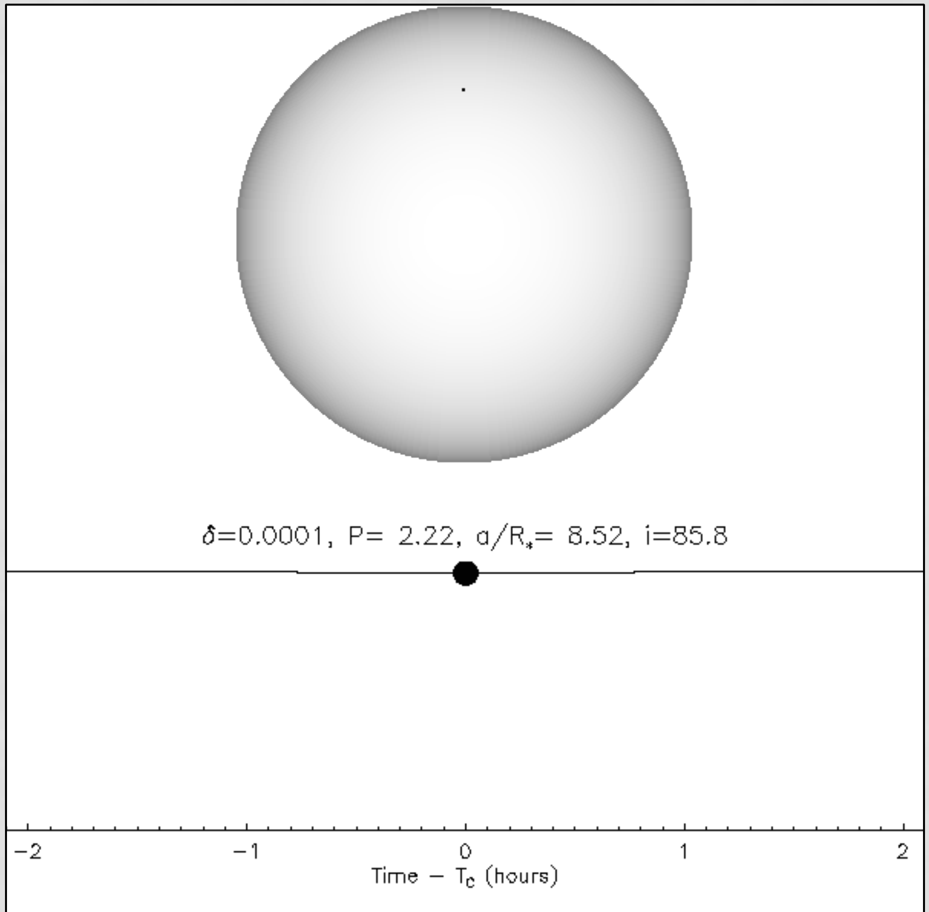
- Rejected steps
  - The previous step must be copied in its place
  - Inefficient?
  - Unintuitive?
- Boundaries
  - Chains must be allowed to step out of bounds



