Looking for Moons

David M. Kipping[1,2]

[1] Harvard-Smithsonian Center for Astrophysics; [2] Carl Sagan Fellow

Outline of this Talk

I. Background

II. Theory

III. Modeling

IV. Challenges

V. Synthetic Example

I. Background

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

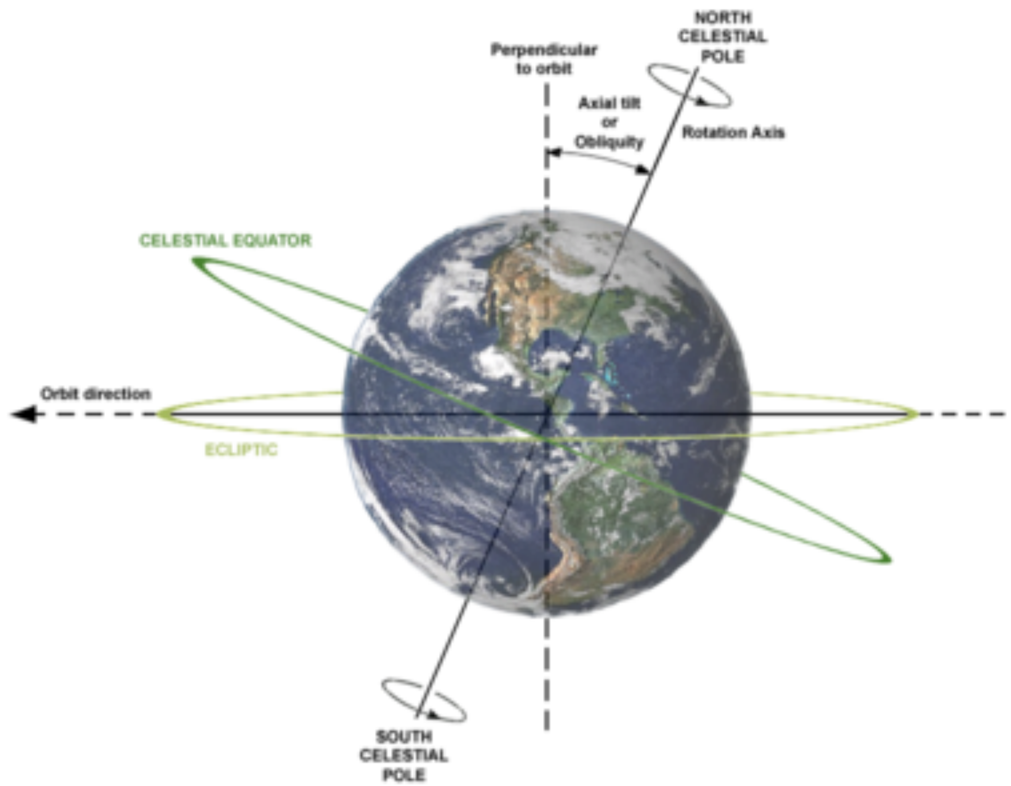
- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

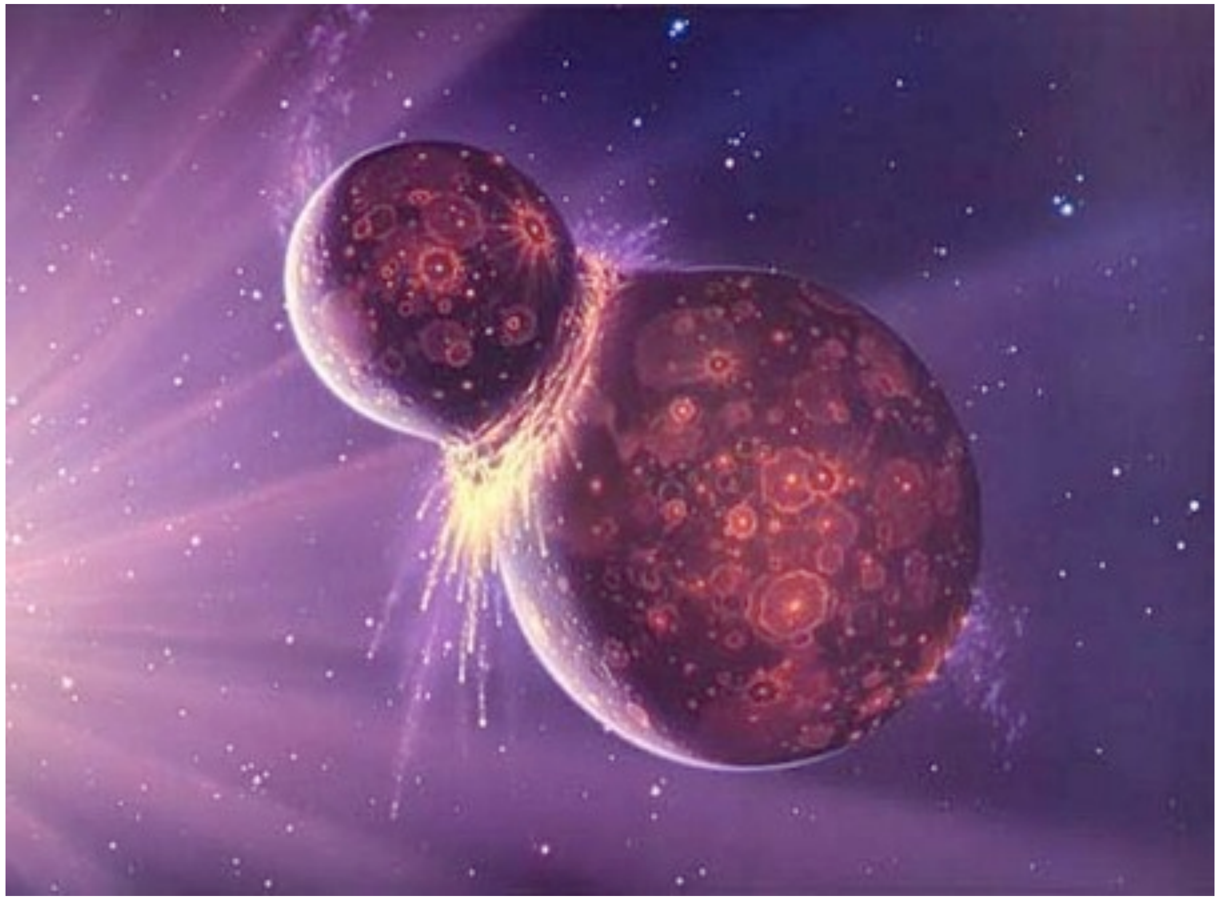


Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory

Why bother?

- Intrinsic habitability
- Extrinsic habitability
- Planet formation theory



Order-of-Magnitude Feasibility

- Roughly, Kepler is sensitive to ~1$R_\oplus$ planets
- ⇒Kepler is sensitive to ~1$R_\oplus$ moons
- We may be able to detect Earth-sized/mass moons

Large Exomoons

- Largest known moon is Ganymede
- $R=0.413\ R_{\oplus}$; $M=0.025\ M_{\oplus}$
- Not Earth-sized or mass => likely undetectable

Large Regular Moons?

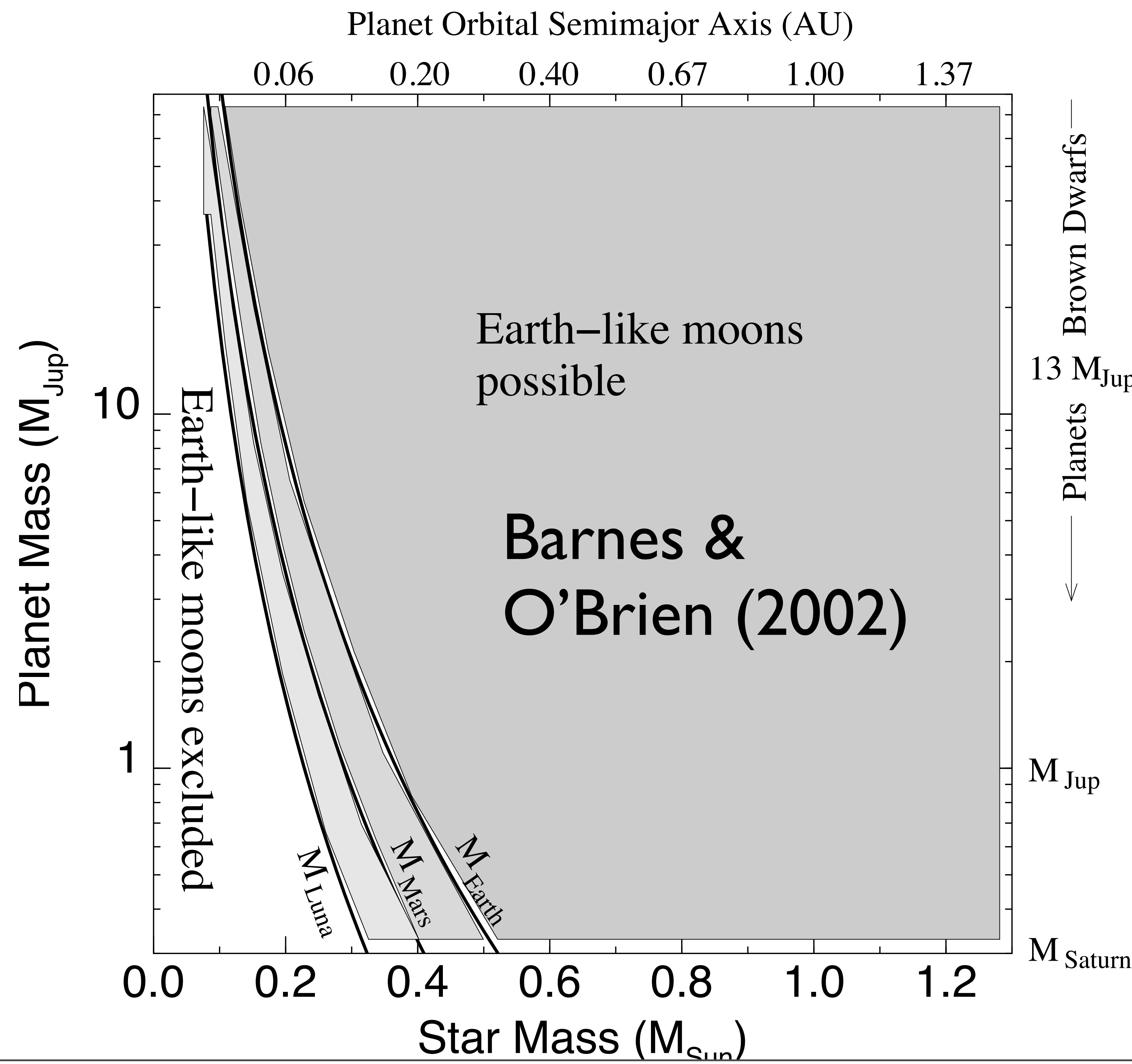
- *Two classes of satellites: Regular & Irregular*
- Regular satellites form in orbit of host planet
- Examples: Galilean satellites, Titan
- Canup & Ward (2004) argue this process limits $\sum m_i \lesssim (2 * 10^{-4}) M_P$
- Bad news for large moons

Large Irregular Moons?

- *Two classes of satellites: Regular & Irregular*
- Irregular satellites come from elsewhere
- Examples: Triton, the Moon
- No obvious limit. We only require dynamical stability.
- Good news for large moons?

Large Stable Moons

- Moon must be within the Hill sphere
- Moon tends to spin-in/ out due to tides



Barnes & O'Brien (2002)

Checklist

Checklist

✓ Motivation to look for exomoons

Checklist

✓ Motivation to look for exomoons

✓ Feasible existence of large (detectable) moons

Checklist

✓ Motivation to look for exomoons

✓ Feasible existence of large (detectable) moons

? Viable method to detect such moons

II. Theory

Transits

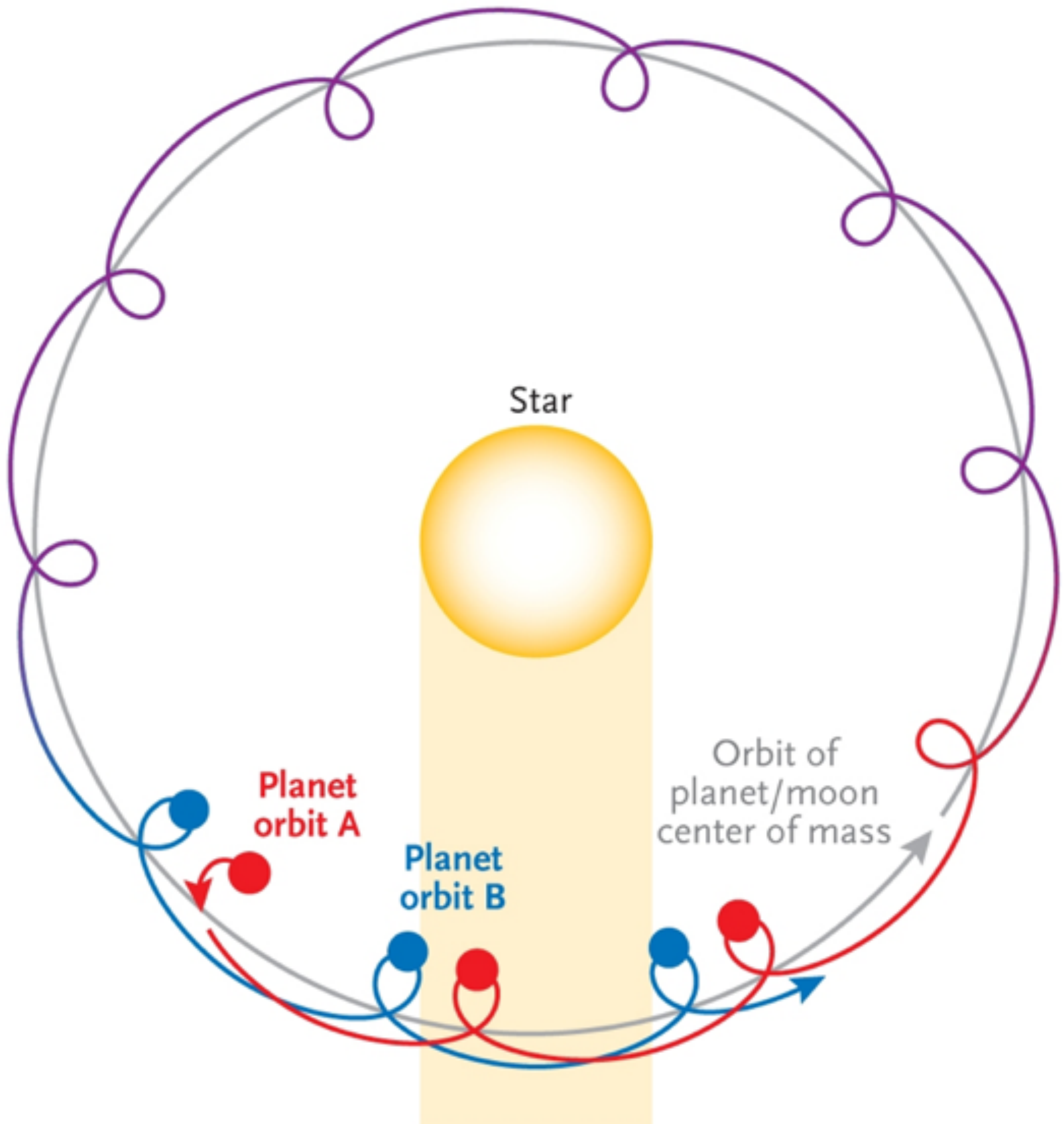
- The remainder of this talk will focus on the transit method.
- ***This is not the only method to detect exomoons.***
- Notably, microlensing is a highly viable method.
- Astrometry, radial velocity, direct imaging, eclipse timing, pulsar timing are less viable.

Observational Consequences

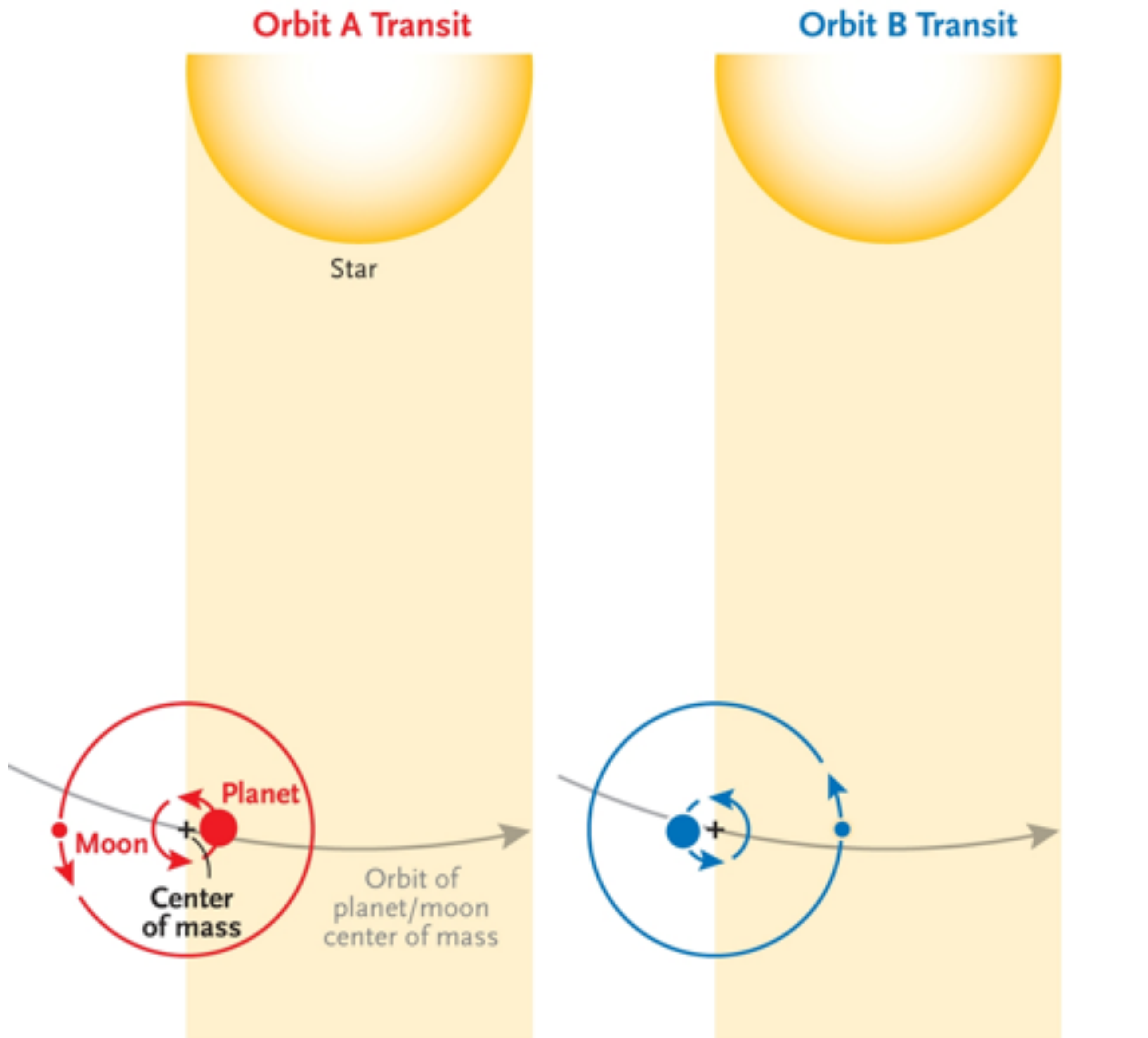
1. Dynamical effects (gives M_S)
2. Eclipse effects (gives R_S)

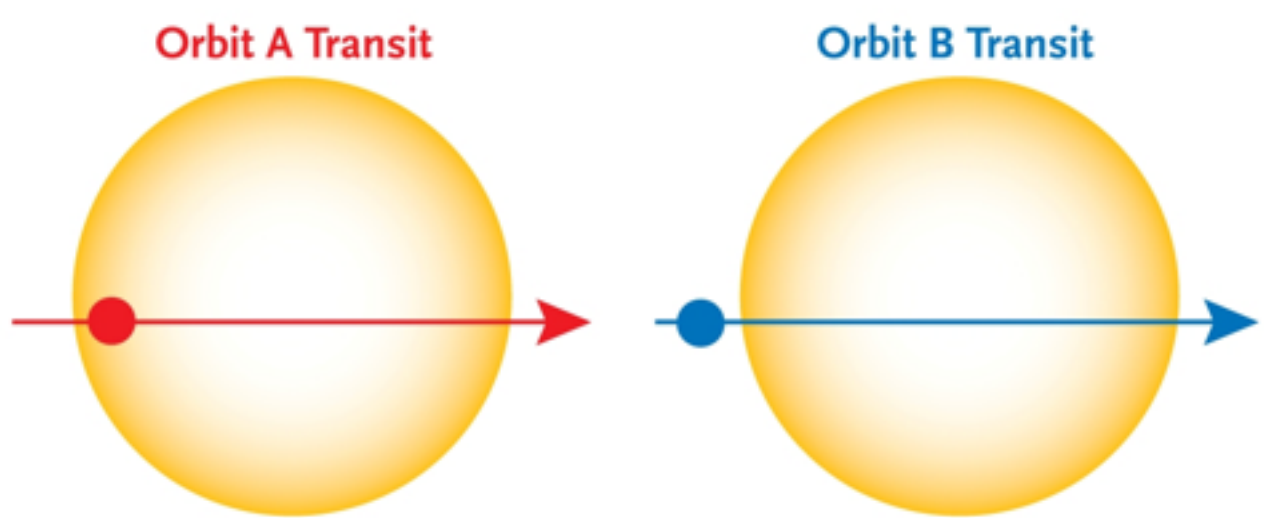
Transit timing variations (TTV)

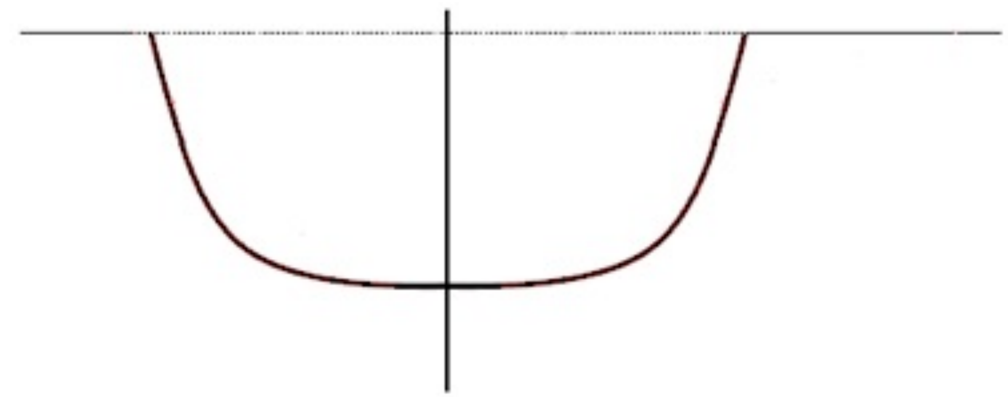
Transit timing variations (TTV)



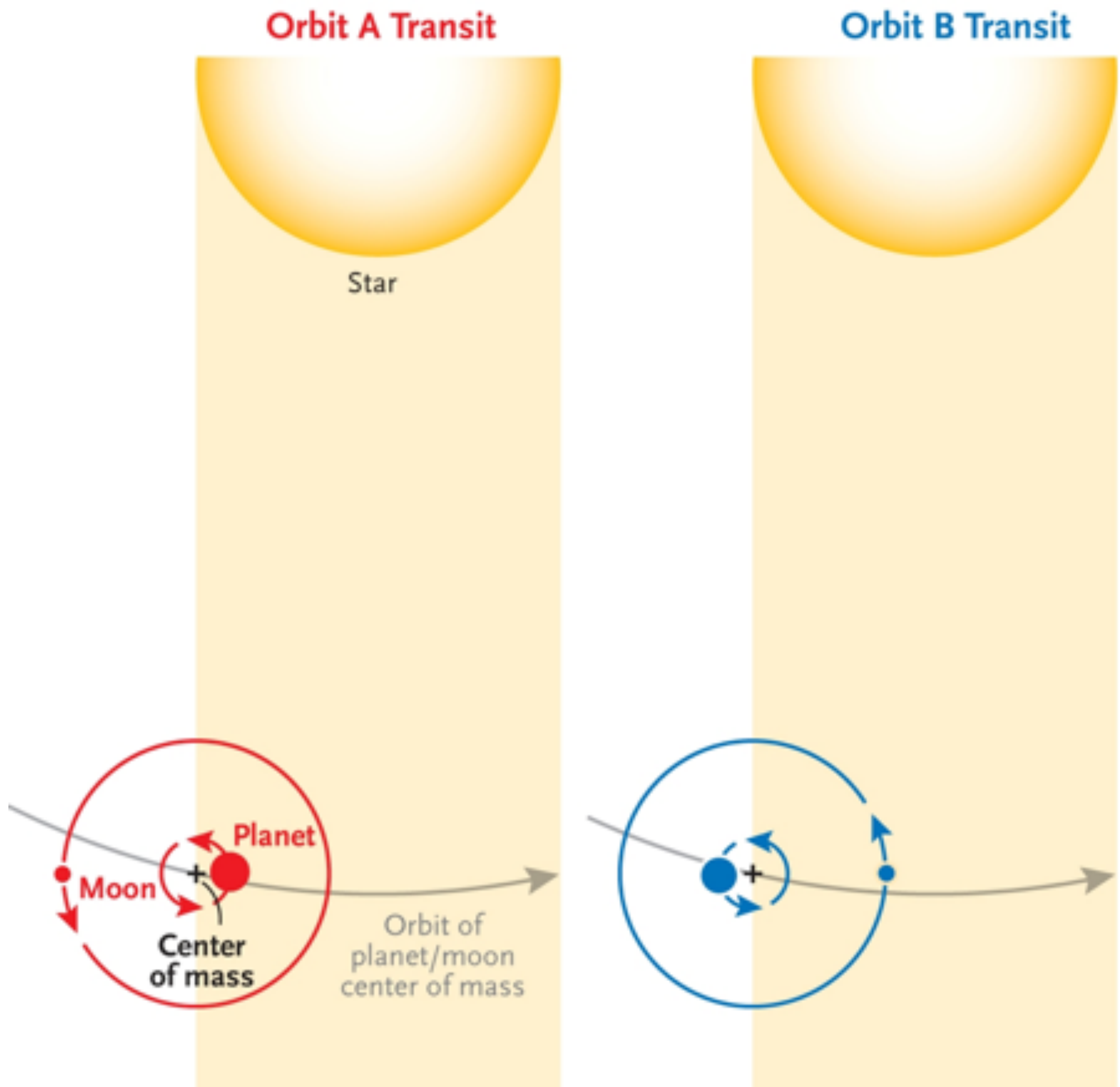
Transit Timing Variation (TTV)

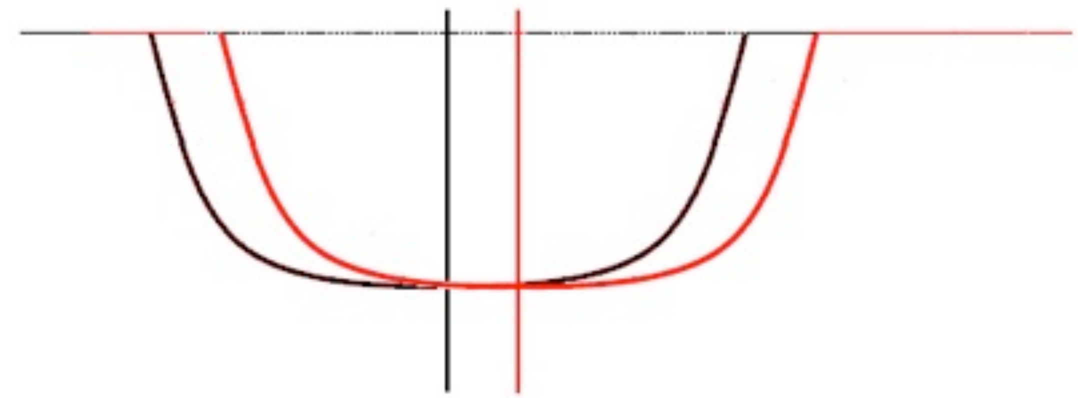


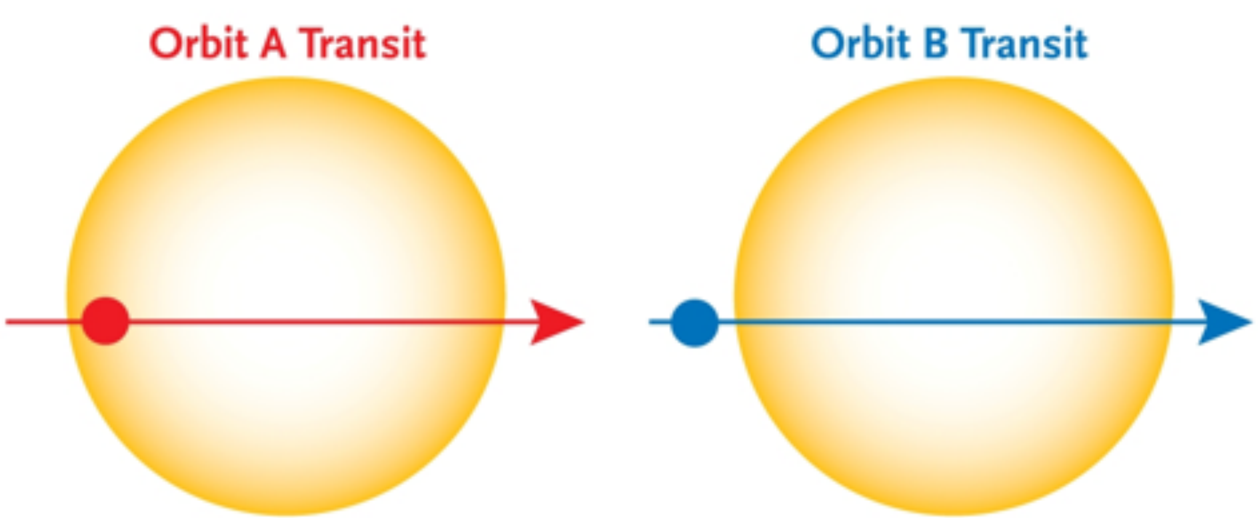




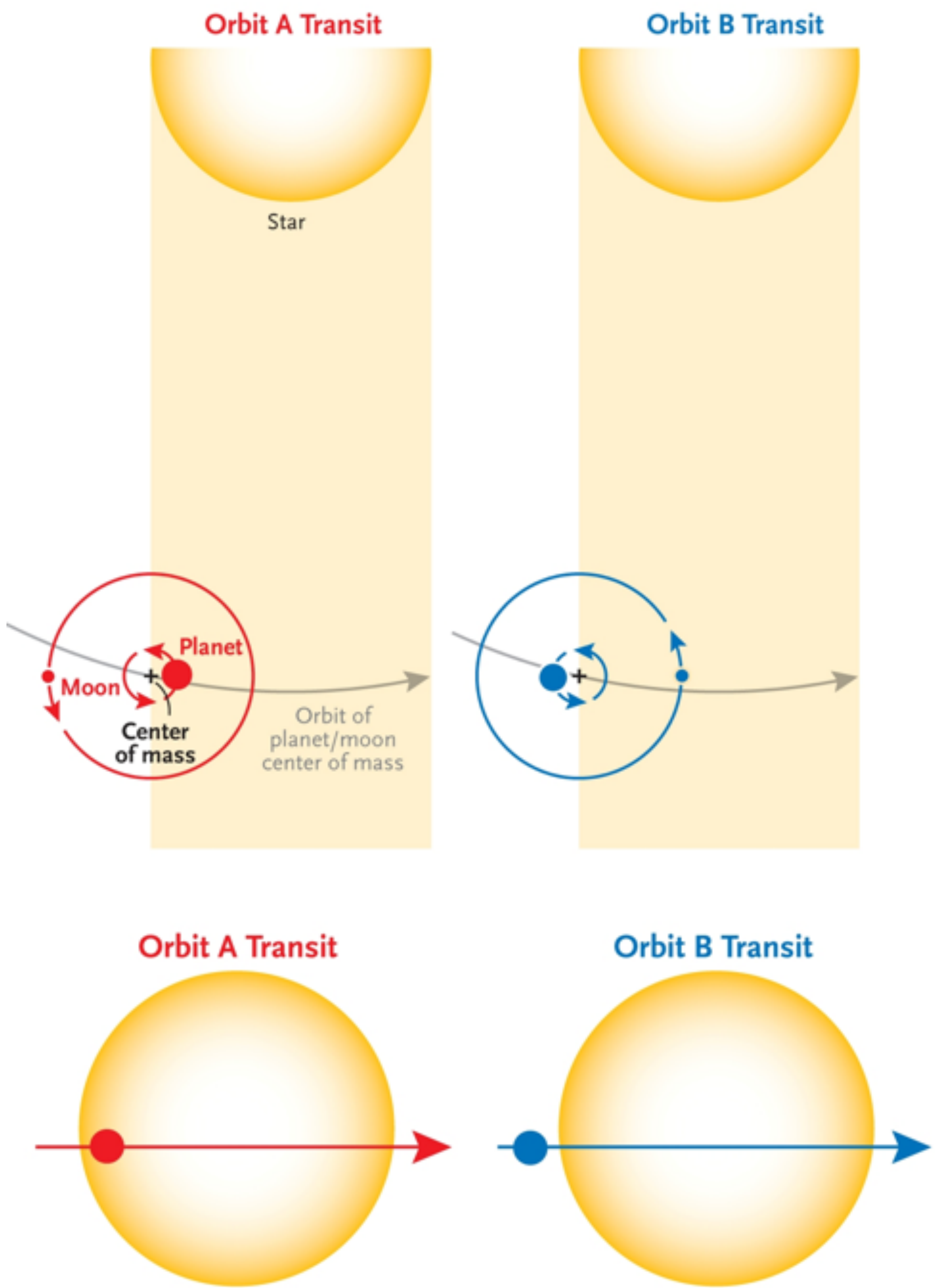
Transit Timing Variation (TTV)

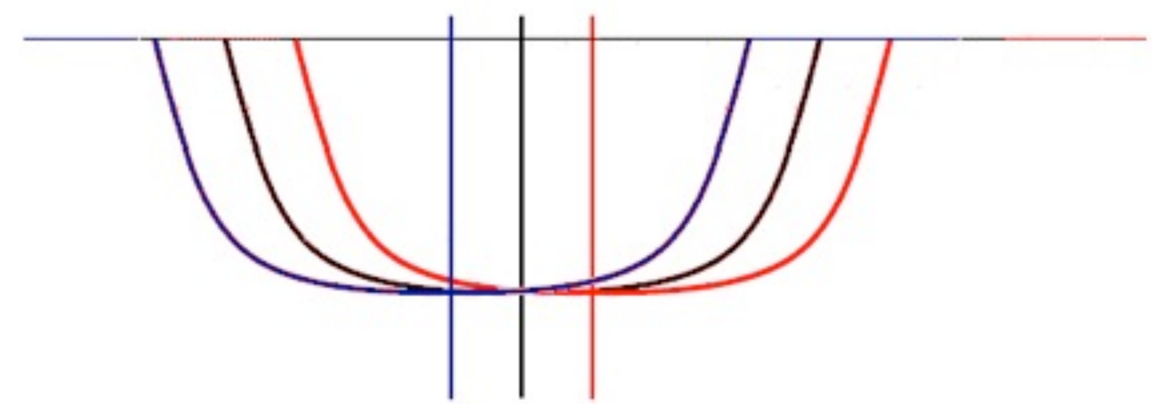




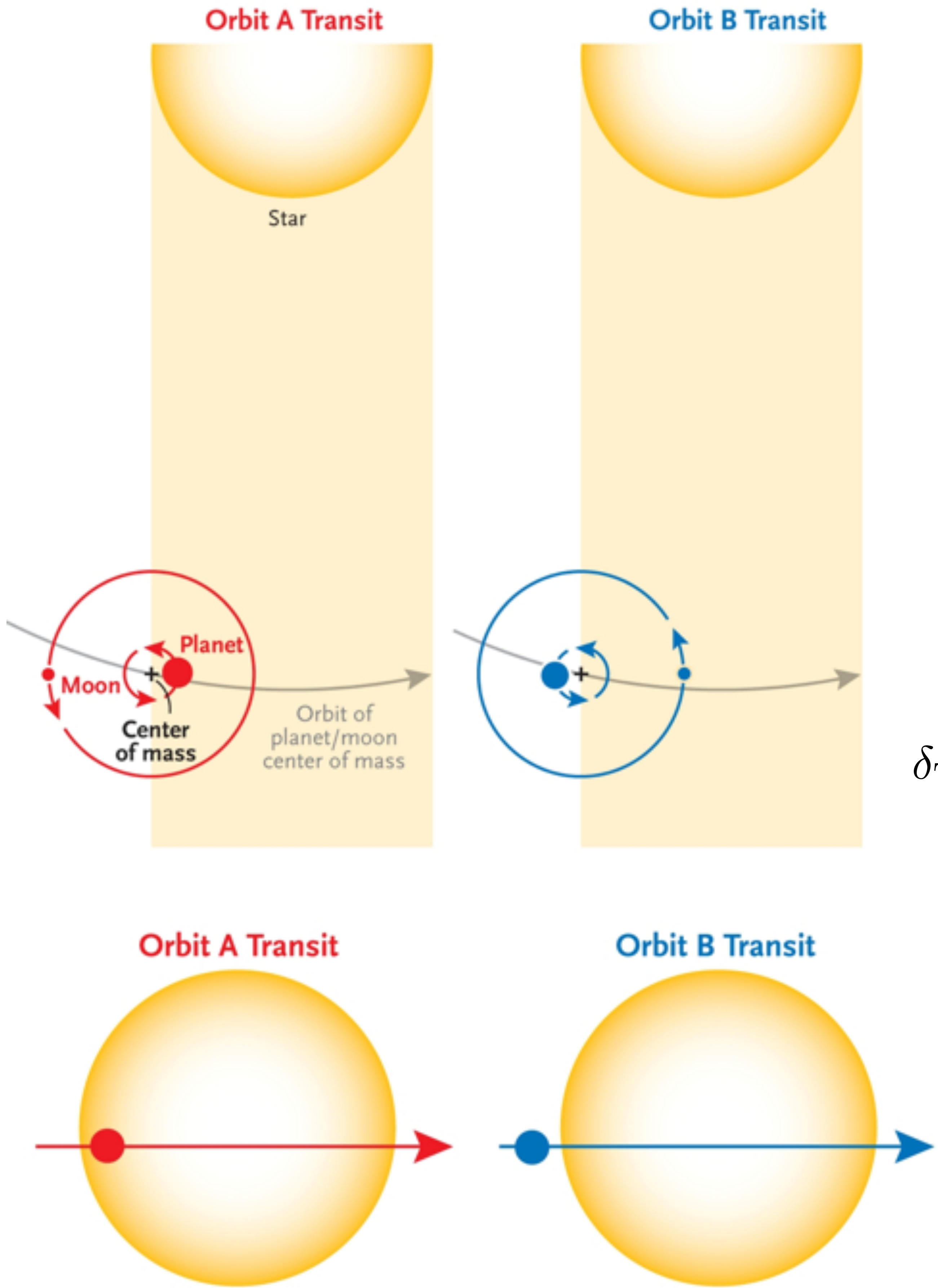


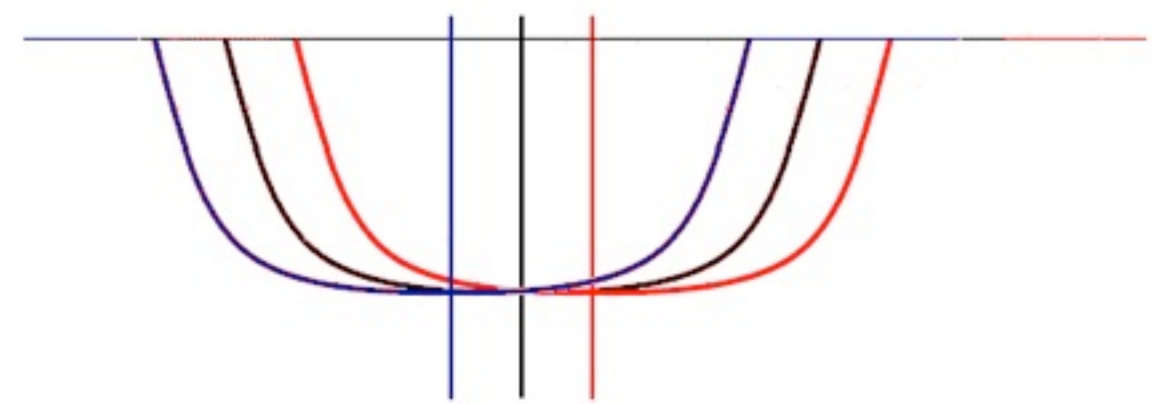
Transit Timing Variation (TTV)



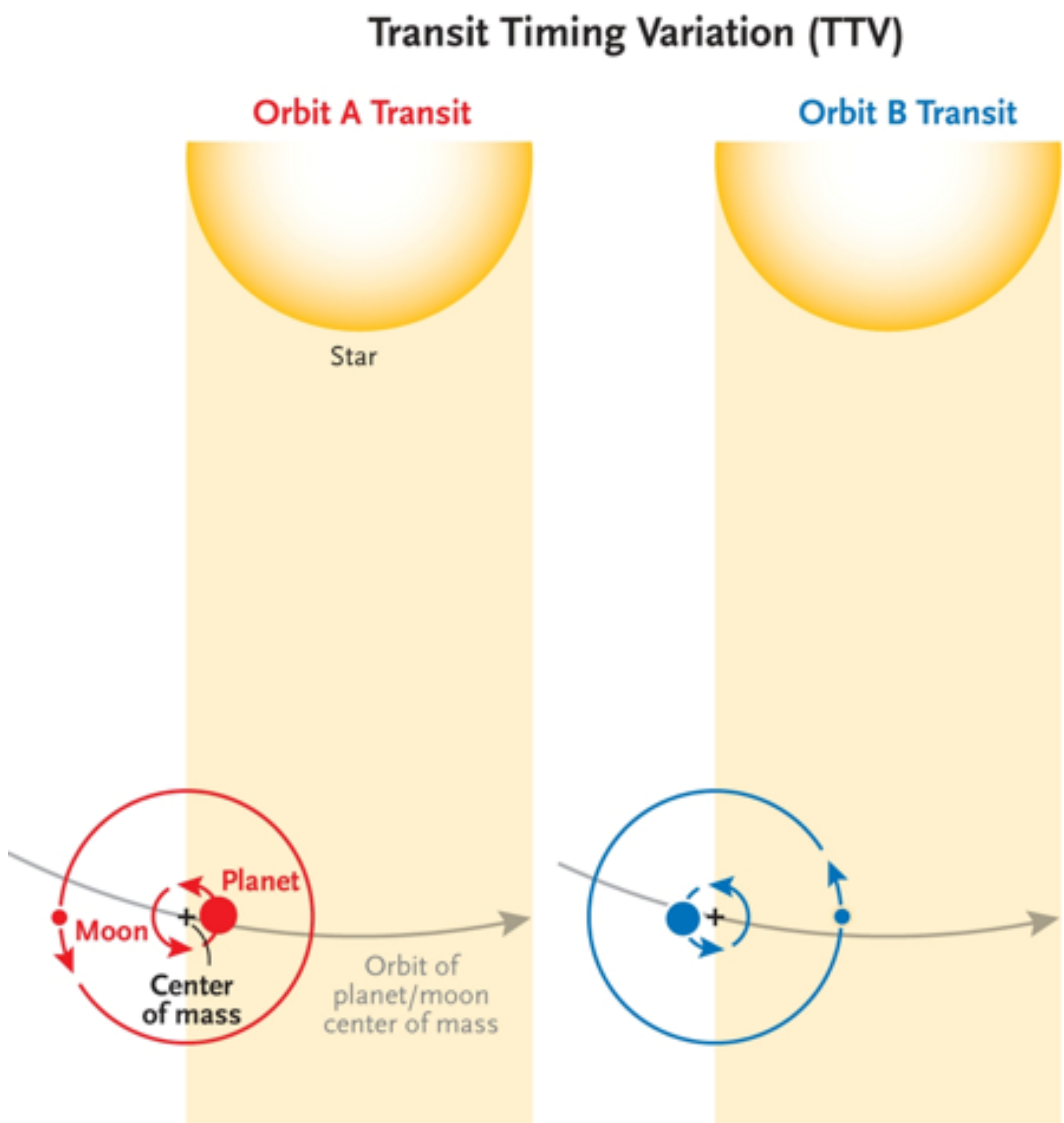


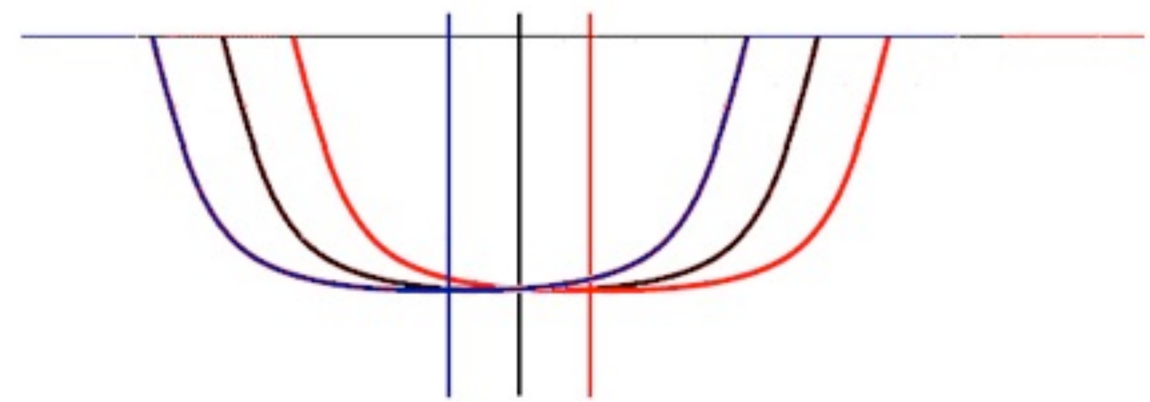
Transit Timing Variation (TTV)