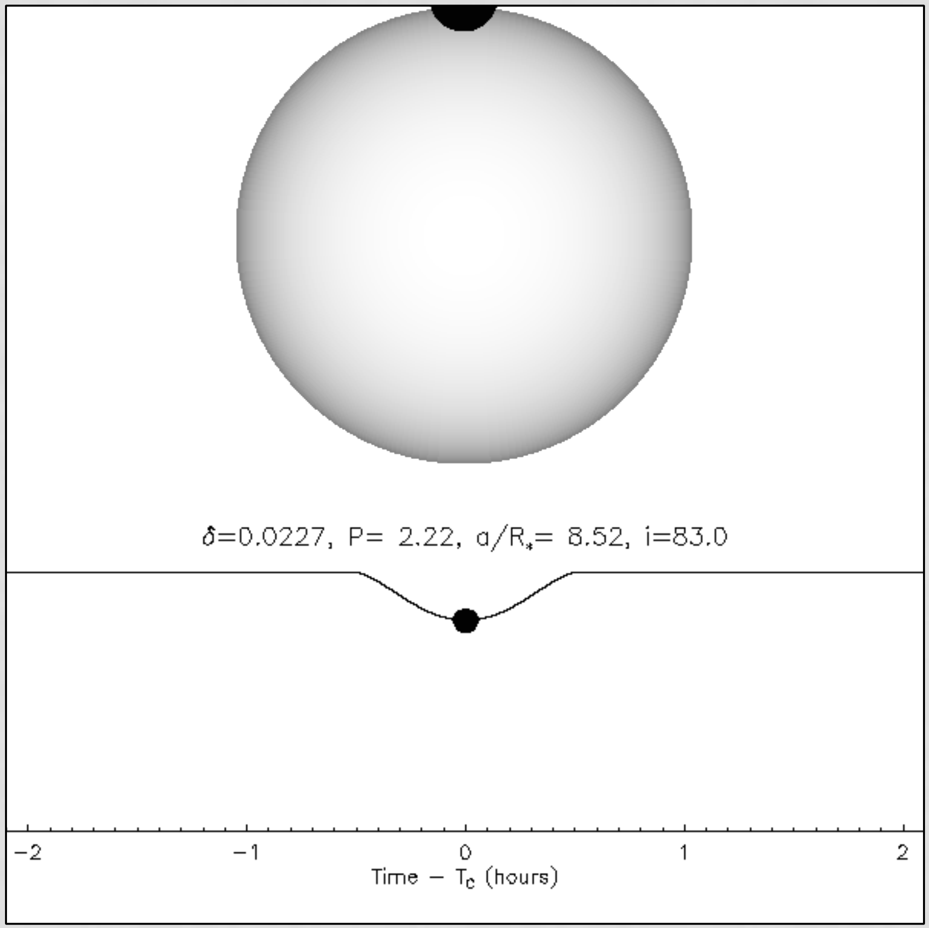
TRANSIT MODEL



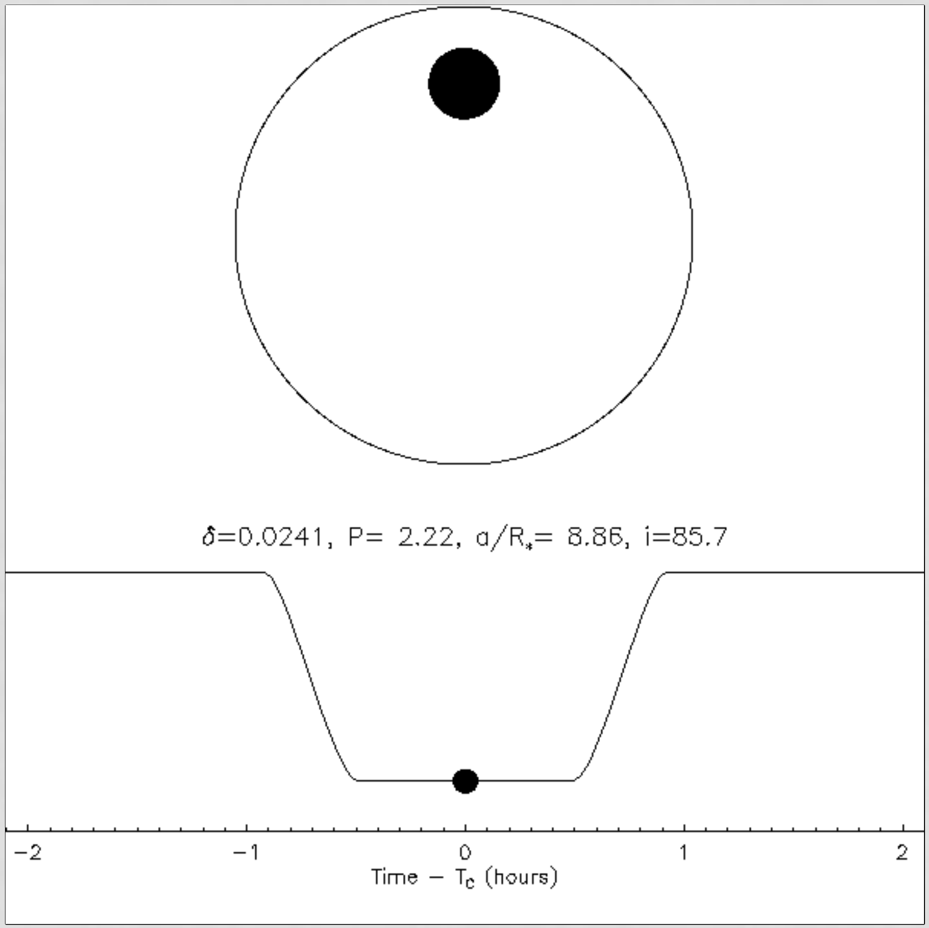
CHANGING R_P/R_*



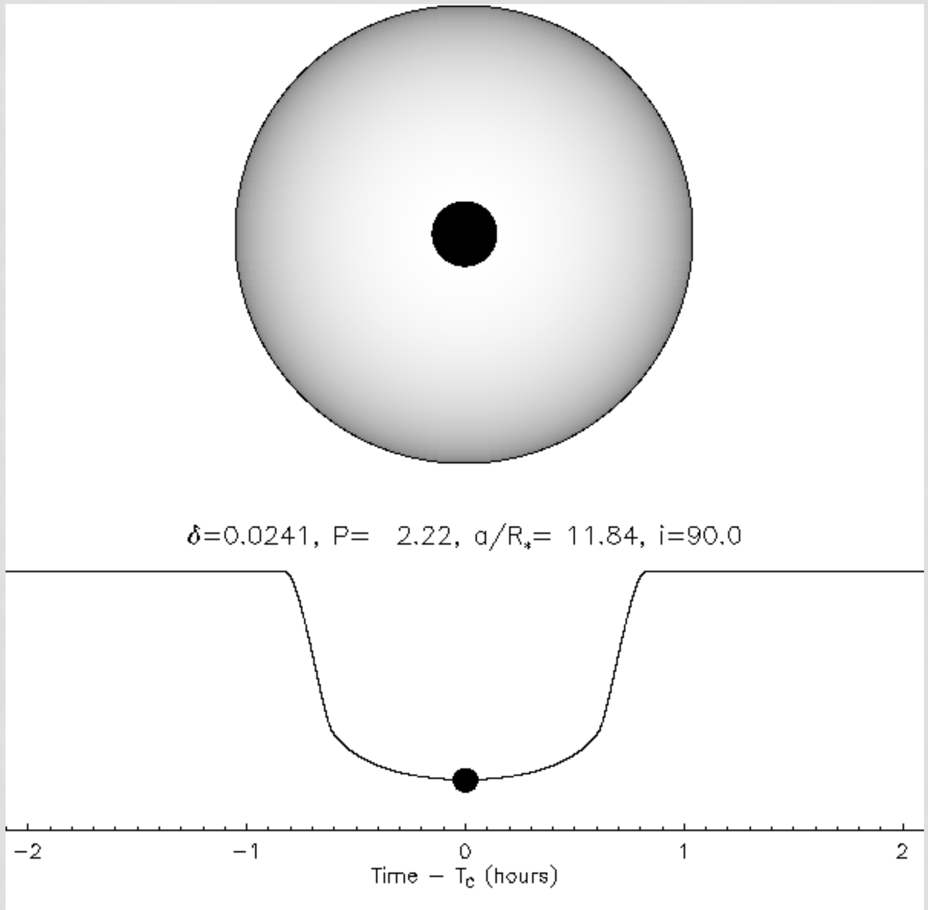
CHANGING *i*



CHANGING LIMB DARKENING

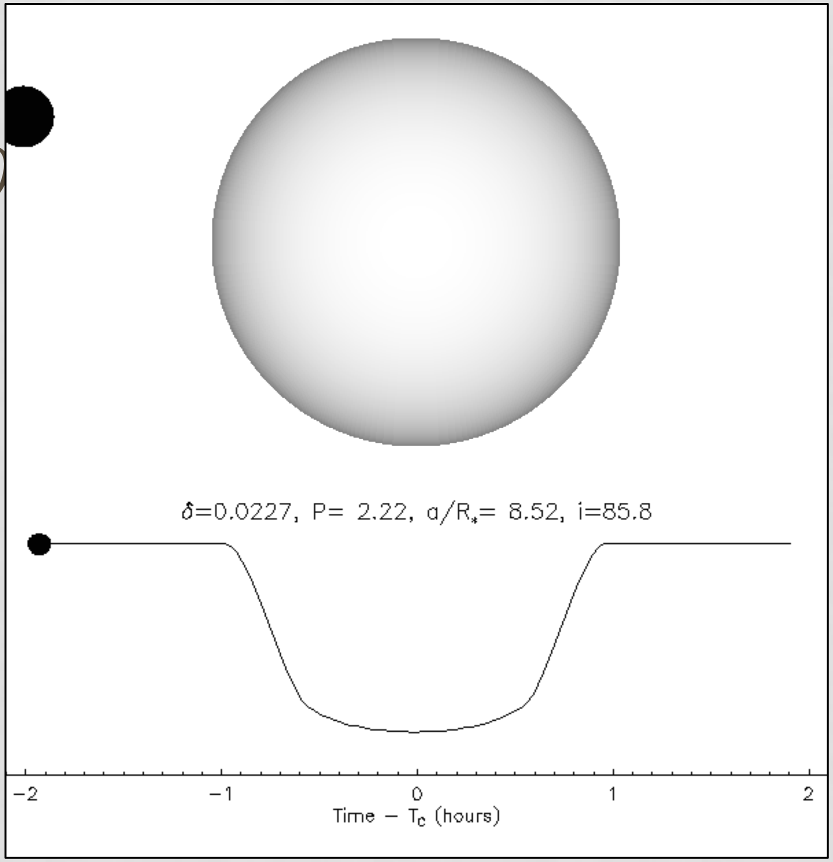


CHANGING τ