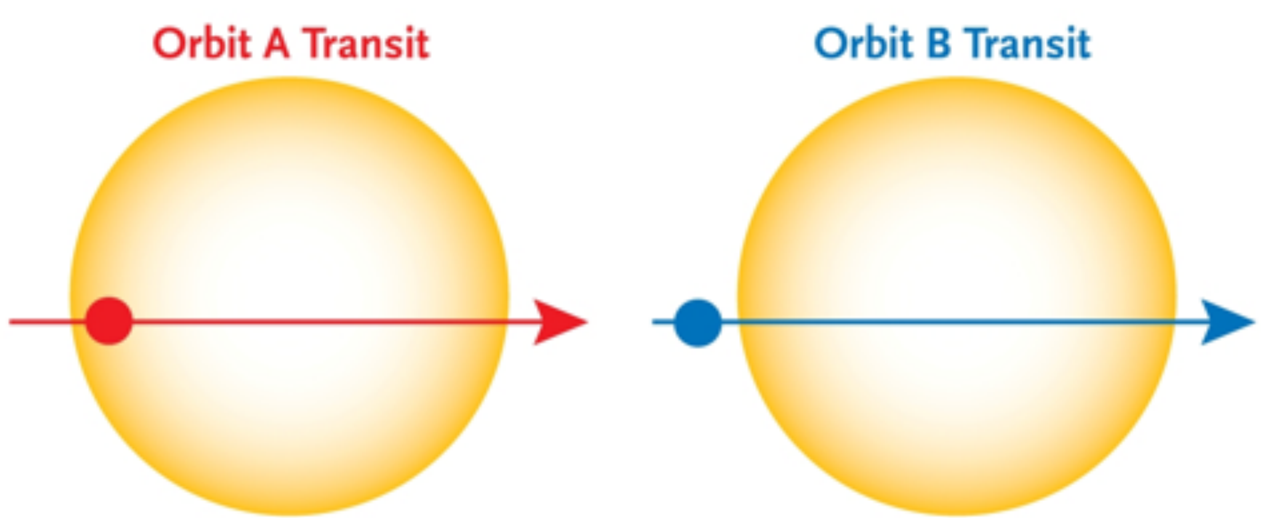


$$\delta_{\mathrm{TTV}} = \frac{a_S M_S P_P}{a_P M_P} \frac{(1-e_S^2)\sqrt{1-e_P^2}}{(1+e_P \sin\omega_P)} \sqrt{\frac{\Phi_{\mathrm{TTV}}}{2\pi}}$$

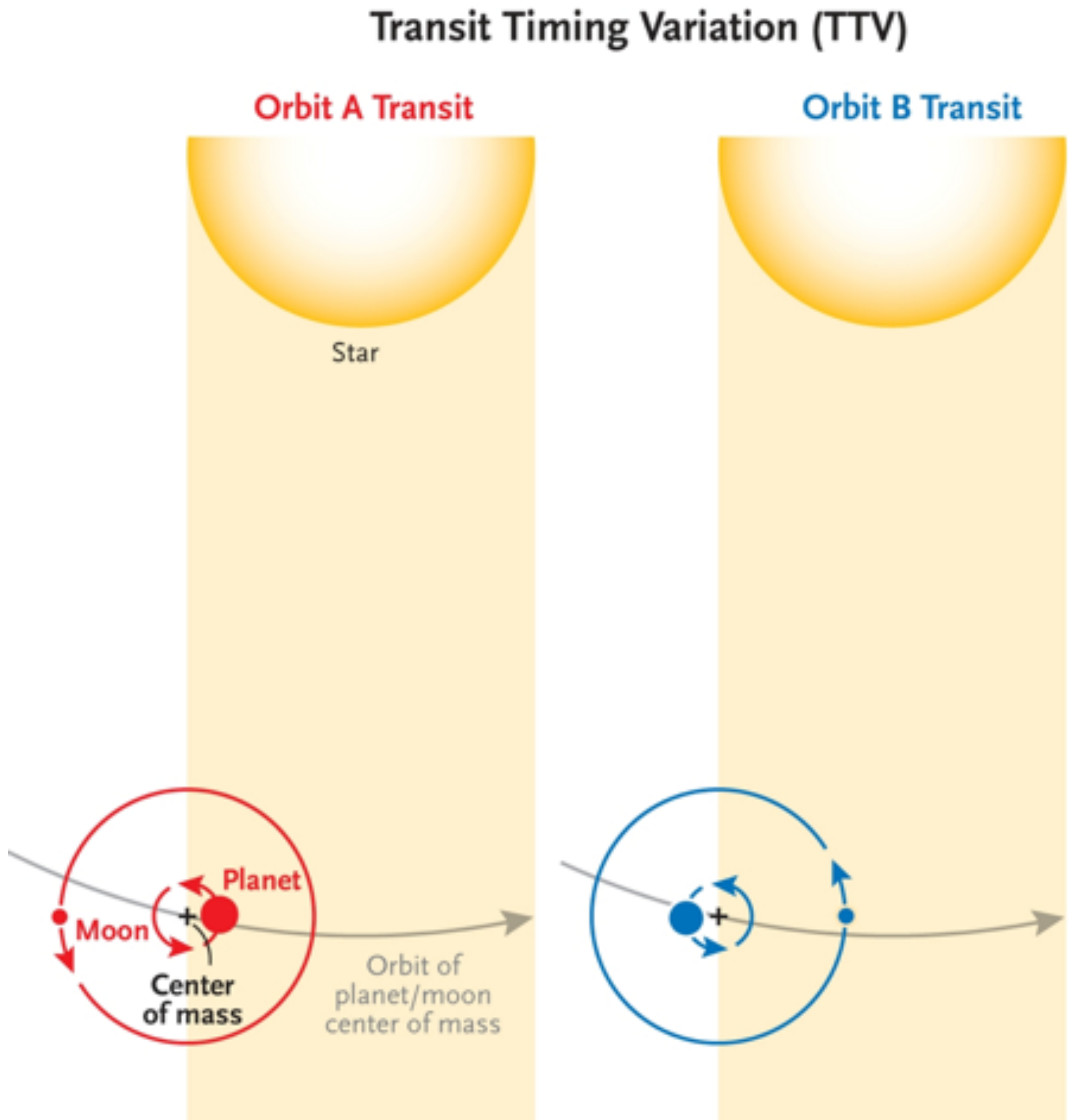




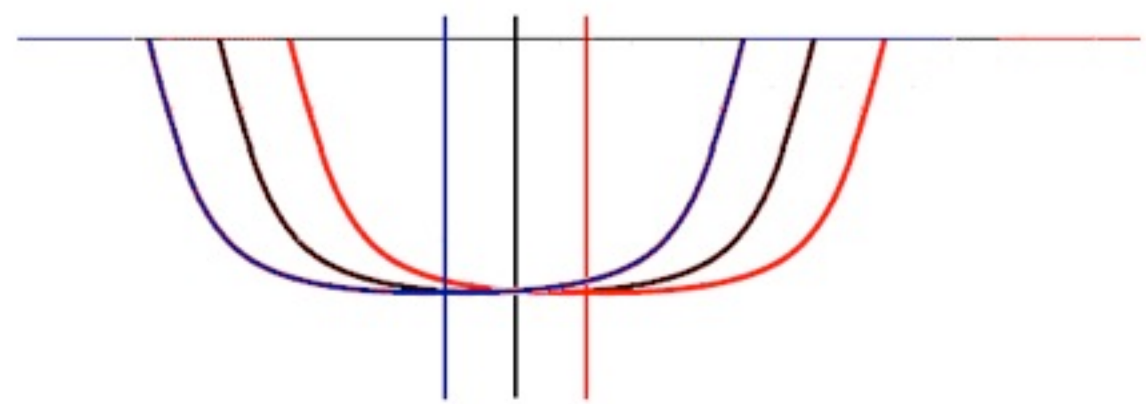
$$\delta_{\rm TTV} = \frac{a_S M_S P_P}{a_P M_P} \frac{(1-e_S^2)\sqrt{1-e_P^2}}{(1+e_P \sin\omega_P)} \sqrt{\frac{\Phi_{\rm TTV}}{2\pi}}$$



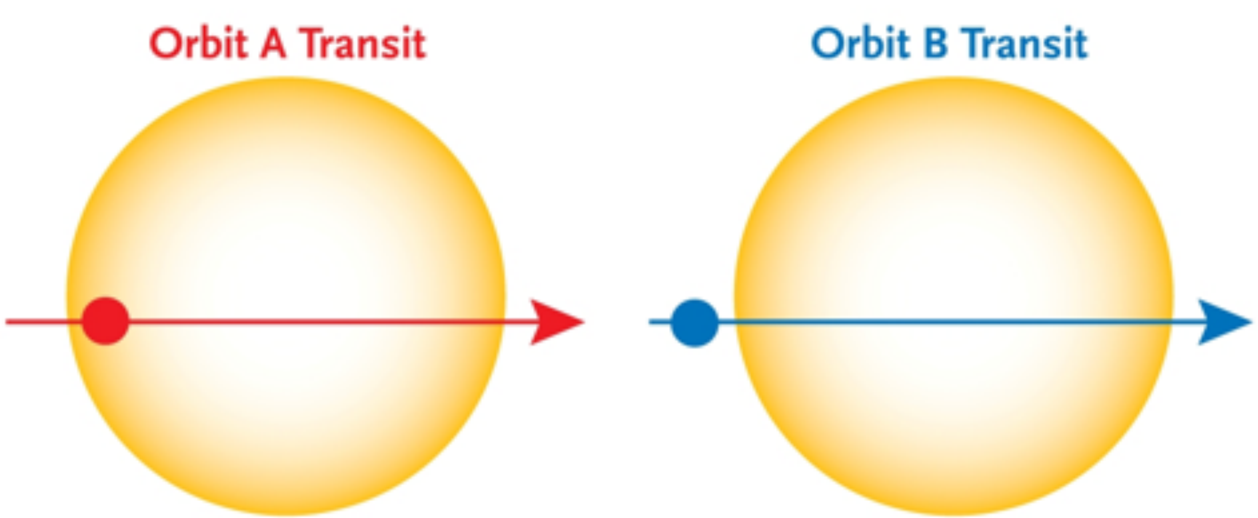
TTV~ $a_S M_S$



TTV is analogous to astrometry



$$\delta_{\mathrm{TTV}} = \frac{a_S M_S P_P}{a_P M_P} \frac{(1-e_S^2)\sqrt{1-e_P^2}}{(1+e_P \sin \omega_P)} \sqrt{\frac{\Phi_{\mathrm{TTV}}}{2\pi}}$$

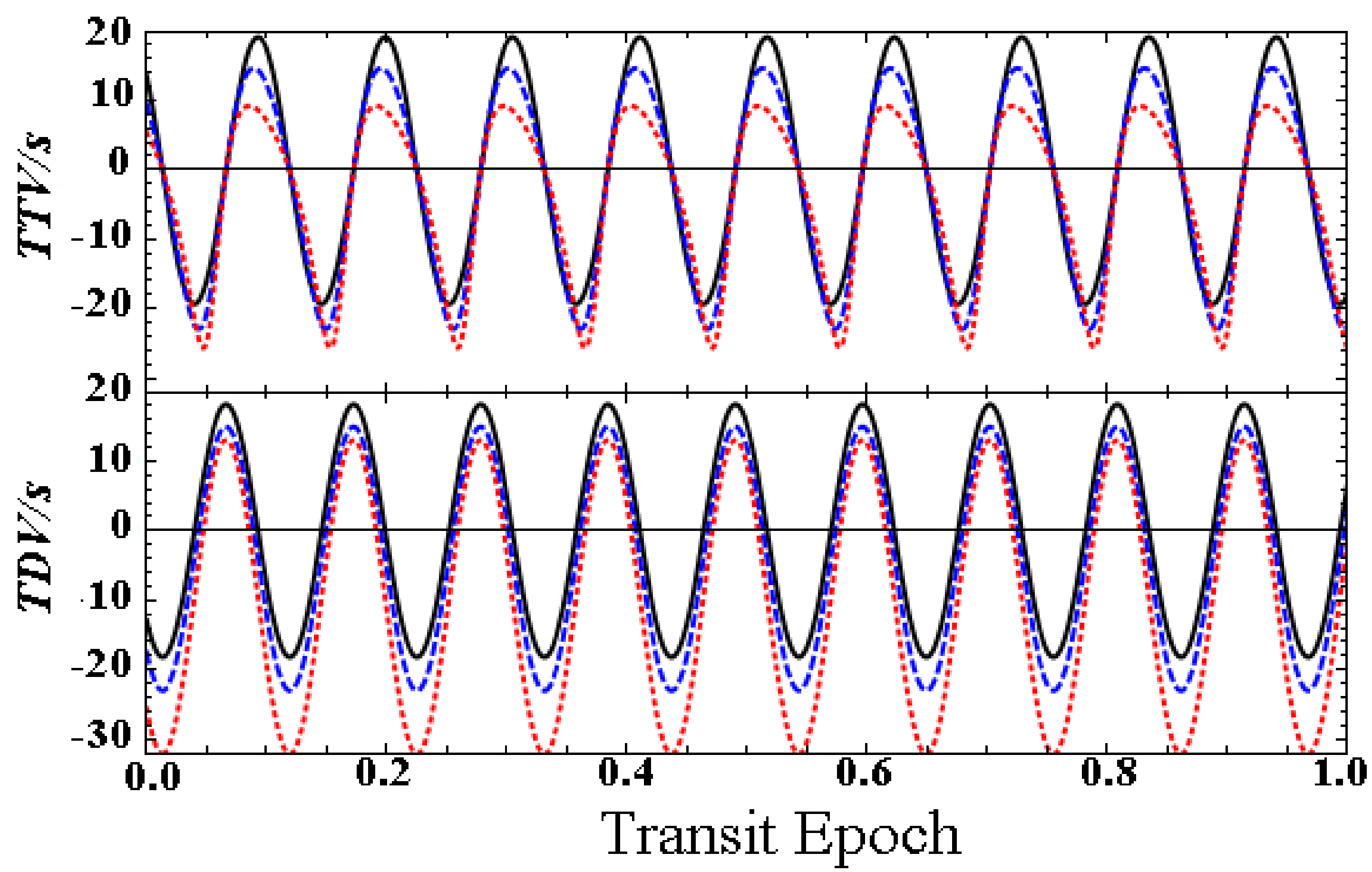


TTV~ $a_S M_S$

Undersampling

- TTV~ $a_S M_S$ and usually one gets a_S by measuring P_S and using Kepler's Third Law.
- But, $P_S < P_P$ and usually $P_S \ll P_P$
- We only measure a TTV once per P_P ⇒heavy undersampling

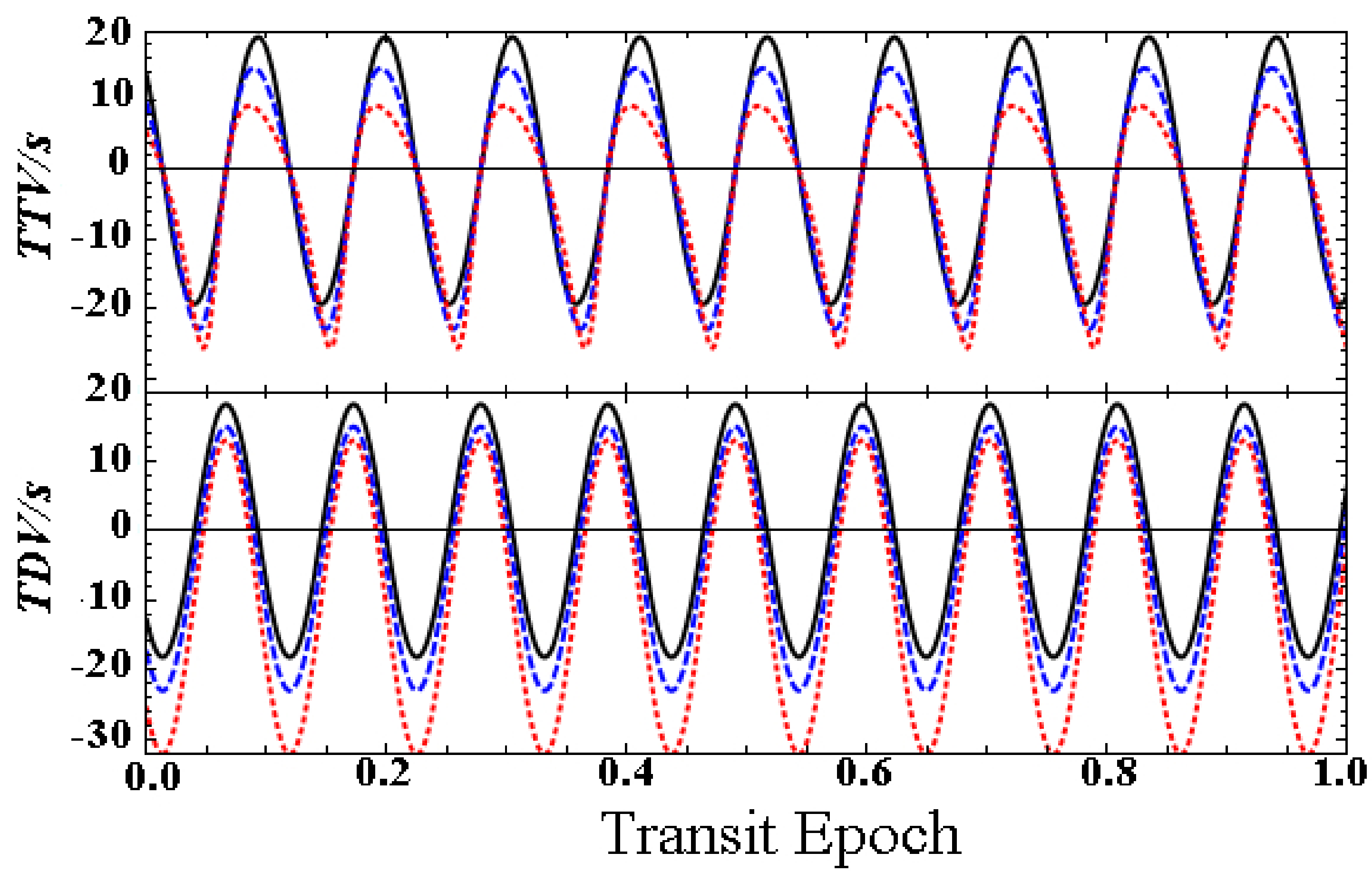
$$P_S = P_P \mathfrak{D}^{3/2}\left(\frac{1}{3}\right)^{1/2}$$



Undersampling

- TTV~ $a_S M_S$ and usually one gets a_S by measuring P_S and using Kepler's Third Law.
- But, $P_S < P_P$ and usually $P_S \ll P_P$
- We only measure a TTV once per P_P ⇒heavy undersampling

$$P_S = P_P \mathfrak{D}^{3/2} \left(\frac{1}{3}\right)^{1/2}$$



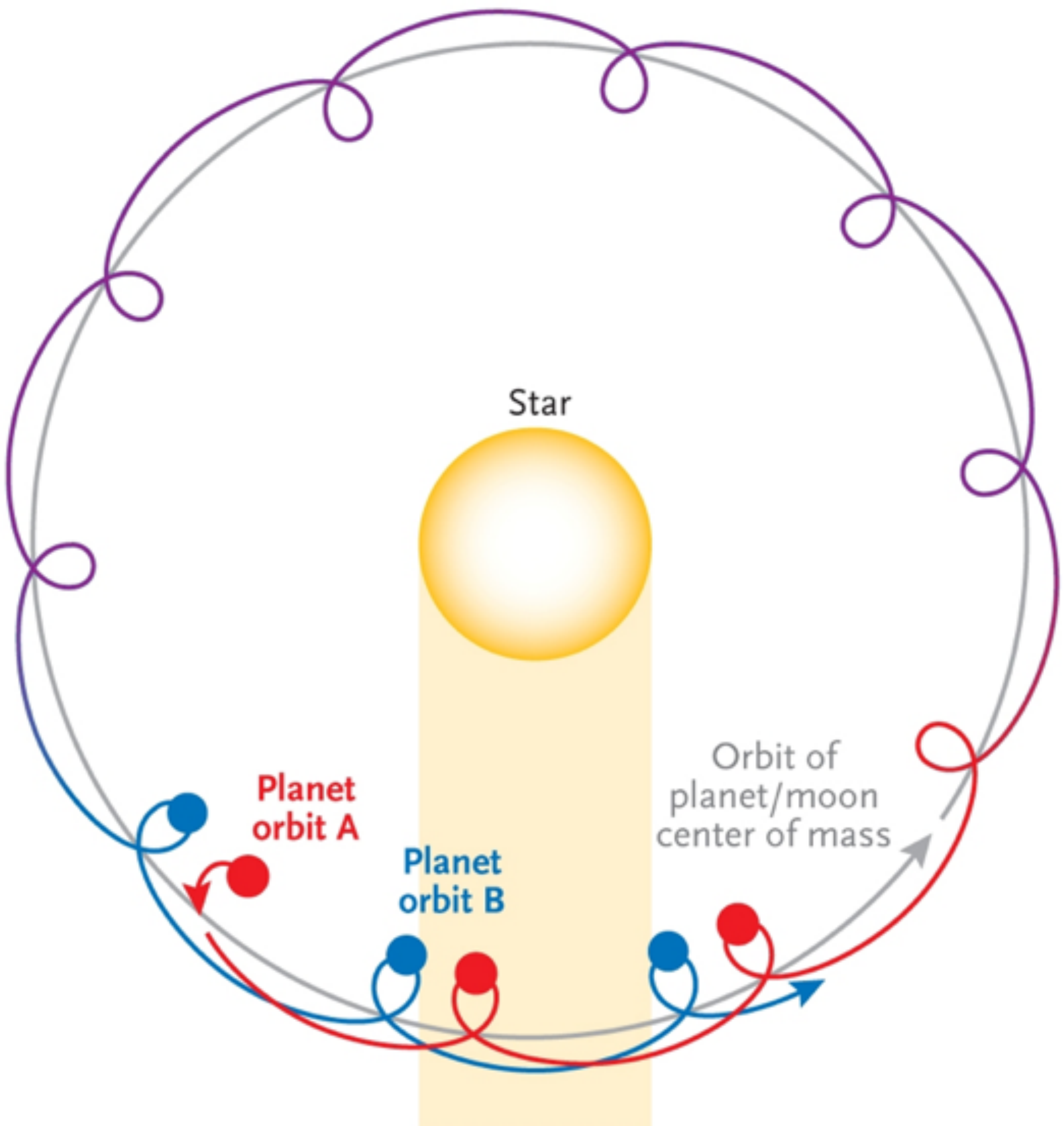
PROBLEM 1:
⇒Cannot measure P_S!