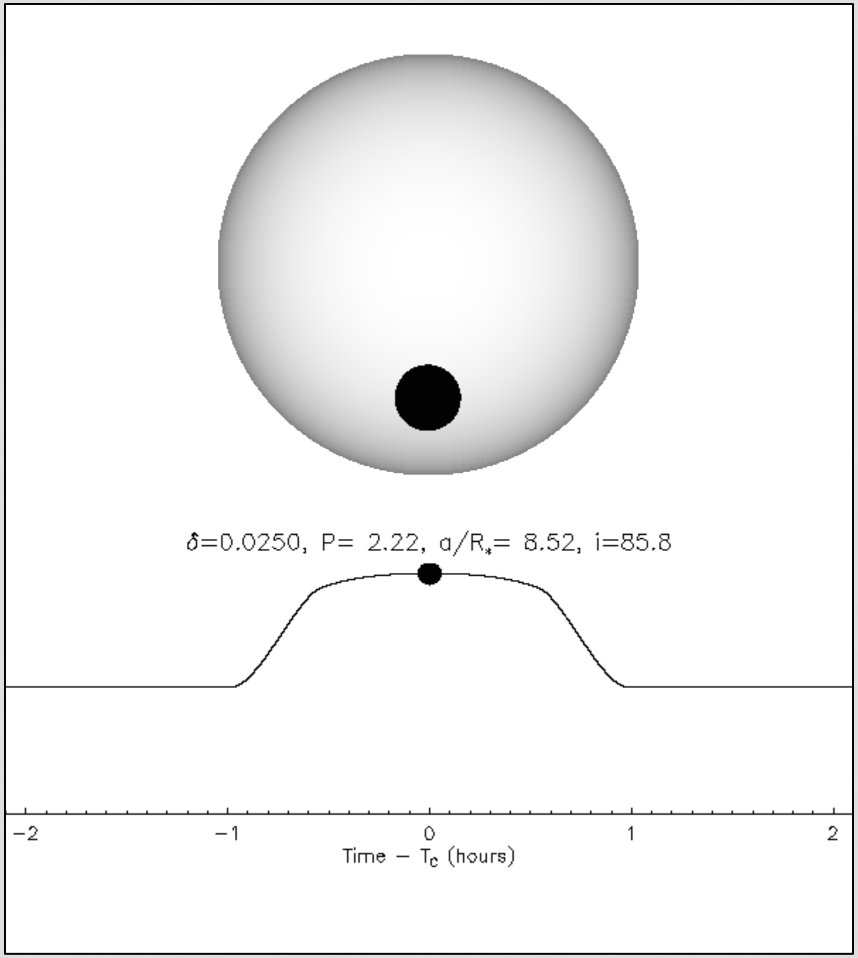
TRANSIT PARAMETERIZATION

- $\log P, T_C, e, \omega_*, R_P/R_*, \cos i, a/R_*, F_0, u_1, u_2$
- $\cos i, a/R_* \Rightarrow \tau, T$?
- $\cos i, a/R_* \Rightarrow T_T, T_F$?
- $R_P/R_* \Rightarrow \delta \sim (R_P/R_*)$



OTHER BIASES?