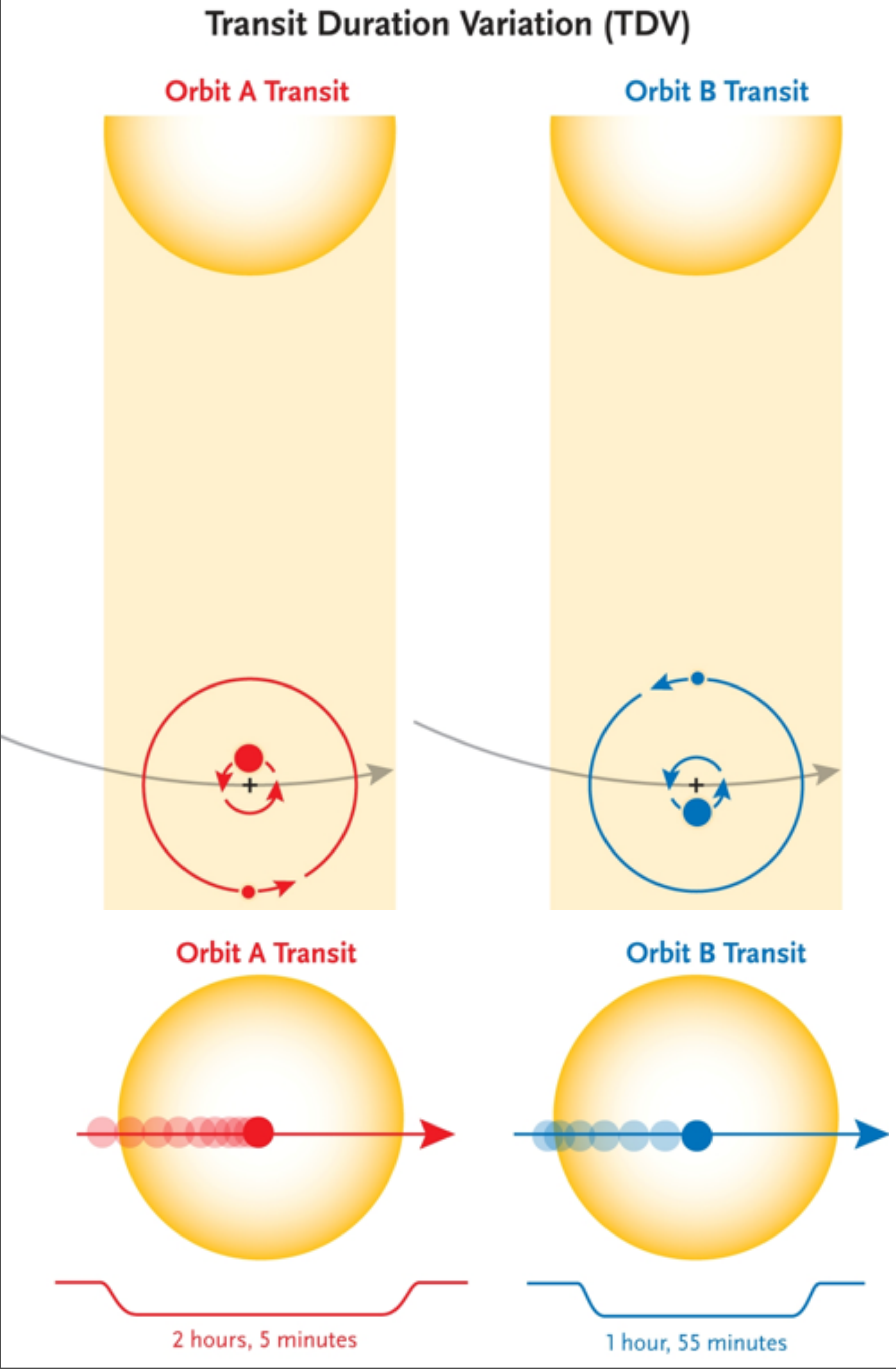
⇒Cannot measure M_S!

Also...

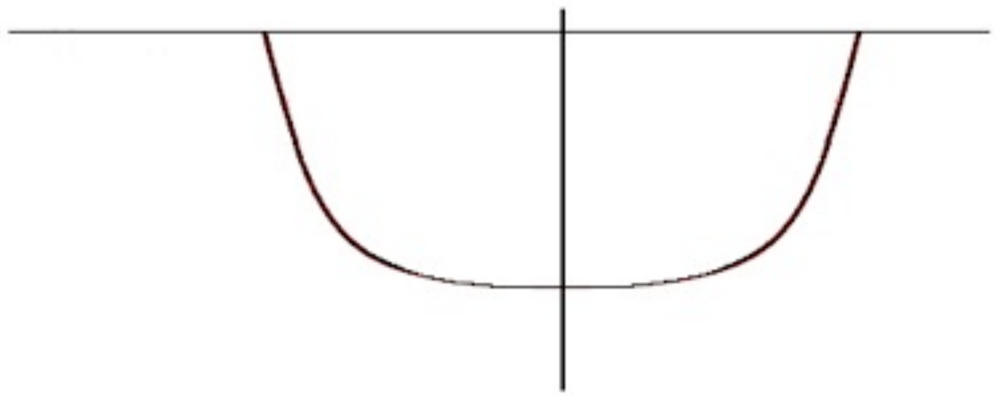
PROBLEM 2:
How do you tell the difference between an exomoon TTV and a second planet TTV?

Velocity-induced transit duration variations (TDV-V)

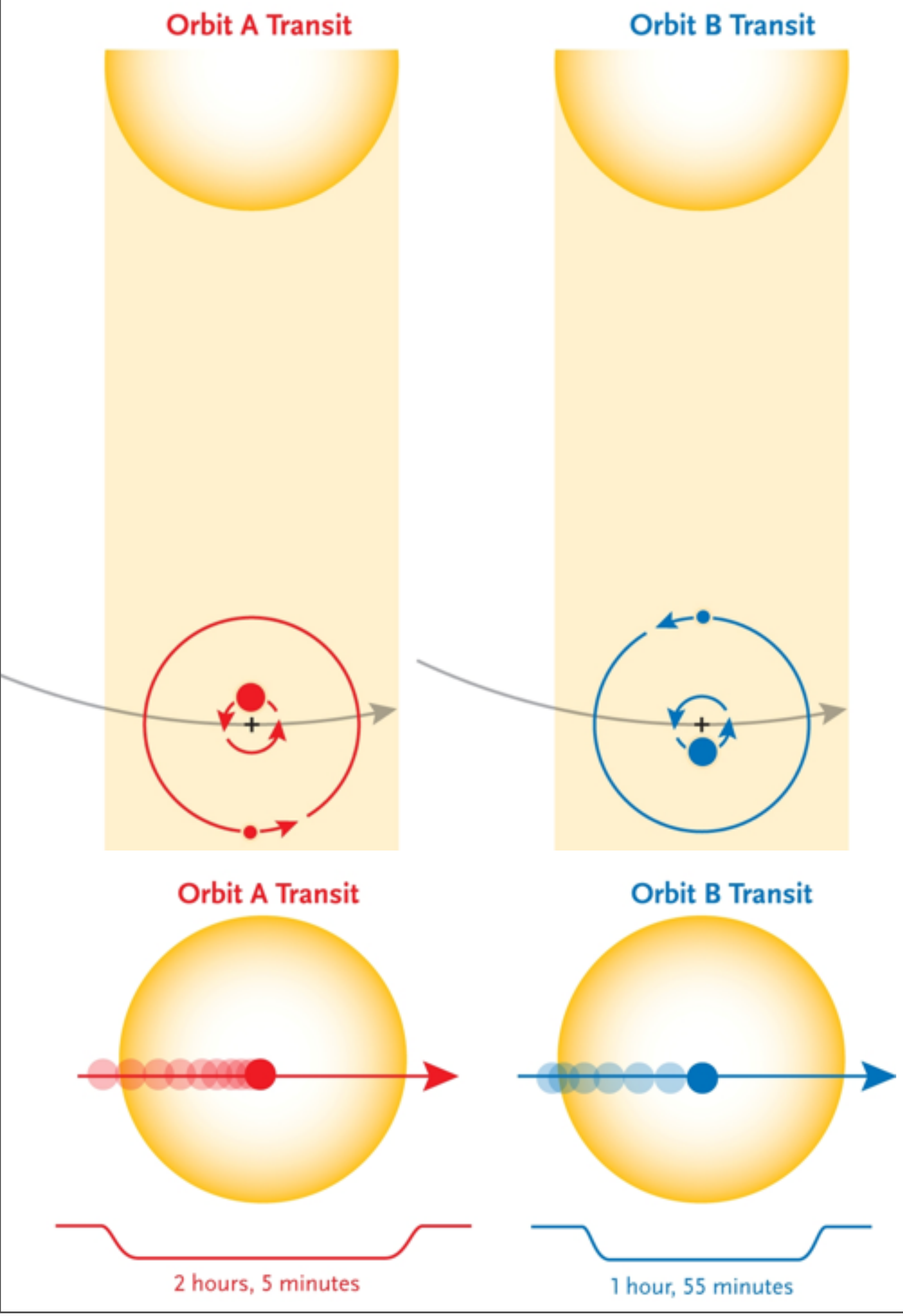


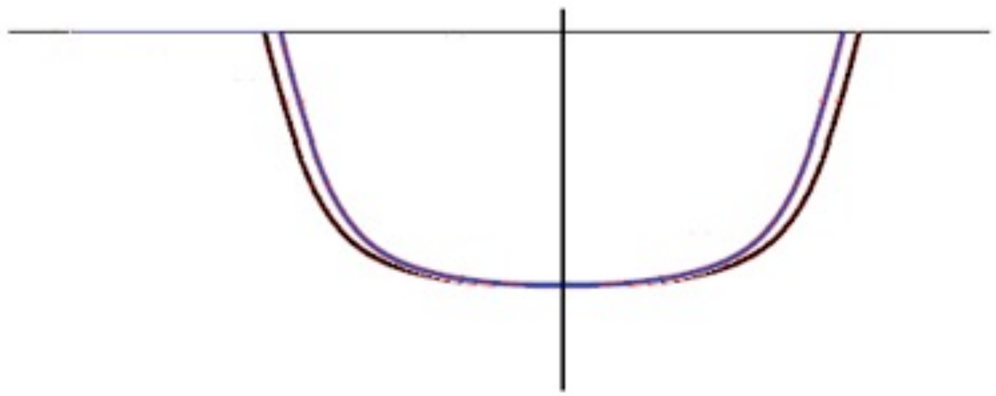


Transit Duration Variation (TDV)
Orbit A Transit
Orbit B Transit
Orbit A Transit
Orbit B Transit
2 hours, 5 minutes
1 hour, 55 minutes

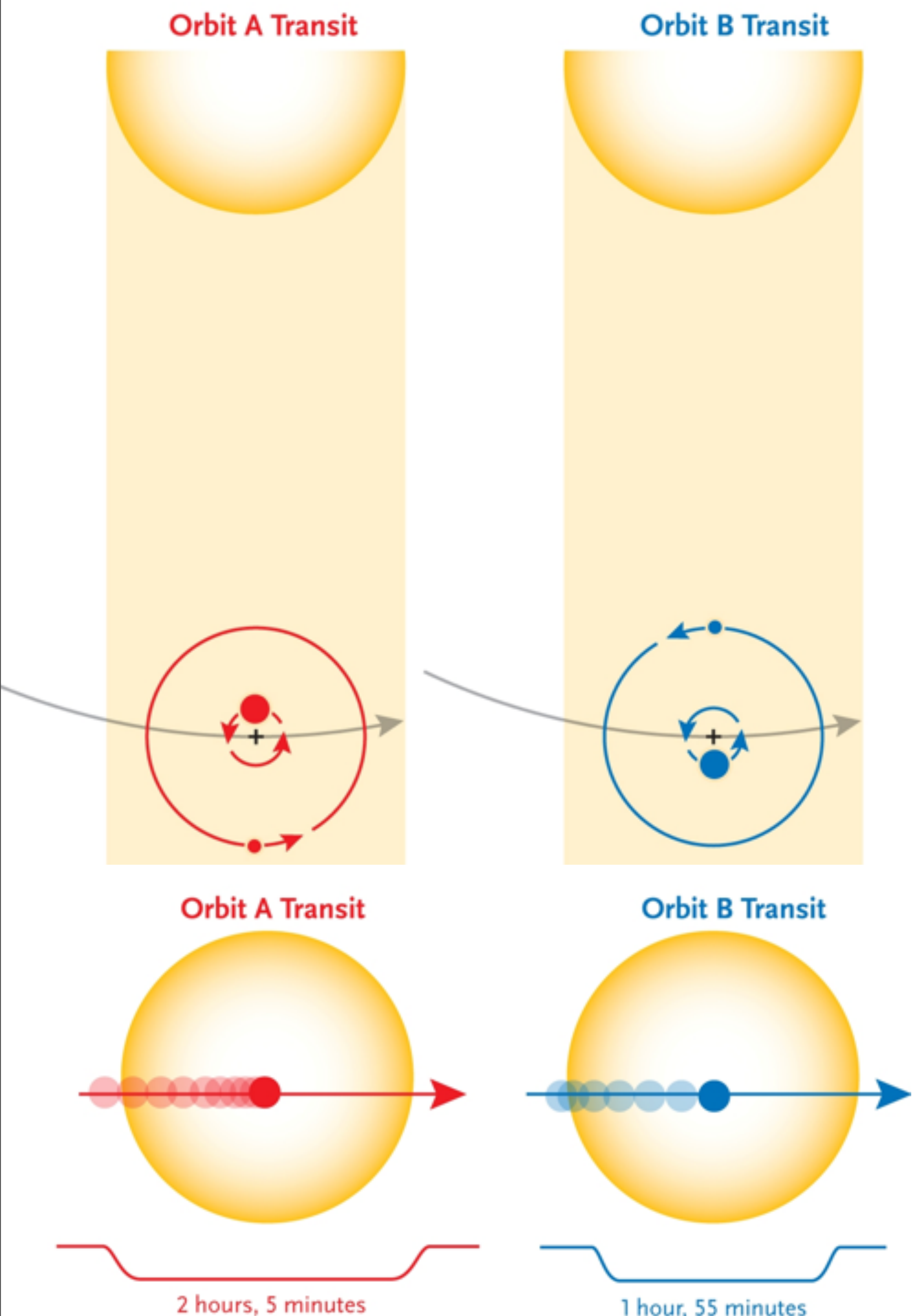


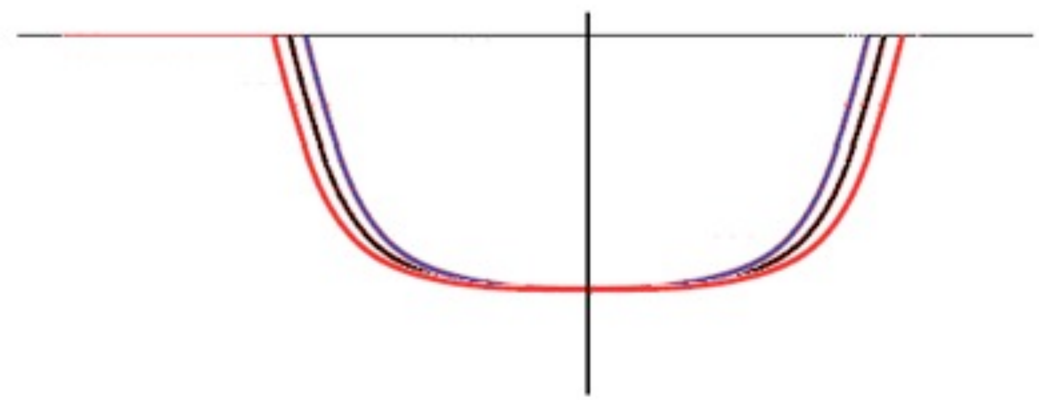
Transit Duration Variation (TDV)



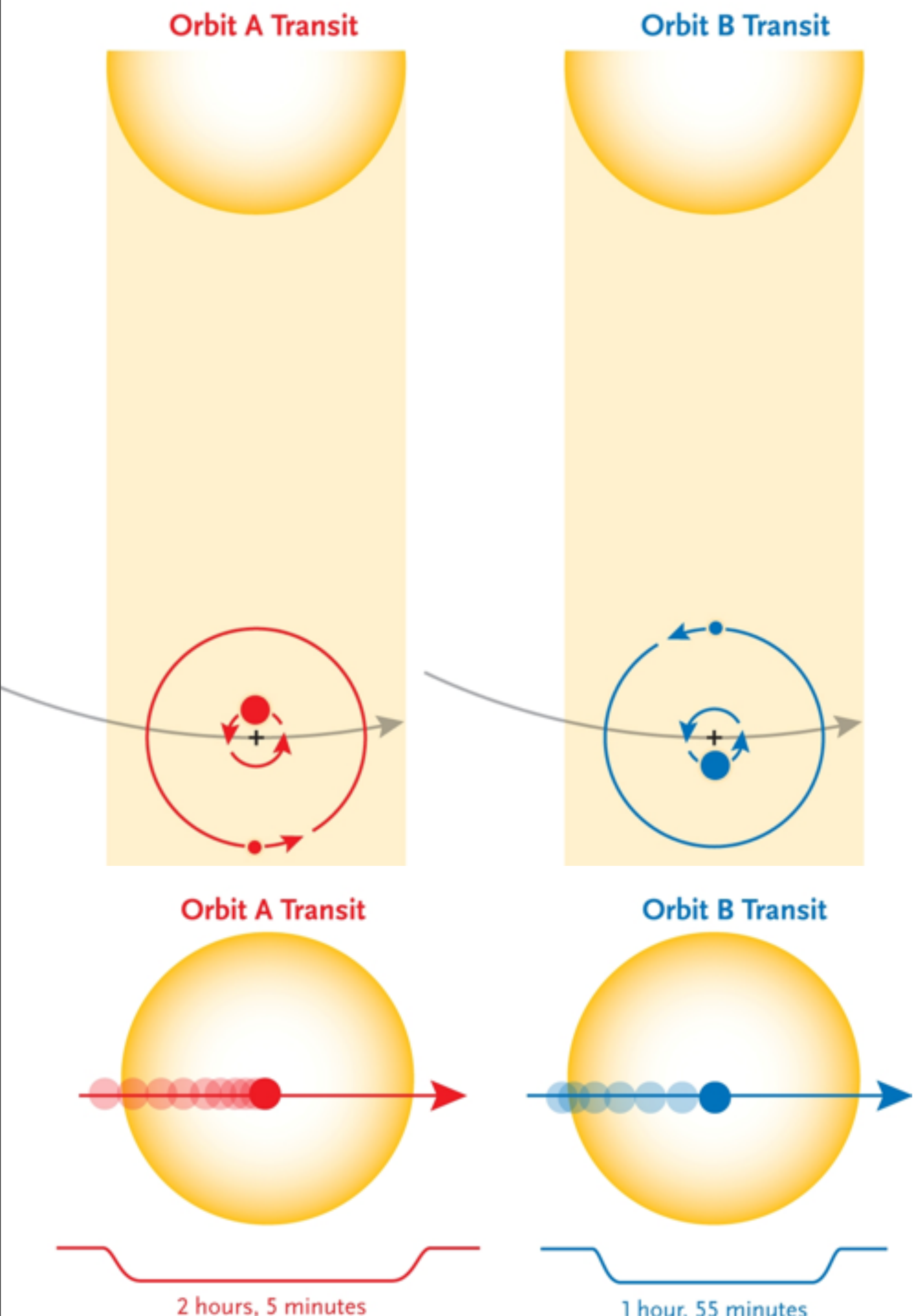


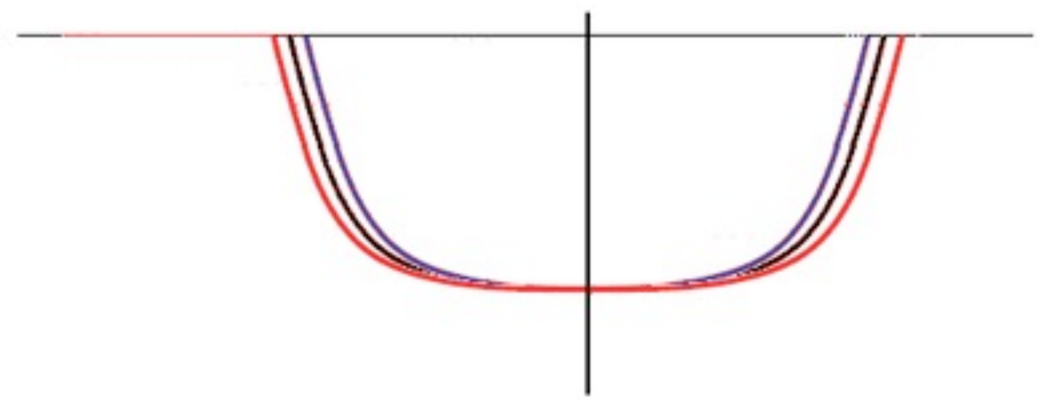
Transit Duration Variation (TDV)