- Other positive-definite parameters:
- R_P / R_*
- $\cos i$



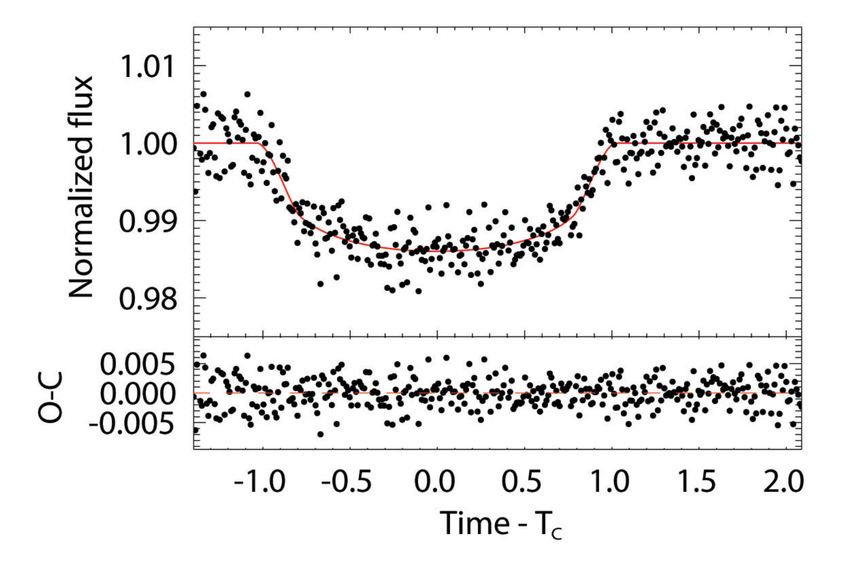
RV + TRANSIT PARAMETERIZATION

- Radial Velocity $\Rightarrow \log P, T_C, e, \omega_*, \log K, \gamma$
  - $K = \left(\frac{2\pi G}{P(M_* + M_P)^2}\right)^{1/3} \frac{M_P \sin i}{\sqrt{1-e^2}}$
- Primary Transit $\Rightarrow \log P, T_C, e, \omega_*, R_P/R_*, \cos i, a/R_*, F_0$
- Kepler's Law: $P^2 = \frac{4\pi^2 a^3}{G(M_* + M_P)}$
- One-parameter family of solutions
- $g_* = GM_*/R_*^2$
- Torres et al., 2010: $\log g_*, [\mathrm{Fe/H}], T_{eff} \Rightarrow M_*, R_*$
- Claret & Bloeman, 2011: $\log g_*, [\mathrm{Fe/H}], T_{eff} \Rightarrow u_1, u_2$

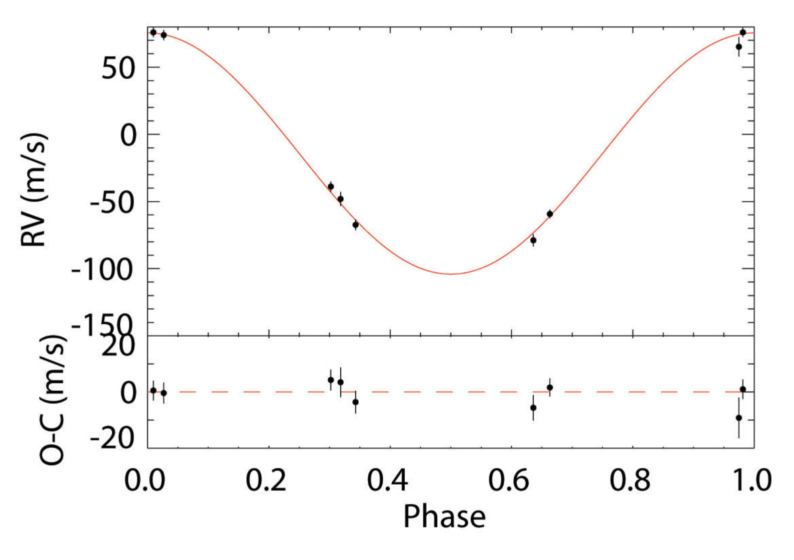
EXOFAST

```
IDL>
IDL> priors = dblarr(2,15)
IDL> priors[1,*] = !values.d_infinity ;; no priors
IDL> priors[*,3] = [0.46235351d0,8.1000453d-6] ;; period prior
IDL> priors[0,10:12] = [4.61d0,5185d0,0.27d0] ;; logg, Teff, [Fe/H]
IDL> priors[1,10:12] = [0.05d0,80d0,0.08d0] ;; errors
IDL> exofast, 'hat3.rv','hat3.flux',band='Sloani',/circular,/noslope,prefix='HAT-P-3b.',priors=priors

Best peaks in the RV fit:
     T_C          Period        ecosw        esinw          K          gamma          slope        chi^2       chi^2/dof
      1.107424     2.509629     0.000000     0.000000   111.702373   -19.695690     0.000000    79.680061     15.936012
      0.032483     3.191384     0.000000     0.000000    88.122421    -8.571517     0.000000    60.684224     12.136845
      1.241369     2.901193     0.000000     0.000000    90.201515   -14.637107     0.000000    31.508634      6.301727
      0.604251     2.985977     0.000000     0.000000    89.044937   -12.943334     0.000000    34.917544      6.983509
      1.105912     2.727992     0.000000     0.000000    95.094301   -17.178435     0.000000    27.056734      5.411347
Chi^2/dof = 5.4113468
Scaling errors by 2.4935620

Transit fit:
Chi^2/dof = 8.5535456
Scaling errors by 2.9273553

EXOFAST_DEMC:  4.01% done; acceptance rate = 23.54%; time left: 37.53 minutes

EXOFAST_DEMC: done in 1.57 minutes; took 23.54% of trial steps
IDL>
```

BEST FITS

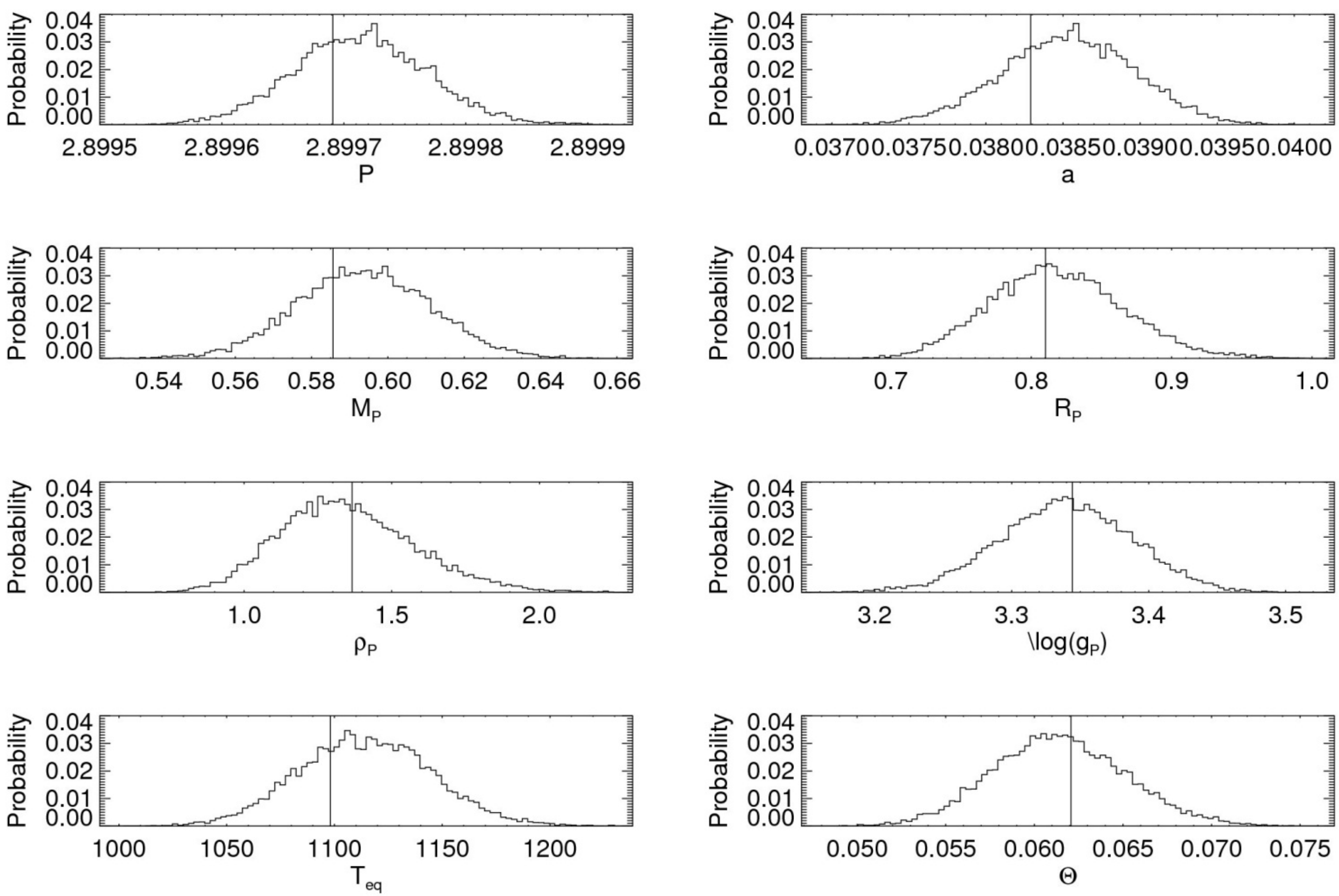


Normalized flux
1.01
1.00
0.99
0.98
O-C
0.005
0.000
-0.005
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
Time - T$_C$