Transit Duration Variation (TDV)

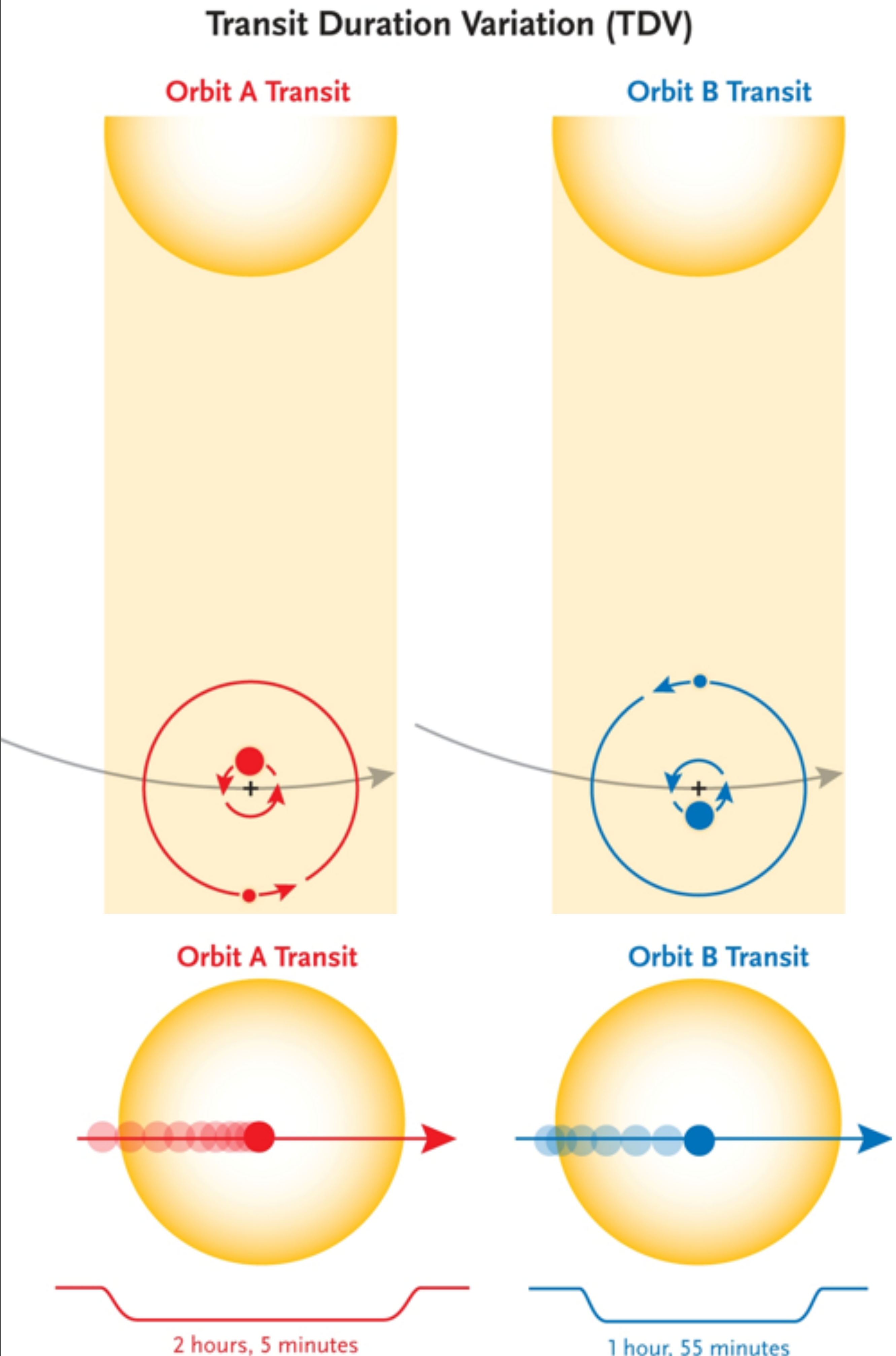




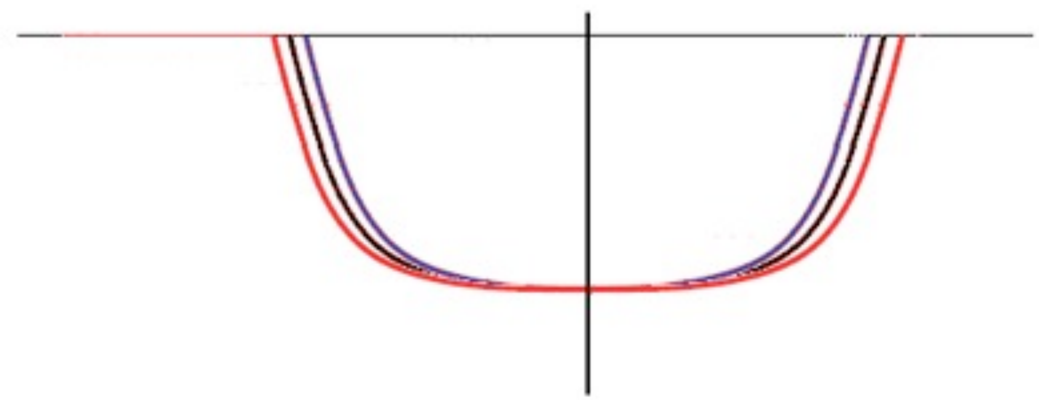
$$\delta_{\mathrm{TDV-V}} = \tilde{T}_B \left(\frac{a_S M_S P_P}{a_P M_P P_S}\right)\left(\frac{\sqrt{1-e_P^2}}{\sqrt{1-e_S^2}(1+e_P \sin\omega_P)}\right)\sqrt{\frac{\Phi_{\mathrm{TDV-V}}}{2\pi}}$$

TDV-V~ $a_S M_S / P_S$

TDV-V~ $a_S^{-1/2} M_S$



TDV-V is analogous to radial velocity



$$\delta_{\mathrm{TDV-V}} = \tilde{T}_B \left(\frac{a_S M_S P_P}{a_P M_P P_S}\right) \left(\frac{\sqrt{1-e_P^2}}{\sqrt{1-e_S^2}(1+e_P \sin\omega_P)}\right) \sqrt{\frac{\Phi_{\mathrm{TDV-V}}}{2\pi}}$$

$\mathrm{TDV\text{-}V} \sim a_S M_S / P_S$

$\mathrm{TDV\text{-}V} \sim a_S^{-1/2} M_S$

TDV-V~ $a_S M_S / P_S$

TTV~ $a_S M_S$

TDV-V~ $a_S^{-1/2} M_S$

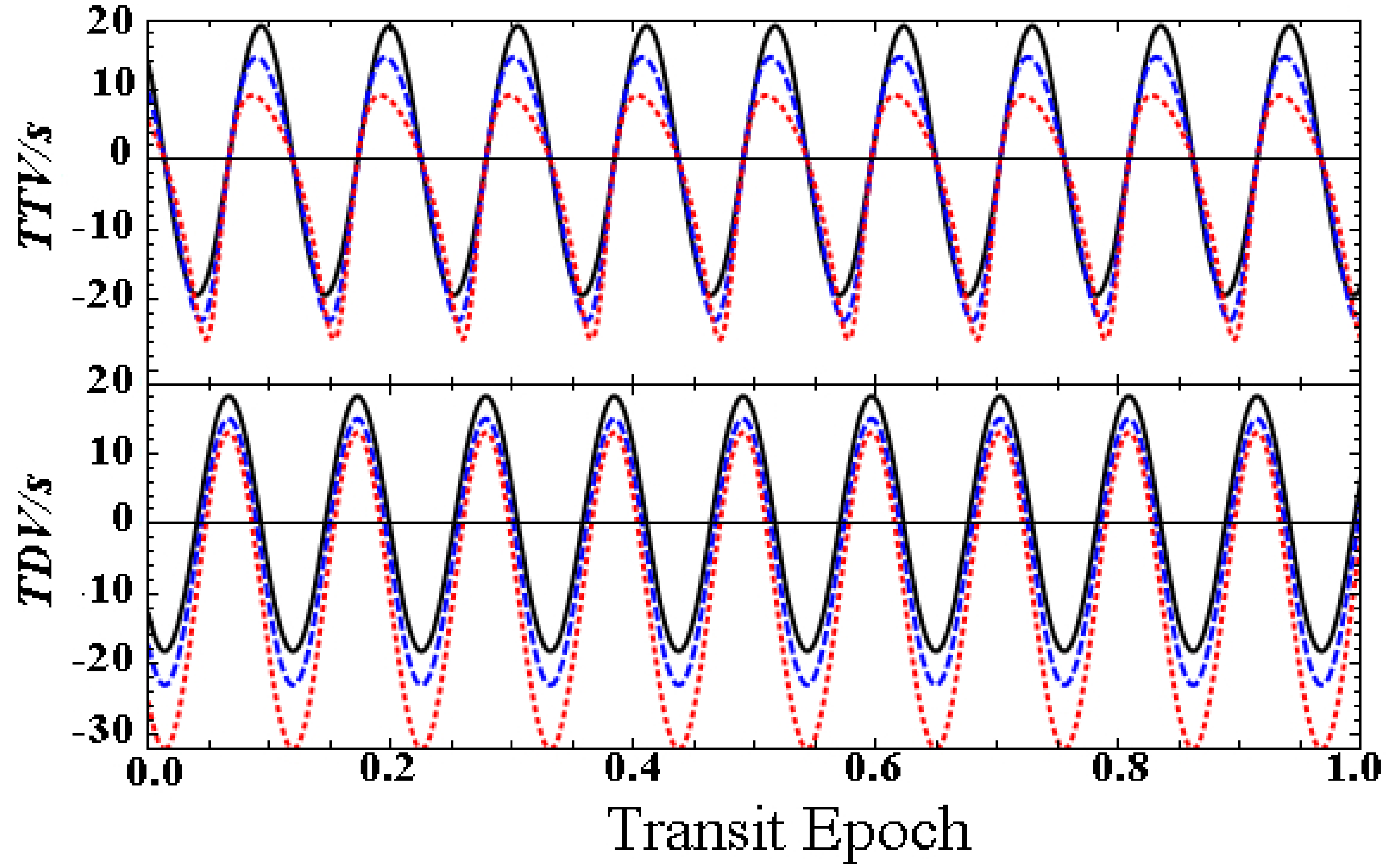
$$\eta = \frac{\delta_{\mathrm{TDV}}}{\delta_{\mathrm{TTV}}}$$

$$\lim_{e_S \to 0} \eta = \frac{\tilde{T}_B}{P_S}$$

TTV and TDV-V allow you determine P_S and thus M_S separately by measuring their amplitudes alone!

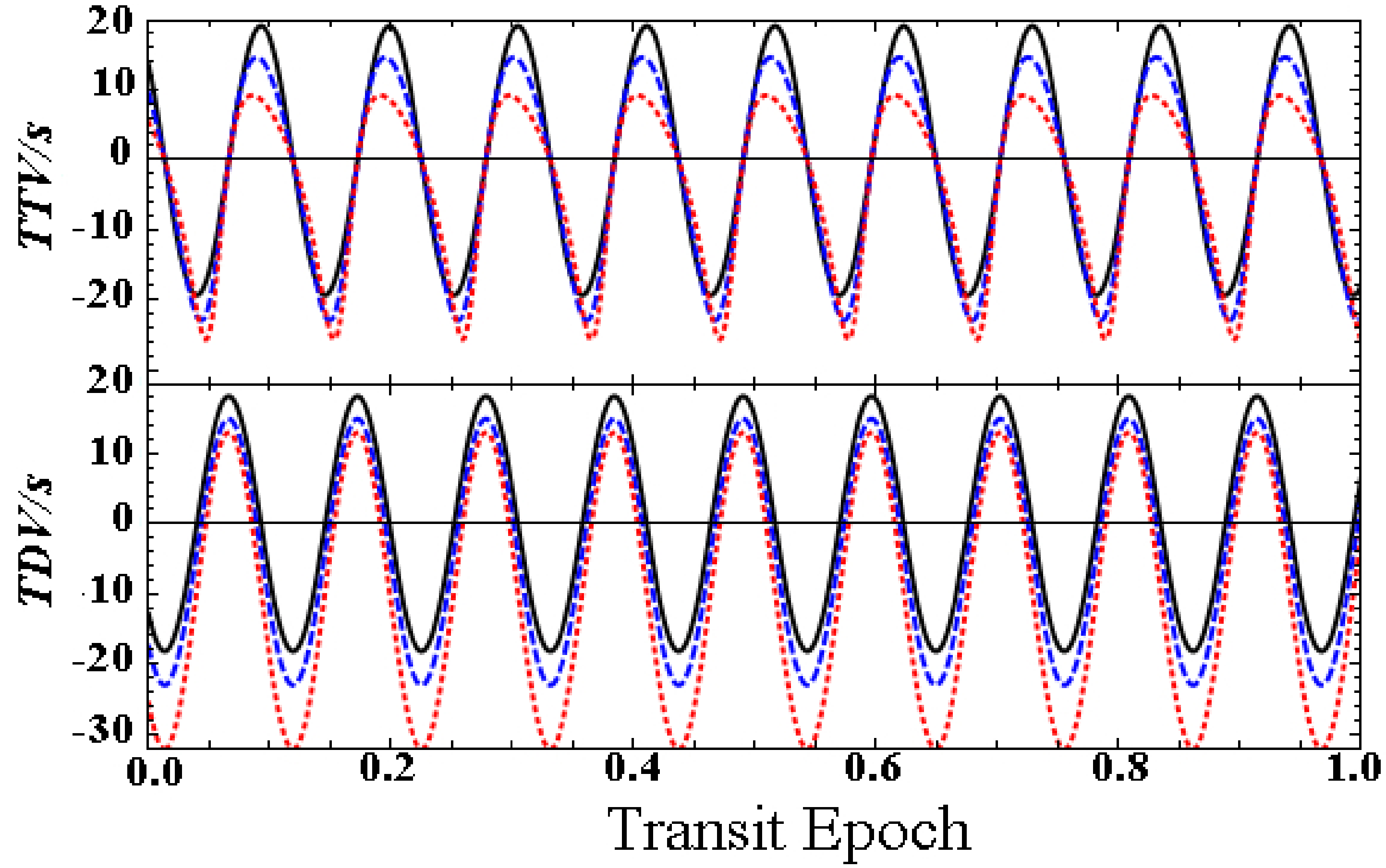
90° Phase Shift

- TTV leads TDV-V by 90 degrees in phase
- A unique signature we can look for



90° Phase Shift

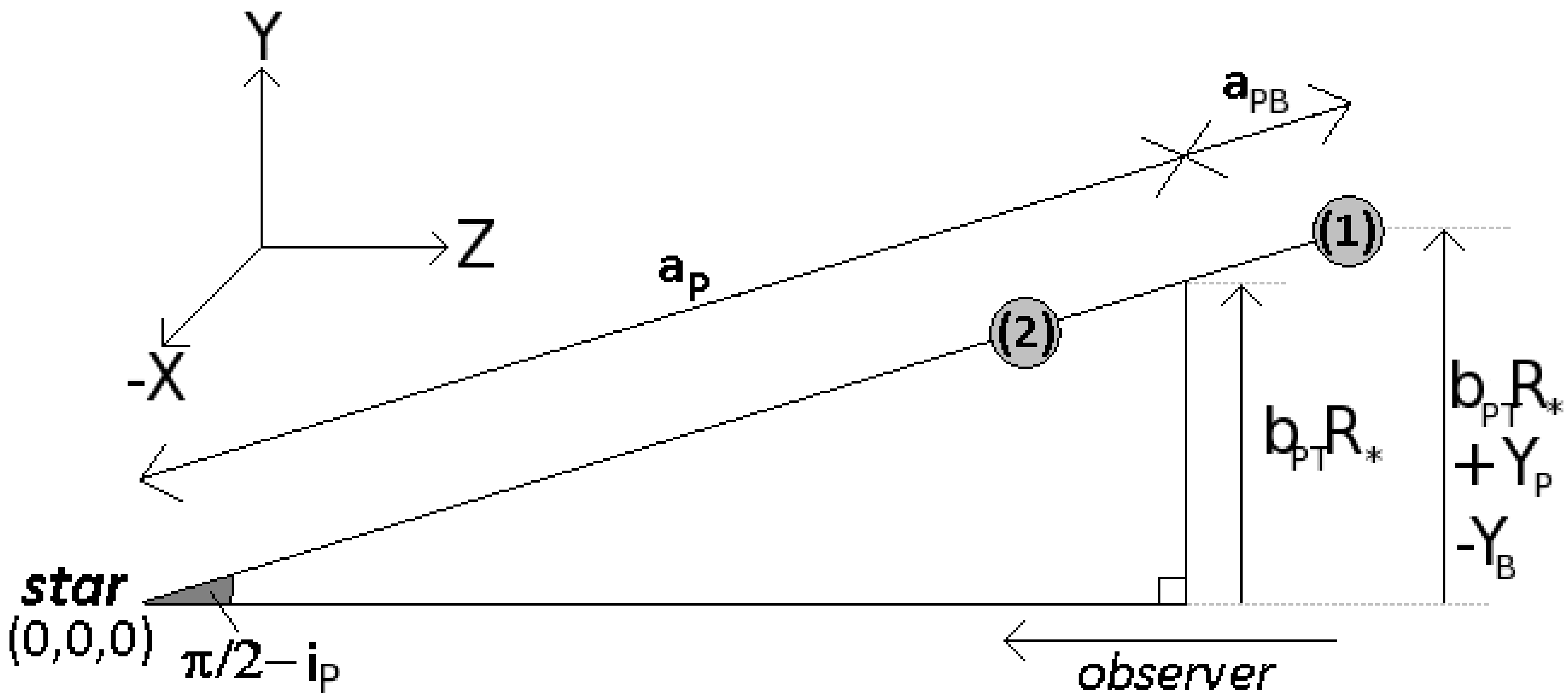
- TTV leads TDV-V by 90 degrees in phase
- A unique signature we can look for



Solves problems 1 and 2

But wait, there's more...

Transit impact parameter induced transit duration variations (TDV-TIP)



Prograde case: TDV-V & TDV-TIP in phase

Retrograde case: TDV-V & TDV-TIP in anti-phase

Does TDV-TIP mess up η?

$$\eta = \frac{\delta_{\mathrm{TDV}}}{\delta_{\mathrm{TTV}}}$$

$$\lim_{e_S \to 0} \eta = \frac{\tilde{T}_B}{P_S}$$

Does TDV-TIP mess up η?

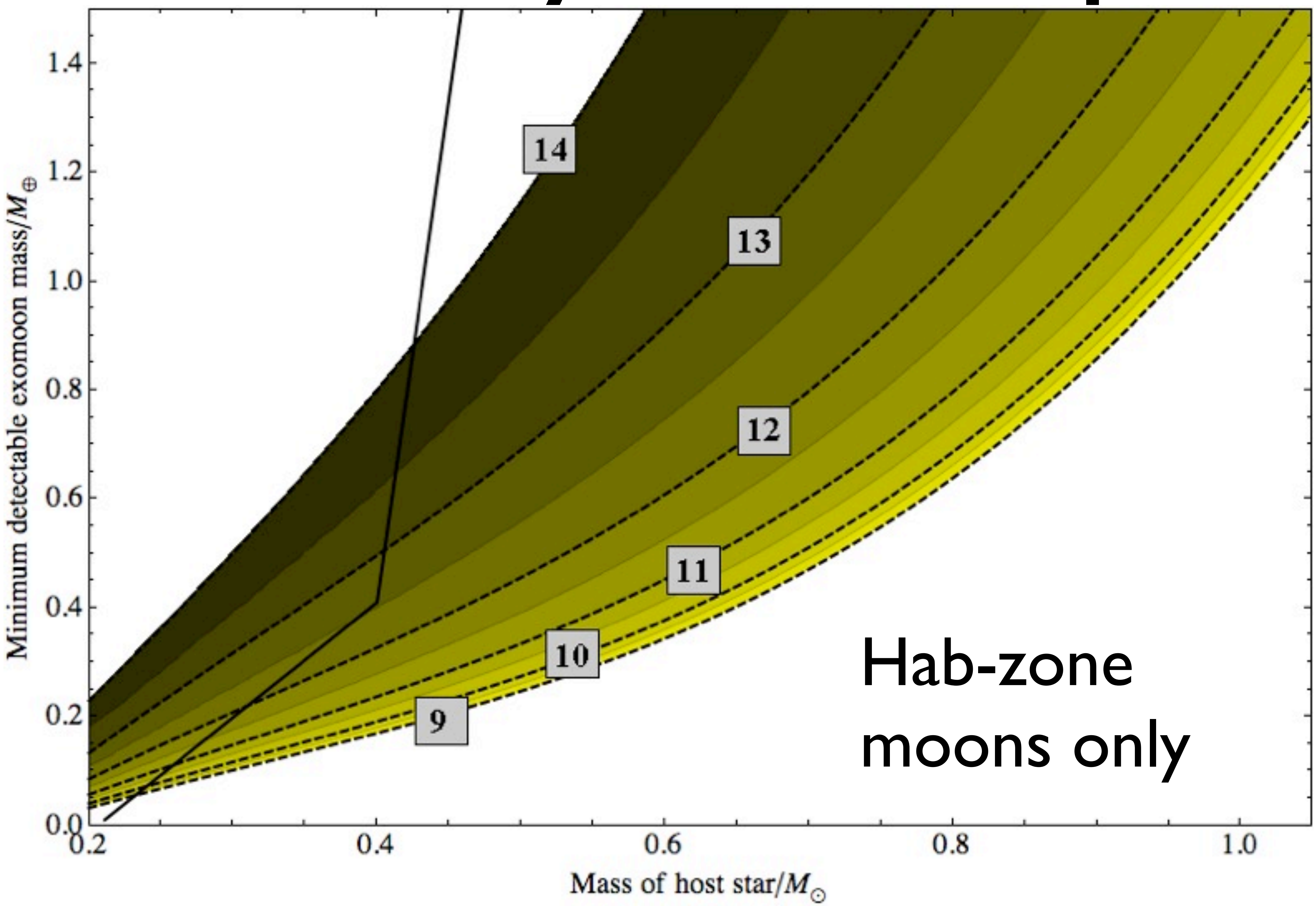
$$\eta = \frac{\delta_{\mathrm{TDV}}}{\delta_{\mathrm{TTV}}}$$

$$\lim_{e_S \to 0} \lim_{i_S \to \pi/2} \eta = \frac{\tilde{T}_B}{P_S} \pm \frac{\tilde{T}_B}{P_B} \left(\frac{b_{P,T}^2}{1 - b_{P,T}^2} \right)$$