RV (m/s)
50
0
-50
-100
-150
O-C (m/s)
20
0
-20
0.0
0.2
0.4
0.6
0.8
1.0
Phase

PARAMETER DISTRIBUTIONS



COVARIANCES

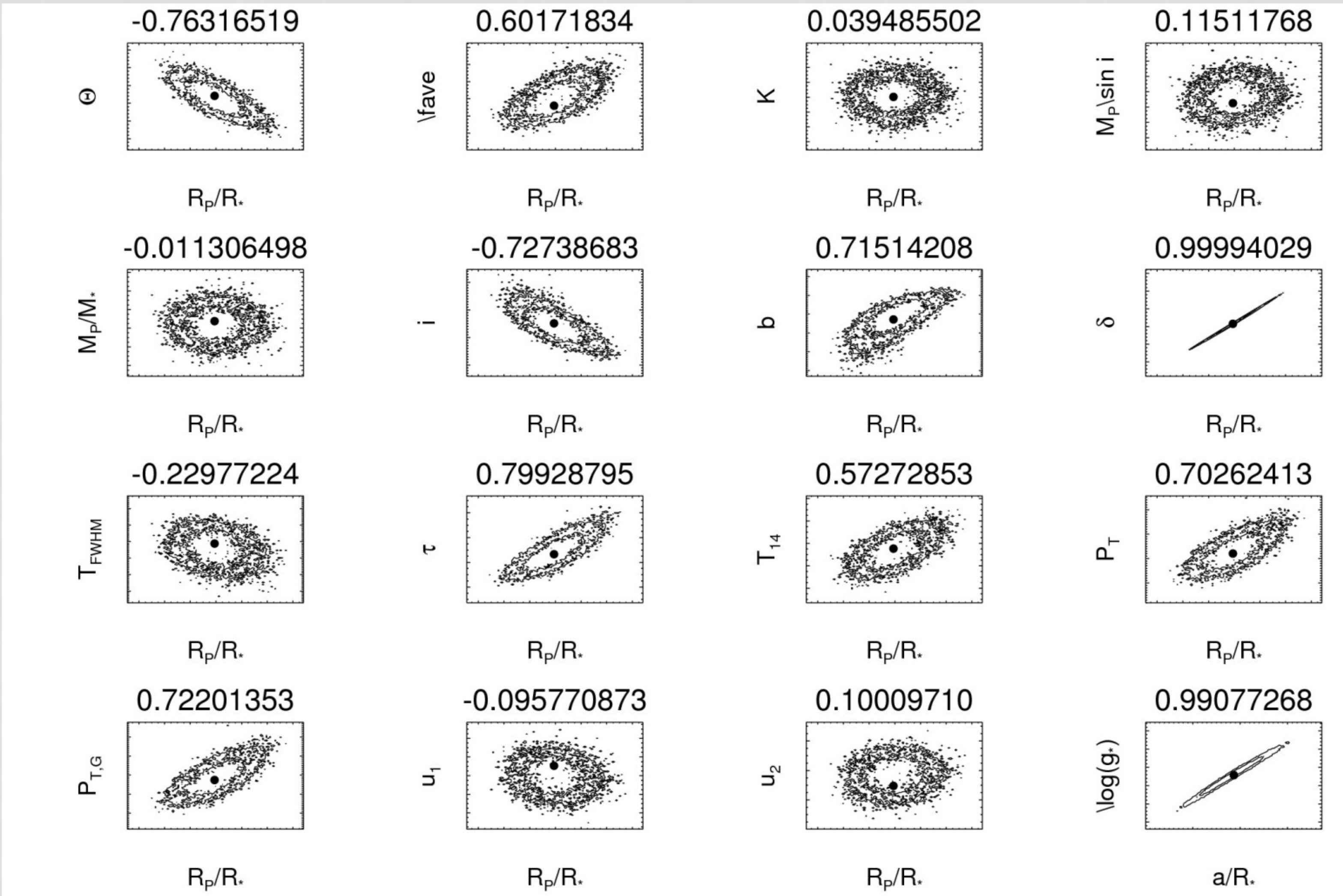


TABLE 1
MEDIAN VALUES AND 68% CONFIDENCE INTERVAL FOR HAT-P-3B.

Parameter	Units	Value
Stellar Parameters:		
M_*	Mass ($M_\odot$)	$0.906^{+0.051}_{-0.047}$
R_*	Radius ($R_\odot$)	$0.771^{+0.046}_{-0.044}$
L_*	Luminosity ($L_\odot$)	$0.384^{+0.059}_{-0.051}$
ρ_*	Density (cgs)	$2.79^{+0.47}_{-0.40}$
$\log(g_*)$	Surface gravity (cgs)	4.621 ± 0.044
$T_{\rm eff}$	Effective temperature (K)	5179^{+79}_{-80}
$[{\rm Fe/H}]$	Metalicity	$0.271^{+0.080}_{-0.079}$
Planetary Parameters:		
P	Period (days)	$2.899703^{+0.000053}_{-0.000054}$
a	Semi-major axis (AU)	$0.03851^{+0.00071}_{-0.00068}$
M_P	Mass (M_J)	$0.590^{+0.025}_{-0.024}$
R_P	Radius (R_J)	$0.826^{+0.061}_{-0.057}$
ρ_P	Density (cgs)	$1.30^{+0.29}_{-0.24}$
$\log(g_P)$	Surface gravity	3.332 ± 0.058
T_{eq}	Equilibrium Temperature (K)	1117 ± 35
Θ	Safronov Number	$0.0607^{+0.0047}_{-0.0044}$
$\langle F \rangle$	Incident flux (10^9 erg s^{-1} cm^{-2})	$0.353^{+0.047}_{-0.042}$
RV Parameters:		
K	RV semi-amplitude (m/s)	89.7 ± 1.9
$M_P \sin i$	Minimum mass (M_J)	$0.590^{+0.025}_{-0.024}$
M_P/M_*	Mass ratio	0.000622 ± 0.000018
γ	Systemic velocity (m/s)	$-14.2^{+1.5}_{-1.4}$
Primary Transit Parameters:		
T_C	Time of transit (BJD_{TDB})	$2454218.76039 \pm 0.00033$
R_P/R_*	Radius of planet in stellar radii	0.1100 ± 0.0019
a/R_*	Semi-major axis in stellar radii	$10.74^{+0.57}_{-0.53}$
i	Inclination (degrees)	$87.37^{+0.62}_{-0.54}$
b	Impact Parameter	$0.494^{+0.071}_{-0.097}$
δ	Transit depth	$0.01211^{+0.00043}_{-0.00041}$
T_{FWHM}	FWHM duration (days)	$0.07470^{+0.00081}_{-0.00083}$
τ	Ingress/egress duration (days)	$0.0109^{+0.0014}_{-0.0012}$
T_{14}	Total duration (days)	$0.0857^{+0.0014}_{-0.0013}$
P_T	A priori non-grazing transit probability	$0.0639^{+0.0071}_{-0.0065}$
$P_{T,G}$	A priori transit probability	$0.0797^{+0.0092}_{-0.0083}$
u_1	linear limb-darkening coefficient	0.421 ± 0.014
u_2	quadratic limb-darkening coefficient	$0.2159^{+0.0086}_{-0.0088}$
F_0	Baseline flux	$1.00629^{+0.00019}_{-0.00020}$
Secondary Eclipse Parameters:		
T_S	Time of eclipse (BJD_{TDB})	$2454220.21024^{+0.00033}_{-0.00034}$