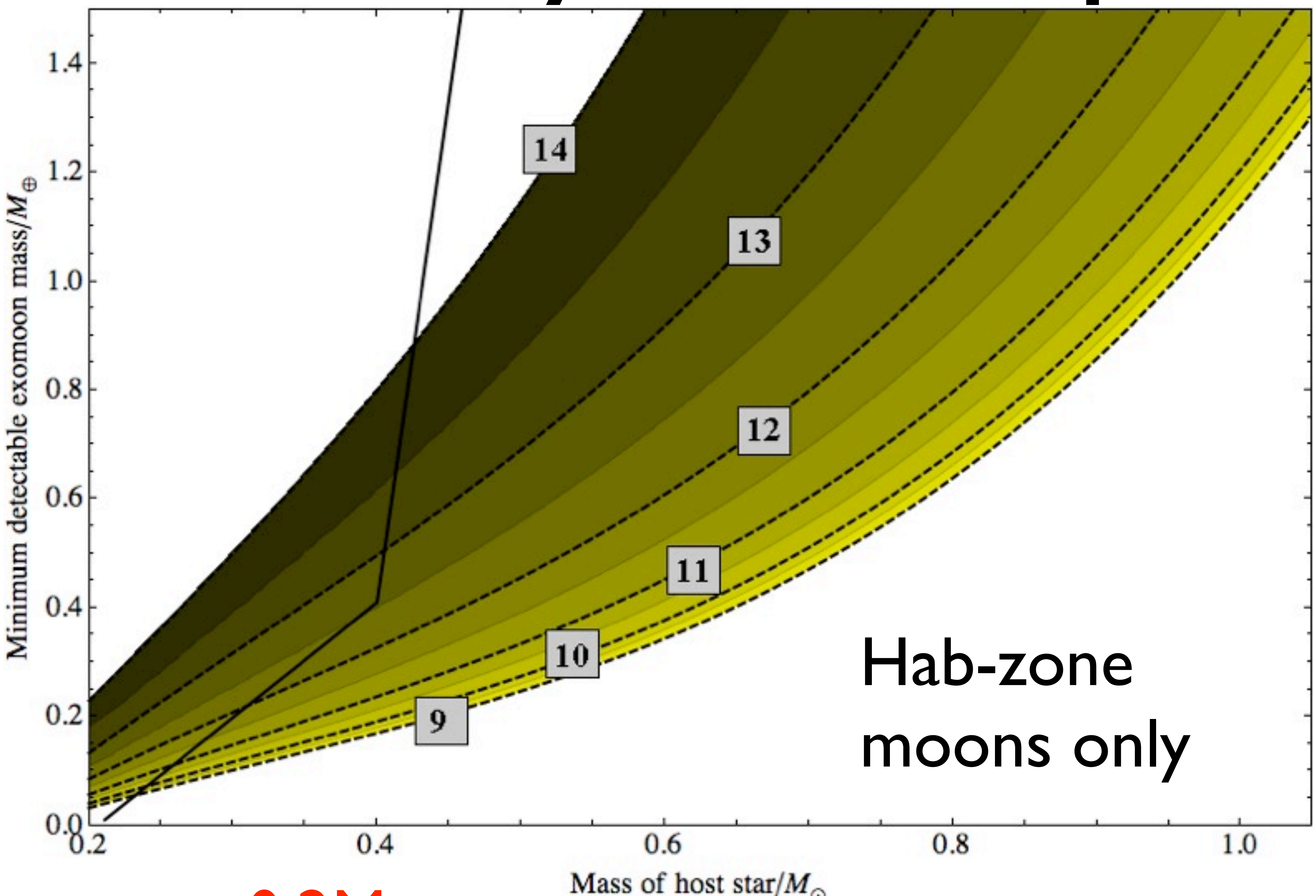
⇒We can distinguish between

prograde & retrograde moons!

Feasibility with Kepler



Feasibility with Kepler



- Best-case: ~$0.2M_\oplus$
- 25,000 stars bright enough to go for $1M_\oplus$

Observational Consequences

1. Dynamical effects (gives M_S)

    (i) Transit timing variations (TTV)

    (ii) Velocity induced transit duration variations (TDV-V)

    (iii) Transit impact parameter induced transit duration variations (TDV-TIP)

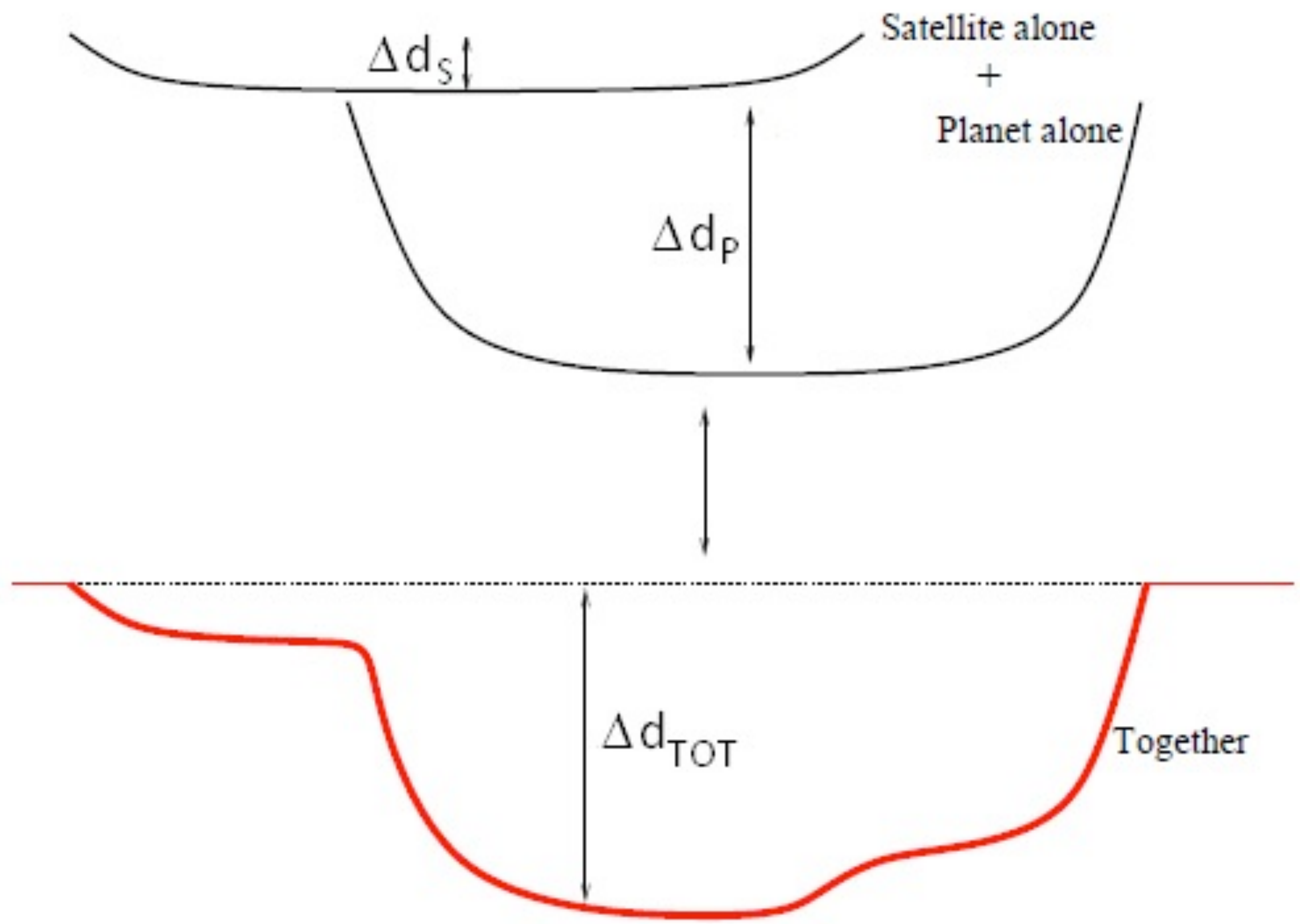
2. Eclipse effects (gives R_S)

Observational Consequences

1. Dynamical effects (gives M_S)
   - (i) Transit timing variations (TTV)
   - (ii) Velocity induced transit duration variations (TDV-V)
   - (iii) Transit impact parameter induced transit duration variations (TDV-TIP)
2. Eclipse effects (gives R_S)

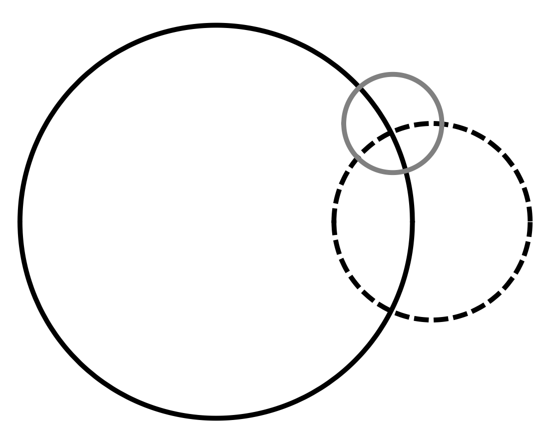
Auxiliary Transits

- At the simplest level, a moon can induce an auxiliary transit...



Mutual Events

- But if the moon, planet and star all overlap, we have a "mutual event".
- Can no longer simply add two signals together.



Star-Planet System

- Light curve is completely described by S_{P*}, the sky-projected planet-star separation (in units of the stellar radius)
- This one parameter exists in 3 states, leading to 3^1=3 cases:

(I) Out-of-transit: $1+p \leq S_{P*} < \infty$

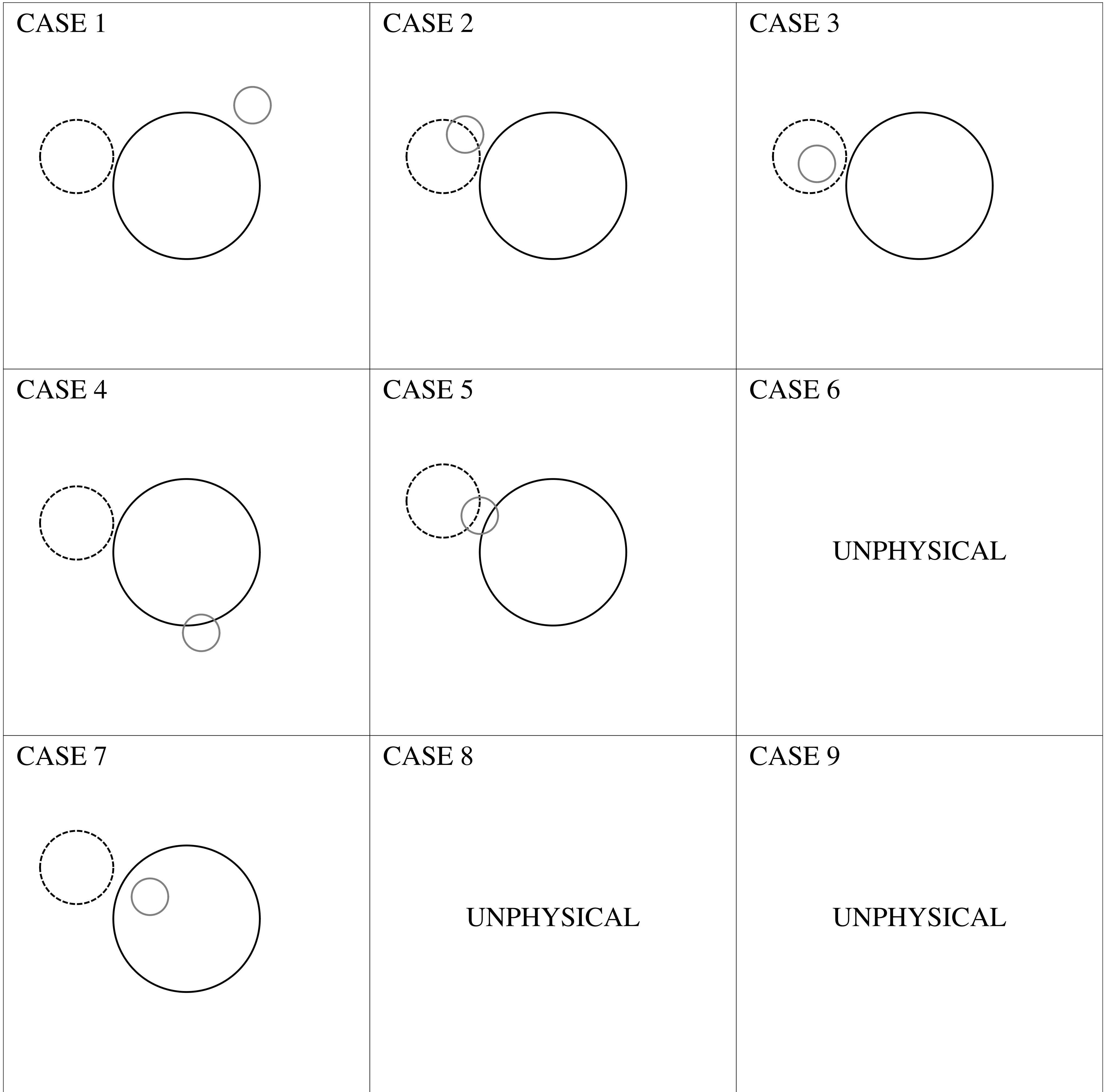
(II) On-the-limb: $1-p \leq S_{P*} < 1+p$

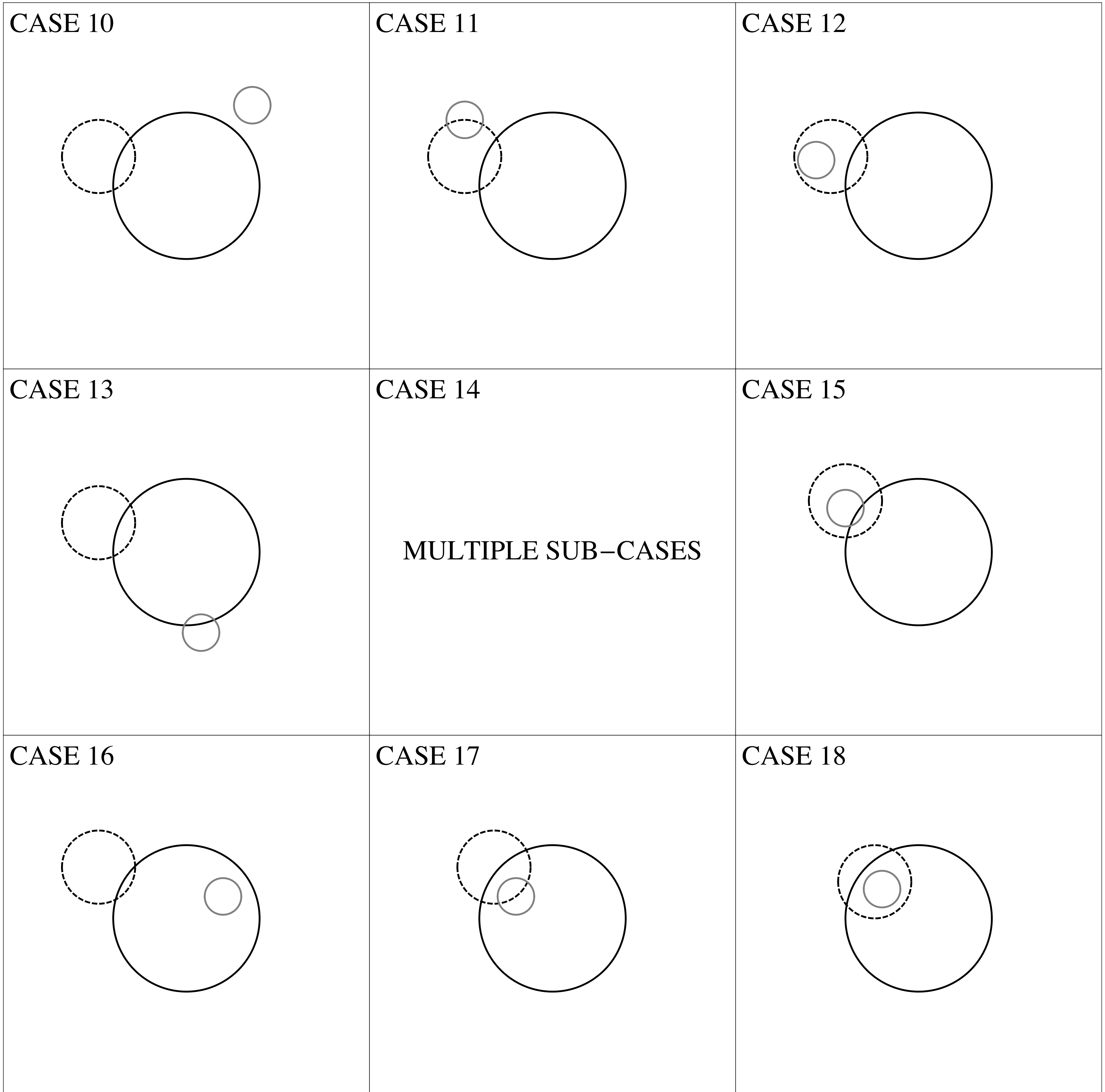
(III) In-transit: $0 \leq S_{P*} < 1-p$

Star-Planet-Moon System

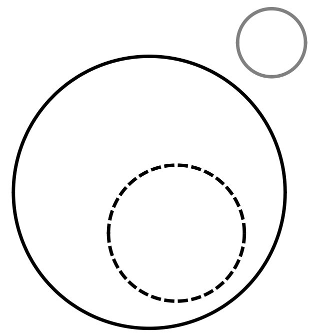
- Light curve now described by 3 parameters: S_{P*}, S_{S*}, S_{PS}
- Each parameter can still be in 3 states each
- Now 3^3 = 27 cases

Case Number [$\mathcal{E}$]	S_P	S_S	S_{PS}	Physical?
1	$S_{P*} \geqslant 1+p$	$S_{S*} \geqslant 1+s$	$S_{PS} \geqslant p+s$	$\surd$
2	$S_{P*} \geqslant 1+p$	$S_{S*} \geqslant 1+s$	$p-s < S_{PS} < p+s$	$\surd$
3	$S_{P*} \geqslant 1+p$	$S_{S*} \geqslant 1+s$	$S_{PS} \leqslant p-s$	$\surd$
4	$S_{P*} \geqslant 1+p$	$1-s < S_{S*} < 1+s$	$S_{PS} \geqslant p+s$	$\surd$
5	$S_{P*} \geqslant 1+p$	$1-s < S_{S*} < 1+s$	$p-s < S_{PS} < p+s$	$\surd$
6	$S_{P*} \geqslant 1+p$	$1-s < S_{S*} < 1+s$	$S_{PS} \leqslant p-s$	$\times$
7	$S_{P*} \geqslant 1+p$	$S_{S*} \leqslant 1-s$	$S_{PS} \geqslant p+s$	$\surd$
8	$S_{P*} \geqslant 1+p$	$S_{S*} \leqslant 1-s$	$p-s < S_{PS} < p+s$	$\times$
9	$S_{P*} \geqslant 1+p$	$S_{S*} \leqslant 1-s$	$S_{PS} \leqslant p-s$	$\times$
10	$1-p < S_{P*} < 1+p$	$S_{S*} \geqslant 1+s$	$S_{PS} \geqslant p+s$	$\surd$
11	$1-p < S_{P*} < 1+p$	$S_{S*} \geqslant 1+s$	$p-s < S_{PS} < p+s$	$\surd$
12	$1-p < S_{P*} < 1+p$	$S_{S*} \geqslant 1+s$	$S_{PS} \leqslant p-s$	$\surd$
13	$1-p < S_{P*} < 1+p$	$1-s < S_{S*} < 1+s$	$S_{PS} \geqslant p+s$	$\surd$
14**	$1-p < S_{P*} < 1+p$	$1-s < S_{S*} < 1+s$	$p-s < S_{PS} < p+s$	$\surd$
15	$1-p < S_{P*} < 1+p$	$1-s < S_{S*} < 1+s$	$S_{PS} \leqslant p-s$	$\surd$
16	$1-p < S_{P*} < 1+p$	$S_{S*} \leqslant 1-s$	$S_{PS} \geqslant p+s$	$\surd$
17*	$1-p < S_{P*} < 1+p$	$S_{S*} \leqslant 1-s$	$p-s < S_{PS} < p+s$	$\surd$
18	$1-p < S_{P*} < 1+p$	$S_{S*} \leqslant 1-s$	$S_{PS} \leqslant p-s$	$\surd$
19	$S_{P*} \leqslant 1-p$	$S_{S*} \geqslant 1+s$	$S_{PS} \geqslant p+s$	$\surd$
20	$S_{P*} \leqslant 1-p$	$S_{S*} \geqslant 1+s$	$p-s < S_{PS} < p+s$	$\times$
21	$S_{P*} \leqslant 1-p$	$S_{S*} \geqslant 1+s$	$S_{PS} \leqslant p-s$	$\times$
22	$S_{P*} \leqslant 1-p$	$1-s < S_{S*} < 1+s$	$S_{PS} \geqslant p+s$	$\surd$
23*	$S_{P*} \leqslant 1-p$	$1-s < S_{S*} < 1+s$	$p-s < S_{PS} < p+s$	$\surd$
24	$S_{P*} \leqslant 1-p$	$1-s < S_{S*} < 1+s$	$S_{PS} \leqslant p-s$	$\times$
25	$S_{P*} \leqslant 1-p$	$S_{S*} \leqslant 1-s$	$S_{PS} \geqslant p+s$	$\surd$
26*	$S_{P*} \leqslant 1-p$	$S_{S*} \leqslant 1-s$	$p-s < S_{PS} < p+s$	$\surd$
27	$S_{P*} \leqslant 1-p$	$S_{S*} \leqslant 1-s$	$S_{PS} \leqslant p-s$	$\surd$