LATEX TABLE

- Round each error to two significant digits
- Round value to precision of error
- LaTeX source code
- Eliminates typos

COMPARISON

$\rho_* = 3M_* / 4\pi R_*^3 = 2.36\ g\ cm^{-3}$

$\rho_* = \frac{3\pi}{GP^2} (a/R_*)^3 = 2.67\ g\ cm^{-3}$

TABLE 2

STELLAR PARAMETERS FOR HAT-P-3

Parameter	Value	Source
T_{eff} (K)	5185 ± 46	SME[a]
[Fe/H]	$+0.27 \pm 0.04$	SME
$\log g_\star$ (cgs)	4.61 ± 0.05	SME
$v \sin i$ (km s^{-1})	0.5 ± 0.5	SME
$M_\star$ ($M_\odot$)	$0.936^{+0.036}_{-0.062}$	Y^2+LC+SME[b]
$R_\star$ ($R_\odot$)	$0.824^{+0.043}_{-0.035}$	Y^2+LC+SME
$L_\star$ ($L_\odot$)	$0.442^{+0.078}_{-0.057}$	Y^2+LC+SME
M_V (mag)	5.86 ± 0.20	Y^2+LC+SME
Age (Gyr)	$0.4^{+6.5}_{-0.3}$	Y^2+LC+SME
Distance (pc)	140 ± 13	Y^2+LC+SME

[a] SME = "Spectroscopy Made Easy" package for analysis of high-resolution spectra (Valenti & Piskunov 1996). See text.

[b] Y^2+LC+SME = Yale-Yonsei isochrones (Yi et al. 2001), light curve parameters, and SME results.

Torres, et. al., 2007

TABLE 3

SPECTROSCOPIC AND LIGHT CURVE SOLUTIONS FOR HAT-P-3, AND INFERRED PLANET PARAMETERS

Parameter	Value
Spectroscopic parameters:	
P[a] (days)	2.899703 ± 0.000054
T_c[a] (HJD − 2,400,000)	$54,218.7594 \pm 0.0029$
K (m s^{-1})	89.1 ± 2.0
γ (m s^{-1})	-14.8 ± 1.5
e	0 (adopted)
Light curve parameters:	
$a/R_\star$	$10.59^{+0.66}_{-0.84}$
$R_p/R_\star$	$0.1109^{+0.002}_{-0.0022}$ [illegible]
$b \equiv a \cos i/R_\star$	$0.51^{+0.11}_{-0.13}$
i (deg)	87.24 ± 0.69
Transit duration (days)	0.0858 ± 0.0020
Planet parameters:	
M_p (M_{Jup})	0.599 ± 0.026
R_p (R_{Jup})	0.890 ± 0.046
ρ_p (g cm^{-3})	1.06 ± 0.17
a (AU)	0.03894 ± 0.00070
$\log g_p$ (cgs)	3.310 ± 0.066

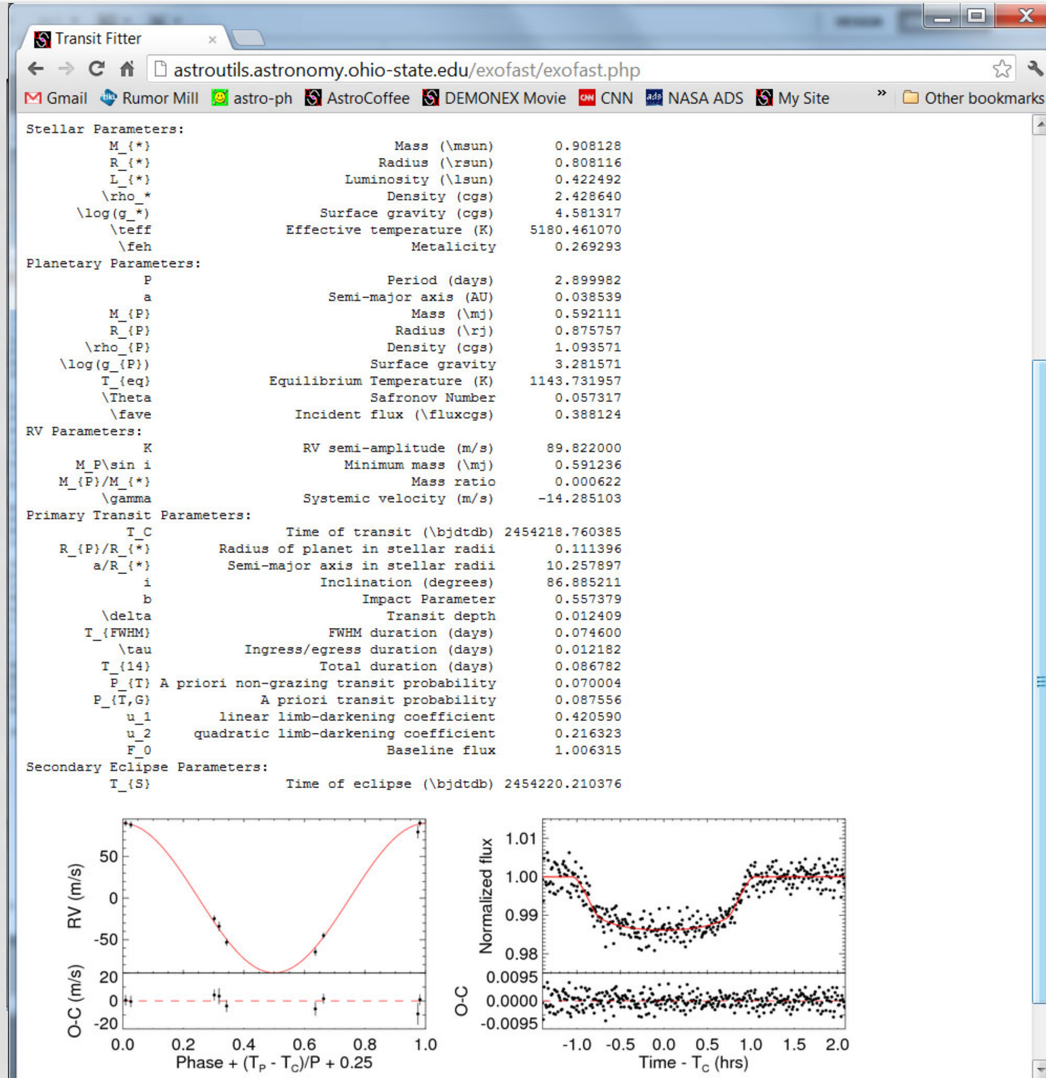
[a] Held fixed from the photometric determination (§ 2).