CASE 1
CASE 2
CASE 3
CASE 4
CASE 5
CASE 6
UNPHYSICAL
CASE 7
CASE 8
UNPHYSICAL
CASE 9
UNPHYSICAL


CASE 10
CASE 11
CASE 12
CASE 13
CASE 14
MULTIPLE SUB−CASES
CASE 15
CASE 16
CASE 17
CASE 18

CASE 19



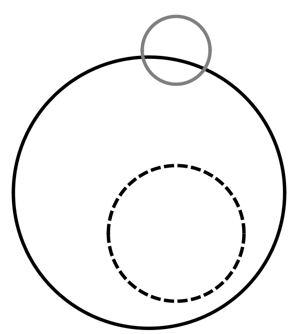
CASE 20

UNPHYSICAL

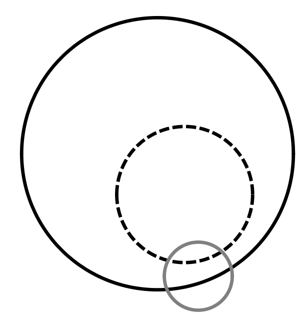
CASE 21

UNPHYSICAL

CASE 22



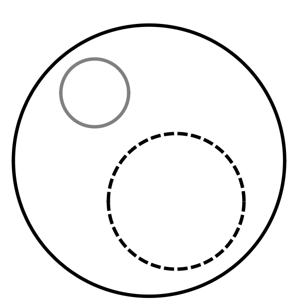
CASE 23



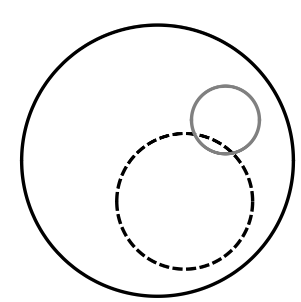
CASE 24

UNPHYSICAL

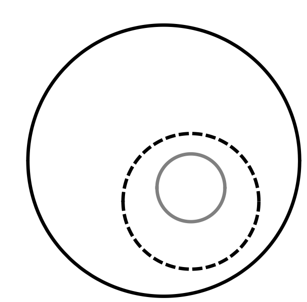
CASE 25



CASE 26



CASE 27

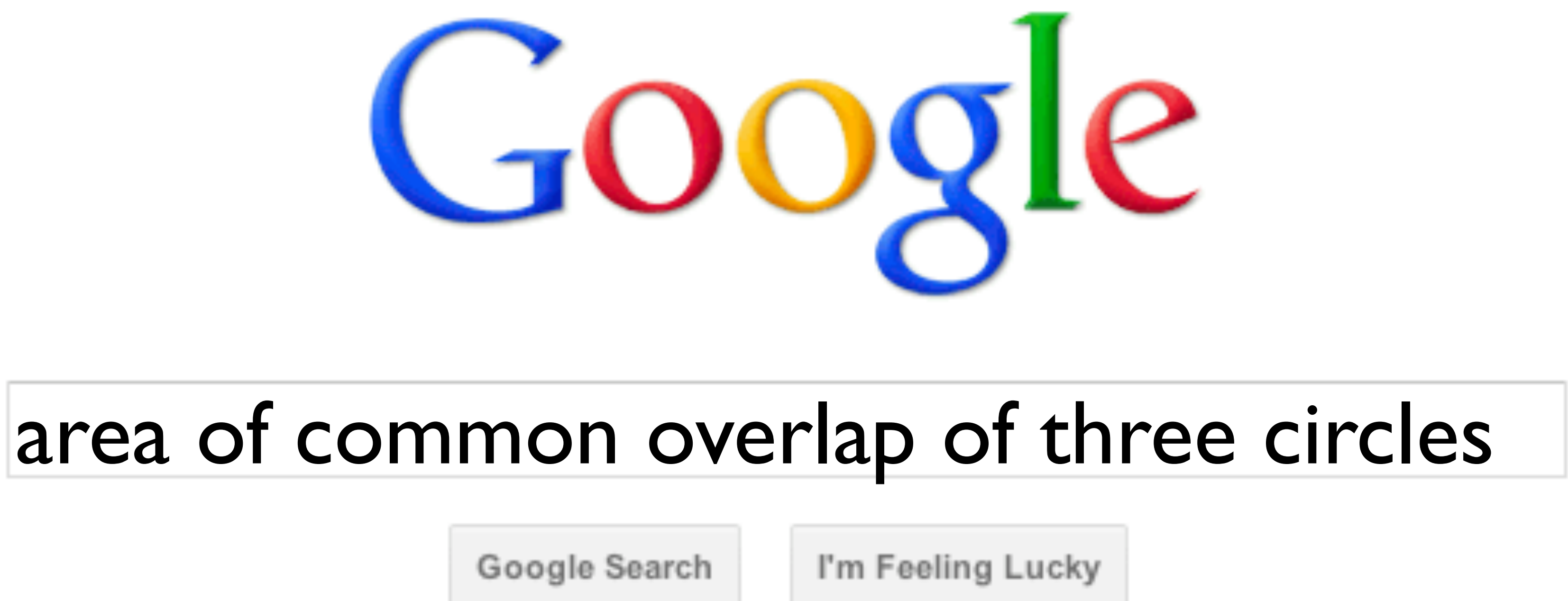


Case 14

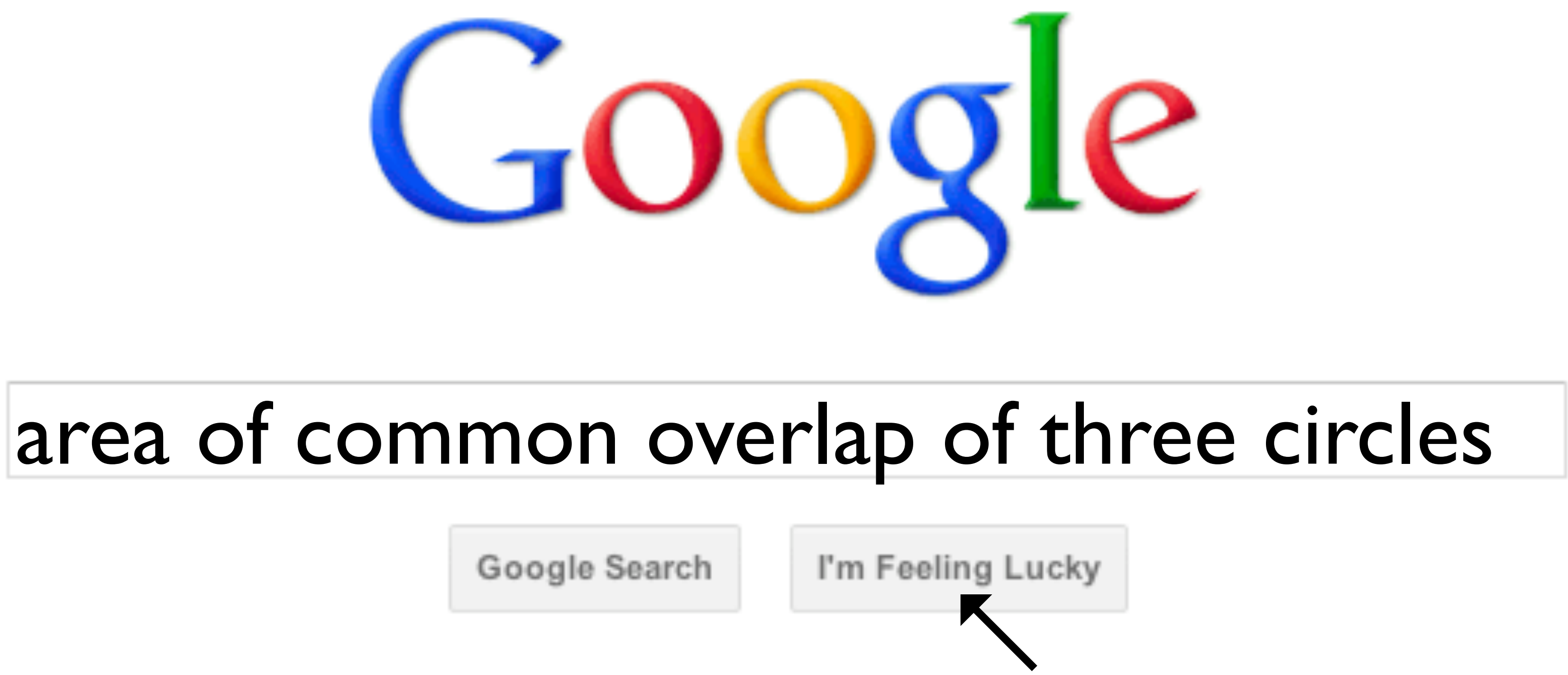
- Case 14 has all three discs on the limb
- We need to calculate the area of overlap
- Could be done numerically, but this would be very slow and inefficient
- Is there an analytic solution?

Some in-depth research is done...

Some in-depth research is done...



Some in-depth research is done...



Australian Government
Department of Defence
Defence Science and
Technology Organisation

Area of Common Overlap of Three Circles

M.P. Fewell

Maritime Operations Division

Defence Science and Technology Organisation

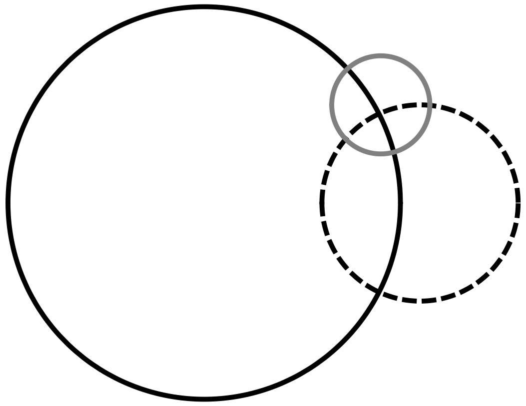
DSTO-TN-0722

ABSTRACT

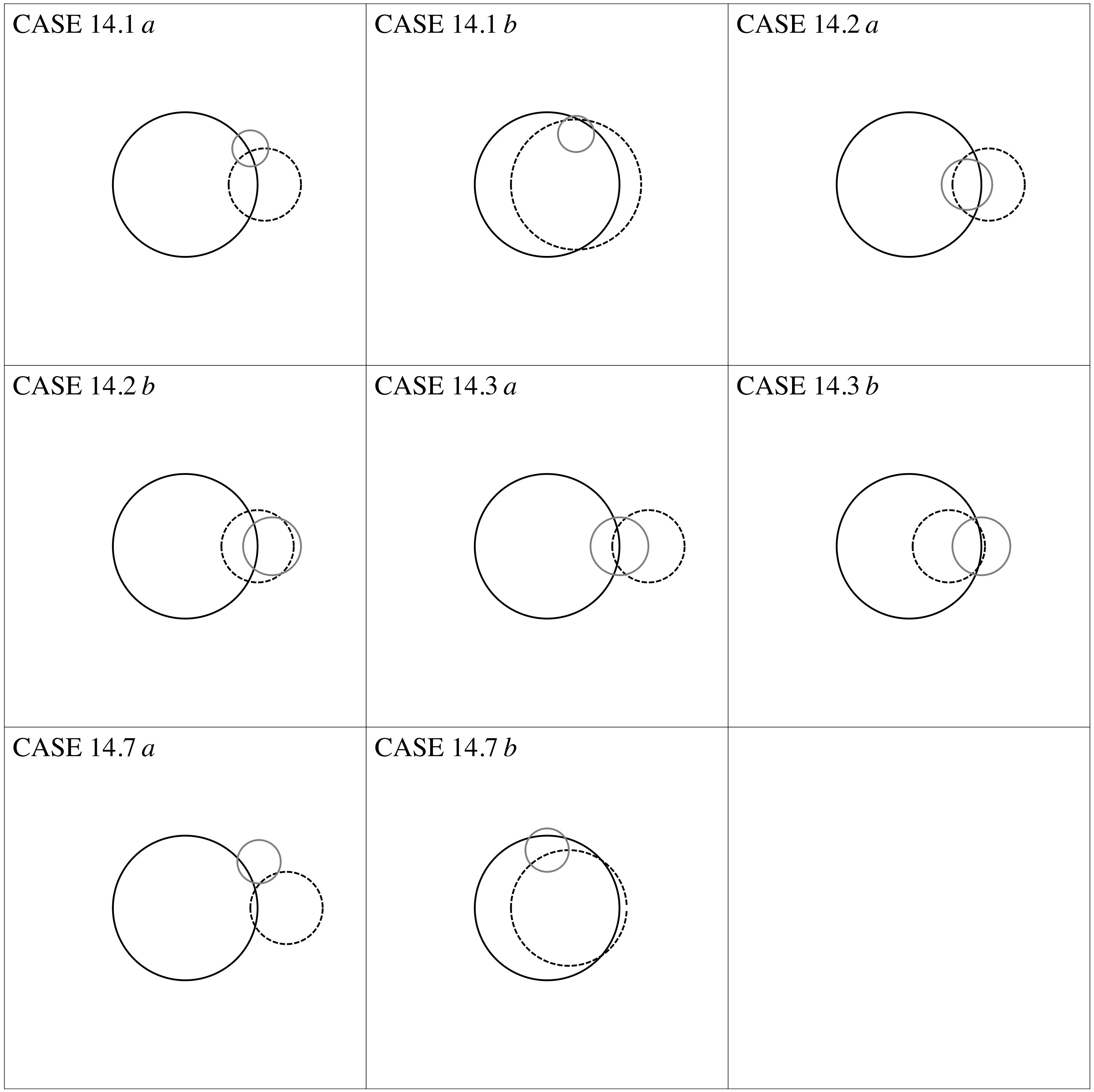
This Note presents the solution to an apparently hitherto unsolved geometrical problem: the derivation of a closed-form algebraic expression of the area of common overlap of three circles, such as can occur in a three-circle Venn diagram. The results presented here have general significance in the corpus of mensuration formulae, and could be of specific use in any quantitative application of the three-circle Venn diagram such as, for example, in search and screening problems.

Fewell (2006) Solution

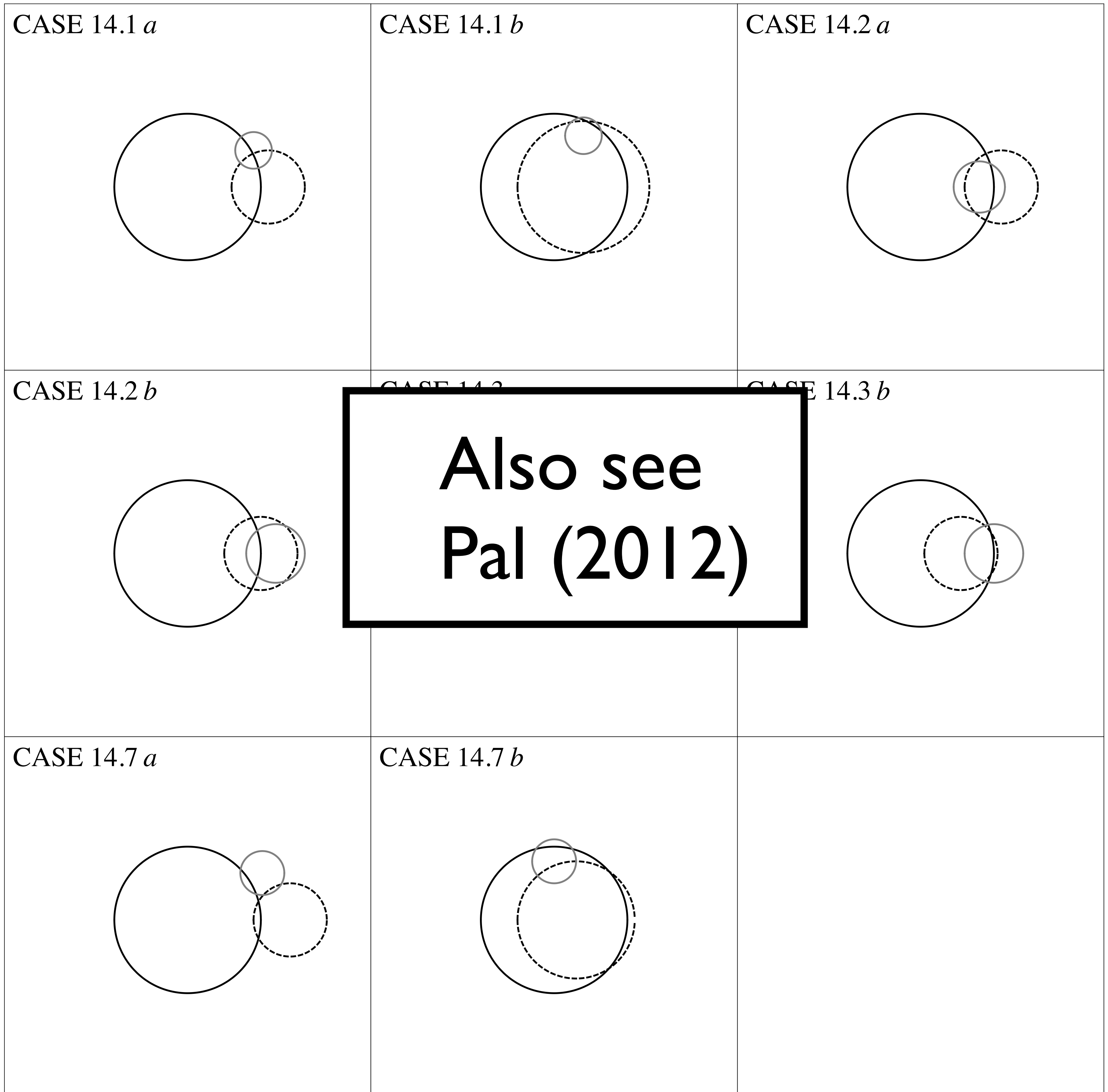
- In 2006, Michael Fewell, presented the first analytic solution to this problem.
- The Fewell solution solves the most critical problem in modeling exomoon signals.



Additional cases solved too



Additional cases solved too



Also see
Pal (2012)

Observational Consequences

1. Dynamical effects (gives M_S)
   (i) Transit timing variations (TTV)
   (ii) Velocity induced transit duration variations (TDV-V)
   (iii) Transit impact parameter induced transit duration variations (TDV-TIP)
2. Eclipse effects (gives R_S)
   (i) Auxiliary transits
   (ii) Mutual events

Moon Detections Allow Us to Measure M_* & R_*

Moon Detections Allow Us to Measure M_* & R_*

How to Weigh a Star Using a Moon

David M. Kipping[1,2]⋆
[1] *Department of Physics and Astronomy, University College London, Gower St., London WC1E 6BT*
[2] *Harvard-Smithsonian Center for Astrophysics, 60, Garden St., Cambridge, MA 02138, USA*

Accepted 2010 September 21. Received 2010 September 17; in original form 2010 August 24

Moon Detections Allow Us to Measure M_* & R_*

How to Weigh a Star Using a Moon

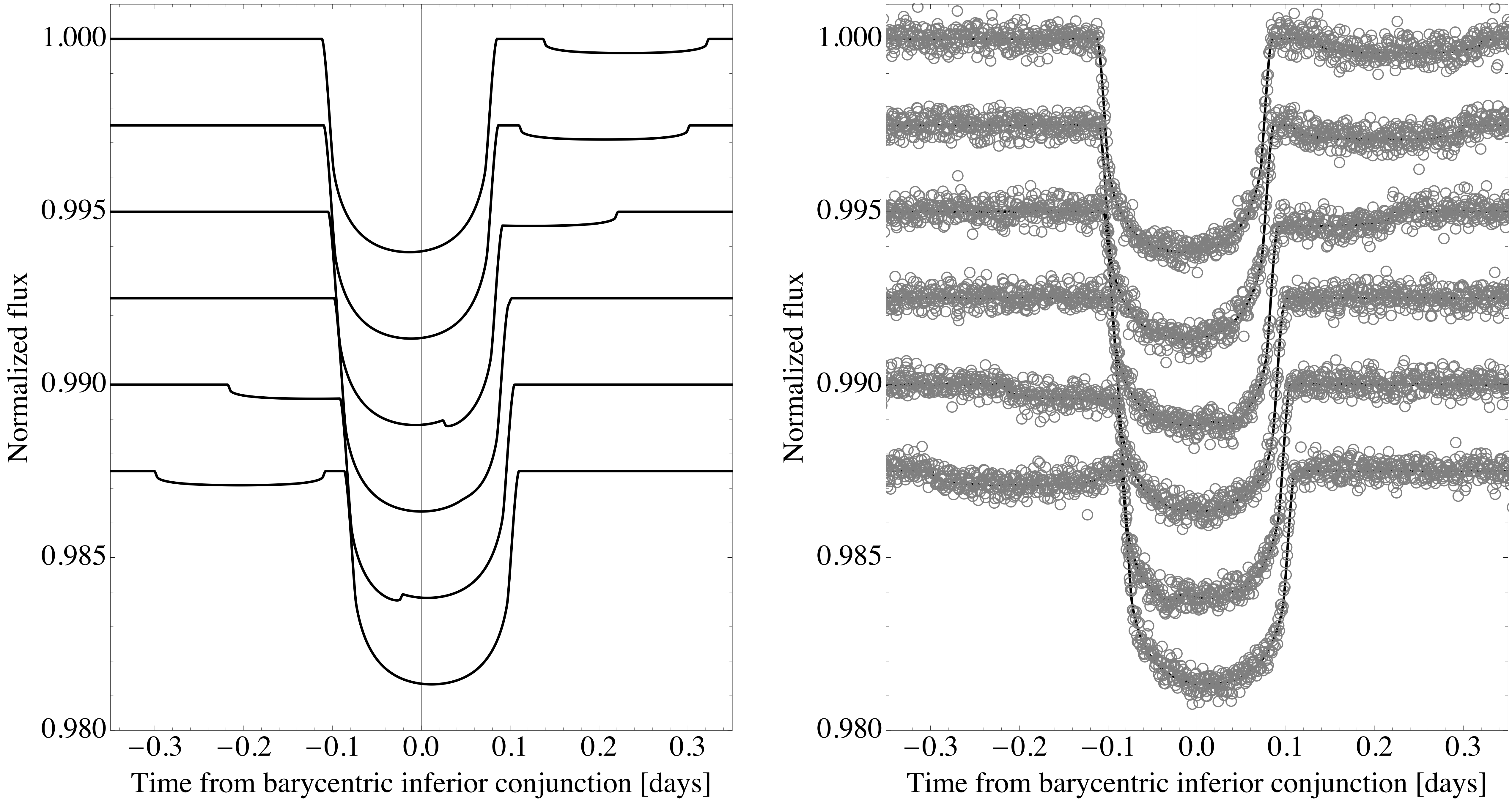
David M. Kipping[1,2⋆]
[1] *Department of Physics and Astronomy, University College London, Gower St., London WC1E 6BT*
[2] *Harvard-Smithsonian Center for Astrophysics, 60, Garden St., Cambridge, MA 02138, USA*