Ours is 10x smaller - typo?

ONLINE TOOLS

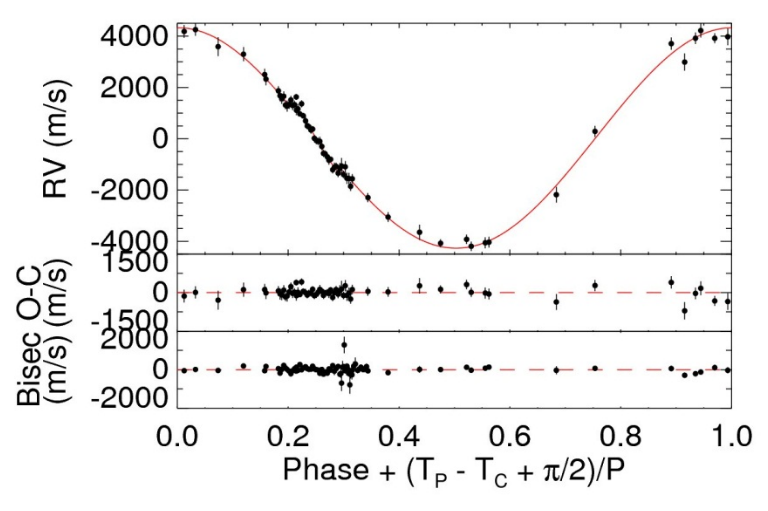
HTTP://ASTROUTILS.ASTRONOMY.OHIO-STATE.EDU/EXOFAST/

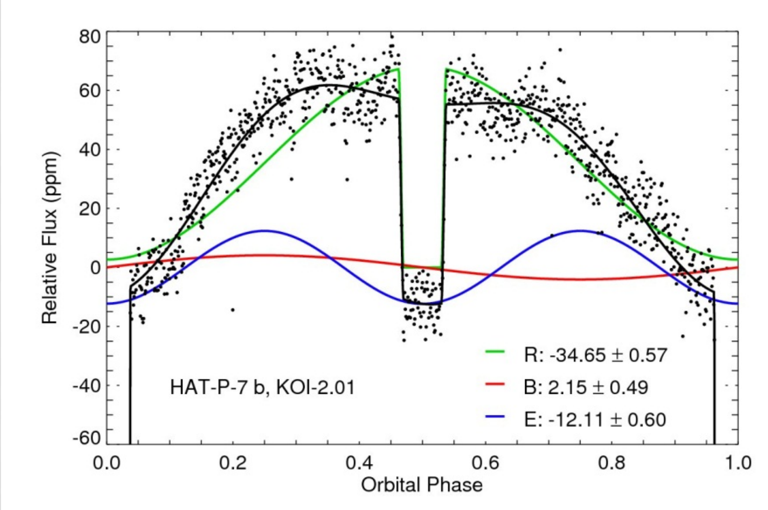
- RV fitting
  - Slope/no slope
  - Circular/eccentric
- Transit fitting
  - Includes detrending
- Ephemeris generator
- Limb darkening
- Linked to exoplanets.org



EXTENSIBLE

- Modular framework
- Add additional effects
  - Rossiter Mclaughlin, TTVs (Siverd et al., 2012)
  - Secondary eclipses, Beaming, reflection, ellipsoidal variations (Shporer, Tuesday)
- Or even completely different models using same core routines





SUMMARY

- Differential Evolution MCMC is the only way to go
- Careful attention to eccentricity; less biased
- Simultaneous fitting
  - takes advantage of all data
  - eliminates overlapping free parameters
  - automatically accounts for covariances
  - Self-consistent (ρ_* is the same from light curve and stellar parameters)
- Created EXOFAST for the community
  - http://astroutils.astronomy.ohio-state.edu/exofast/