Accepted 2010 September 21. Received 2010 September 17; in original form 2010 August 24

$$M_* = \left[\frac{(a_B/R_*)^6}{(a_S/R_*)^9}\right]\left[\frac{P_S^4}{P_B^5}\right]\left[\frac{(1-e_B^2)^{3/2}K_*^3}{2\pi G\sin^3 i_B}\right] \times \left[\frac{(a_B/R_*)^3 P_S^2 - (a_S/R_*)^3(1+\mathcal{M}_{SP})^3 P_B^2}{(1+\mathcal{M}_{SP})^9}\right]$$

$$R_* = \frac{(a_B/R_*)^2\sqrt{1-e_B^2}K_* P_S^2}{2\pi\sin i_B (a_S/R_*)^3(1+\mathcal{M}_{SP})^3 P_B}$$

Neptune in hab-zone of M2 dwarf with **far-out retrograde** Earth-mass and radius moon



~10% uncertainties on M*

~5% uncertainties on R*

Physical params.		
$M_* \ [M_\odot]$	0.400	$0.399^{+0.061}_{-0.064}$
$R_* \ [R_\odot]$	0.500	$0.504^{+0.025}_{-0.029}$
$M_P \ [M_J]$	0.0540	$0.0537^{+0.0055}_{-0.0061}$
$R_P \ [M_J]$	0.346	$0.350^{+0.018}_{-0.020}$
$M_S \ [M_\oplus]$	1.00	$1.05^{+0.13}_{-0.12}$
$R_S \ [R_\oplus]$	1.000	$1.011^{+0.059}_{-0.064}$
$\rho_S \ [\mathrm{g\,cm^{-3}}]$	5.50	$5.62^{+1.03}_{-0.85}$

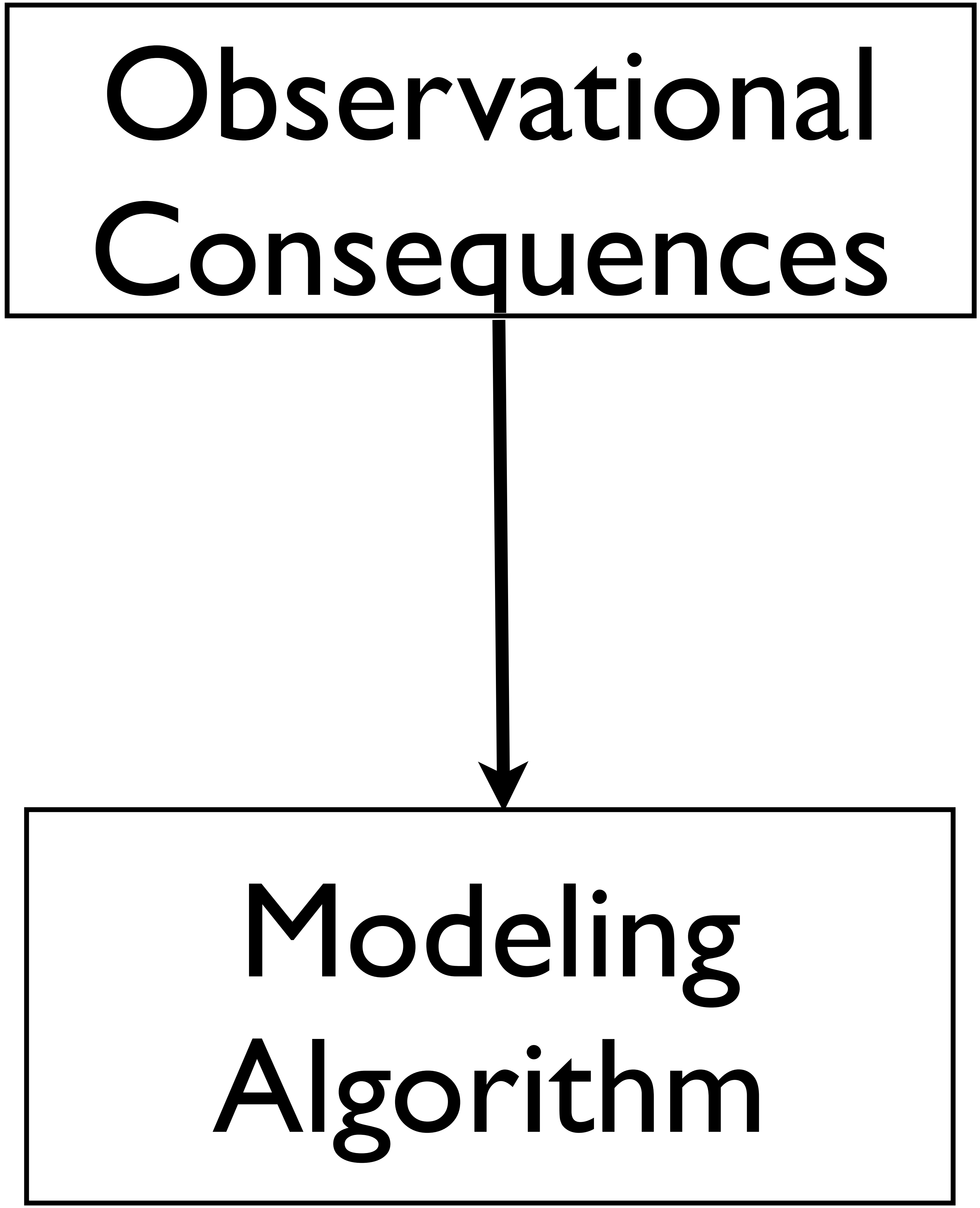
~10% uncertainties on M*

~5% uncertainties on R*

Physical params.		
M_* $[M_\odot]$	0.400	$0.399^{+0.061}_{-0.064}$
R_* $[R_\odot]$	0.500	$0.504^{+0.025}_{-0.029}$
M_P $[M_J]$	0.0540	$0.0537^{+0.0055}_{-0.0061}$
R_P $[M_J]$	0.346	$0.350^{+0.018}_{-0.020}$
M_S $[M_\oplus]$	1.00	$1.05^{+0.13}_{-0.12}$
R_S $[R_\oplus]$	1.000	$1.011^{+0.059}_{-0.064}$
ρ_S $[\mathrm{g\,cm^{-3}}]$	5.50	$5.62^{+1.03}_{-0.85}$

Moons directly yield density of the planet, useful for vetting.

III. Modeling

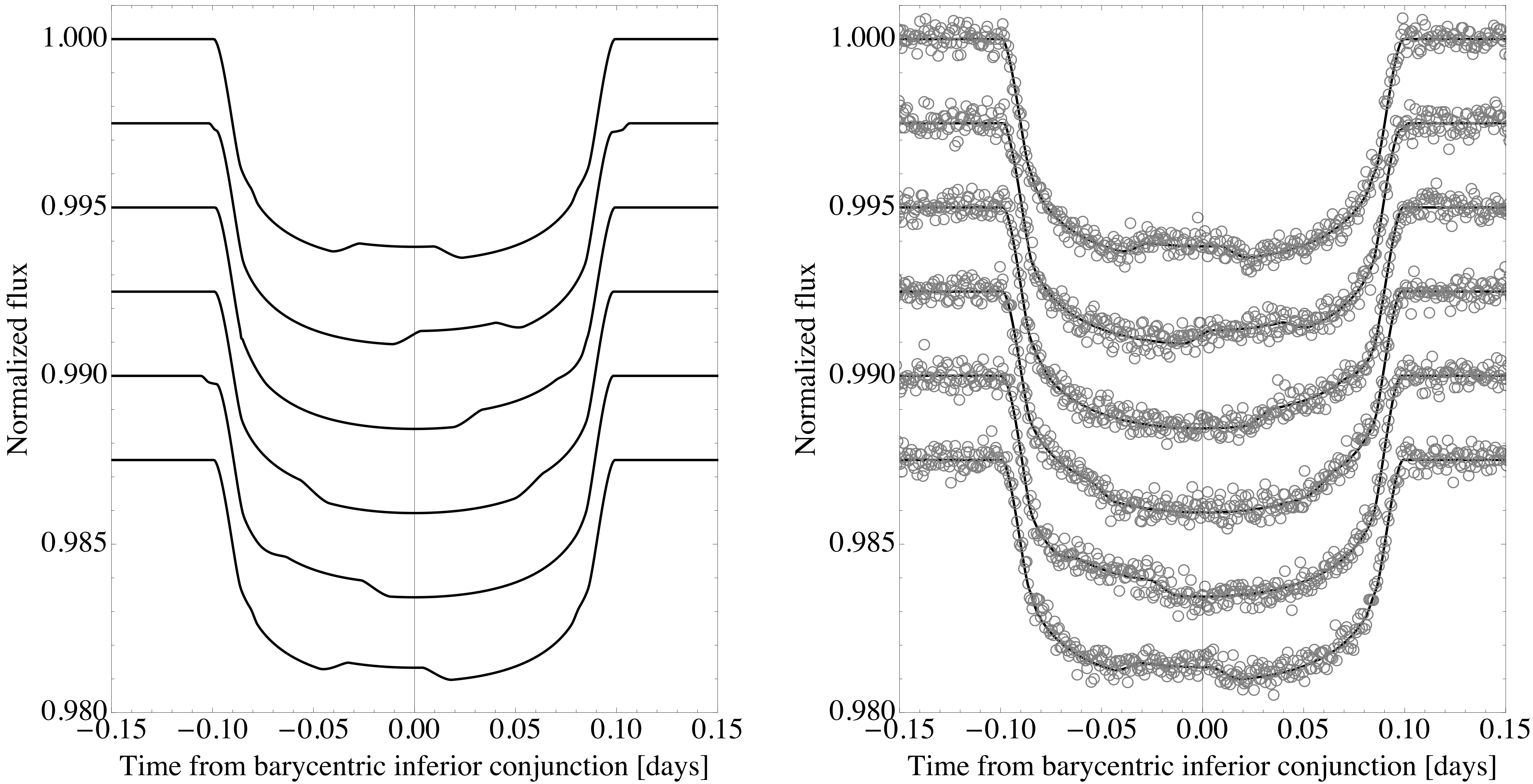

Observational
Consequences
Modeling
Algorithm

Modeling Algorithms

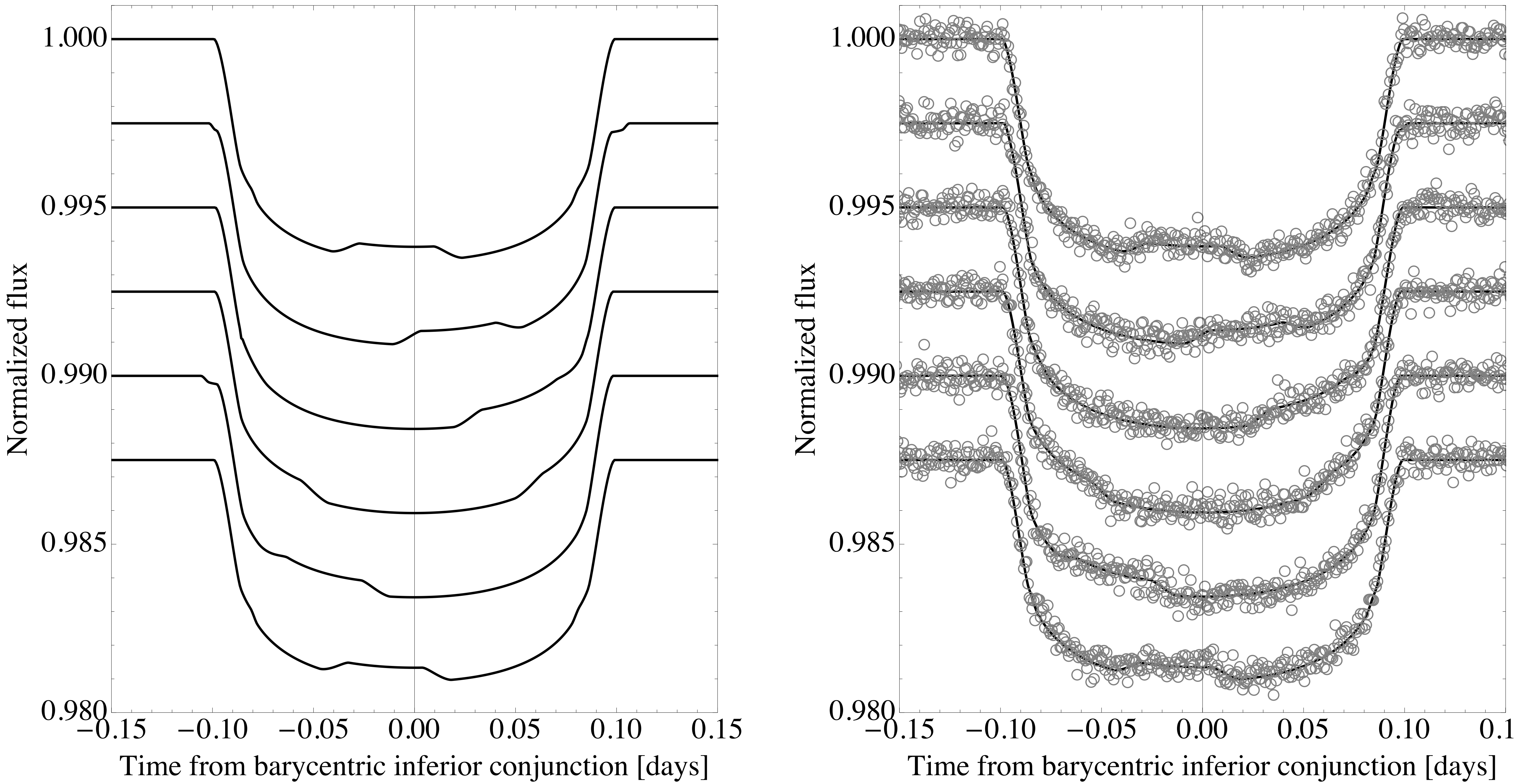
- Kipping (2012); analytic, dynamic algorithm called *LUNA* [analytic solution is public]
- Tusnksi & Valio (2011); circular, coplanar moons only [availability unknown]
- Pal (2012); not specific for moons, simulates mutual events [code is public]
- Sato & Asada (2009); circular, coplanar, no LD [availability unknown]
- Simon, Szabó & Szatmáry (2009); sparse details
- Sartoretti & Schneider (1999); sparse details

Neptune in hab-zone of M2 dwarf with close-in prograde Earth-mass and radius moon

Neptune in hab-zone of M2 dwarf with close-in prograde Earth-mass and radius moon

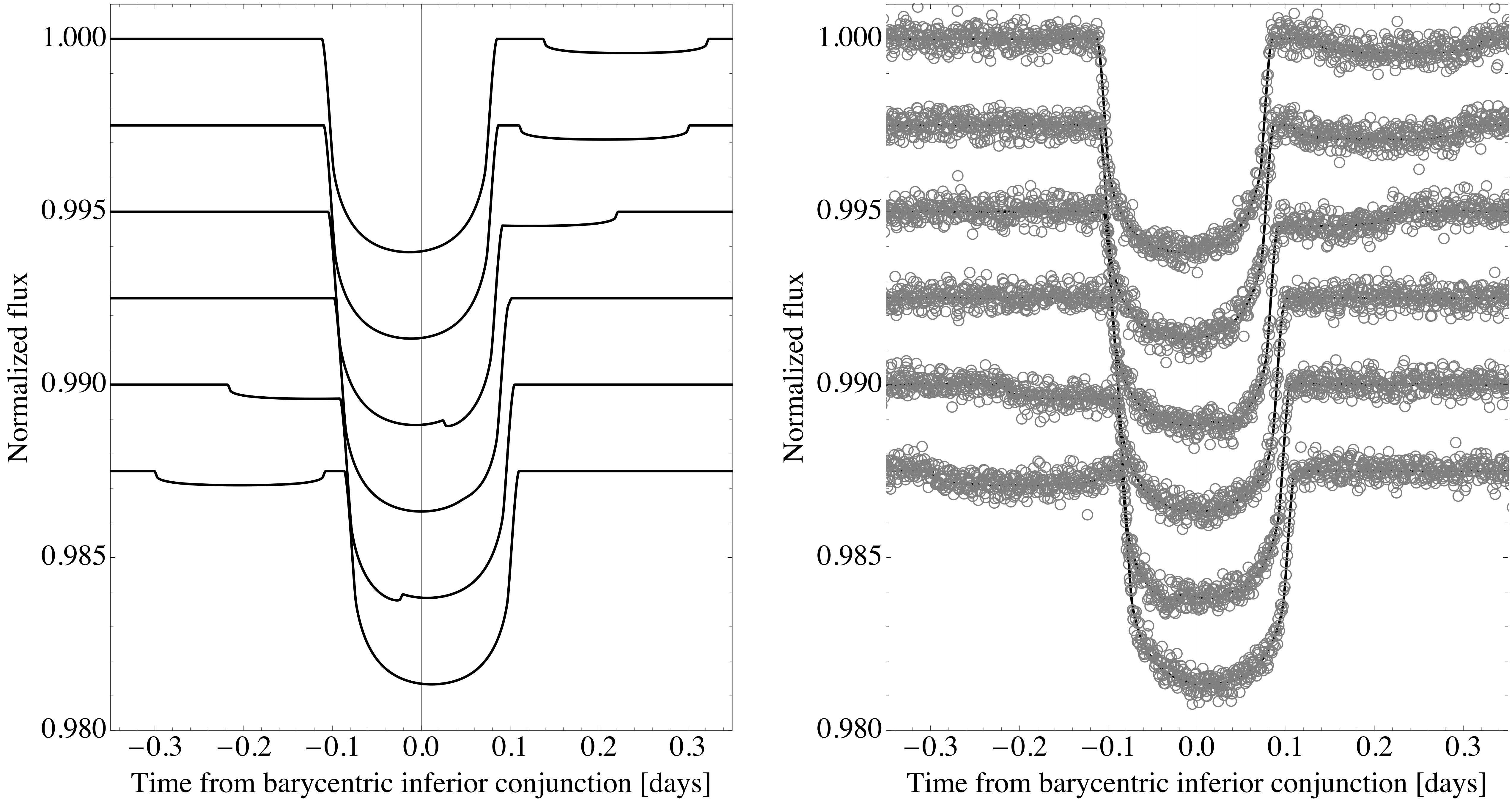


Neptune in hab-zone of M2 dwarf with close-in prograde Earth-mass and radius moon

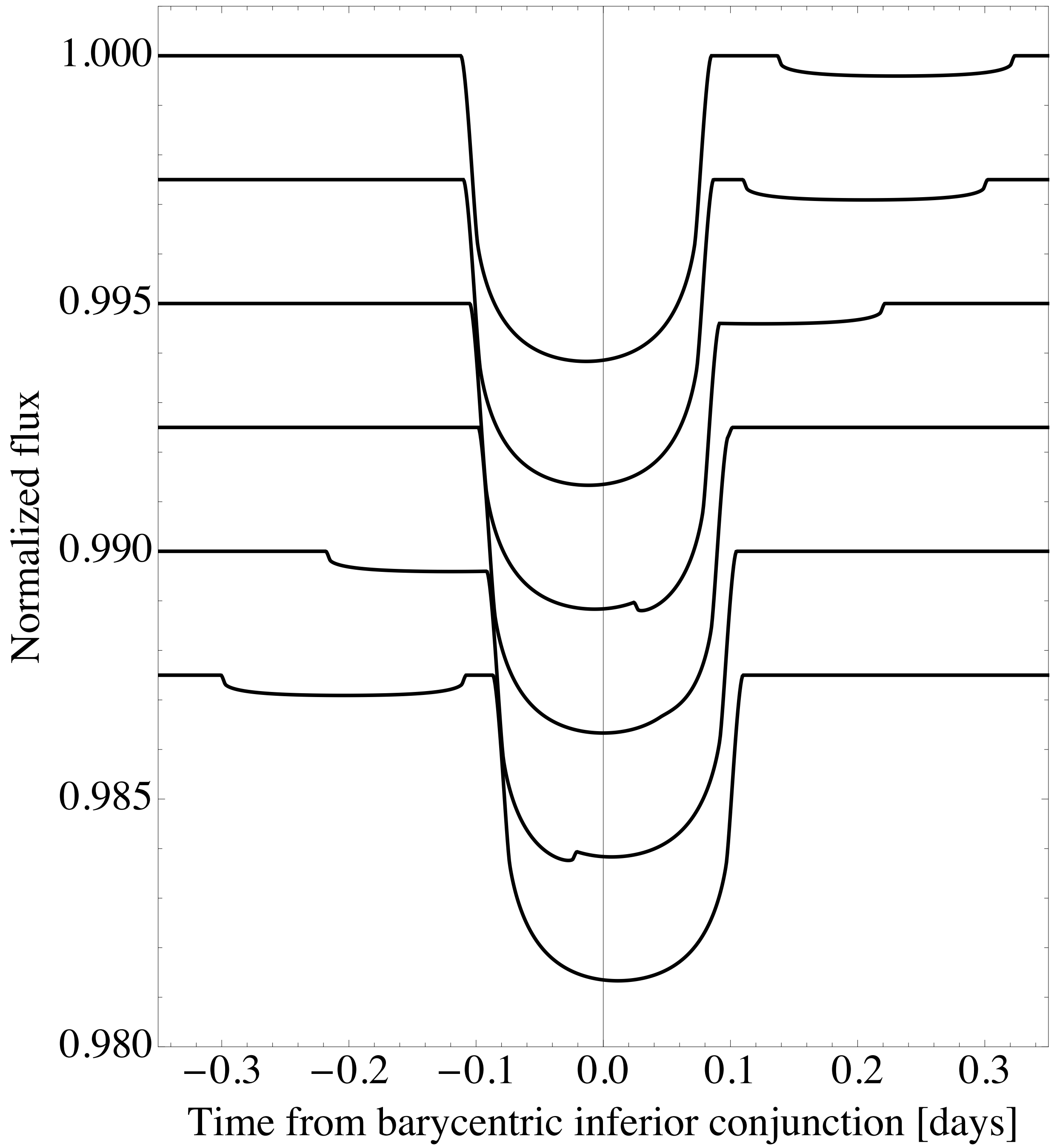


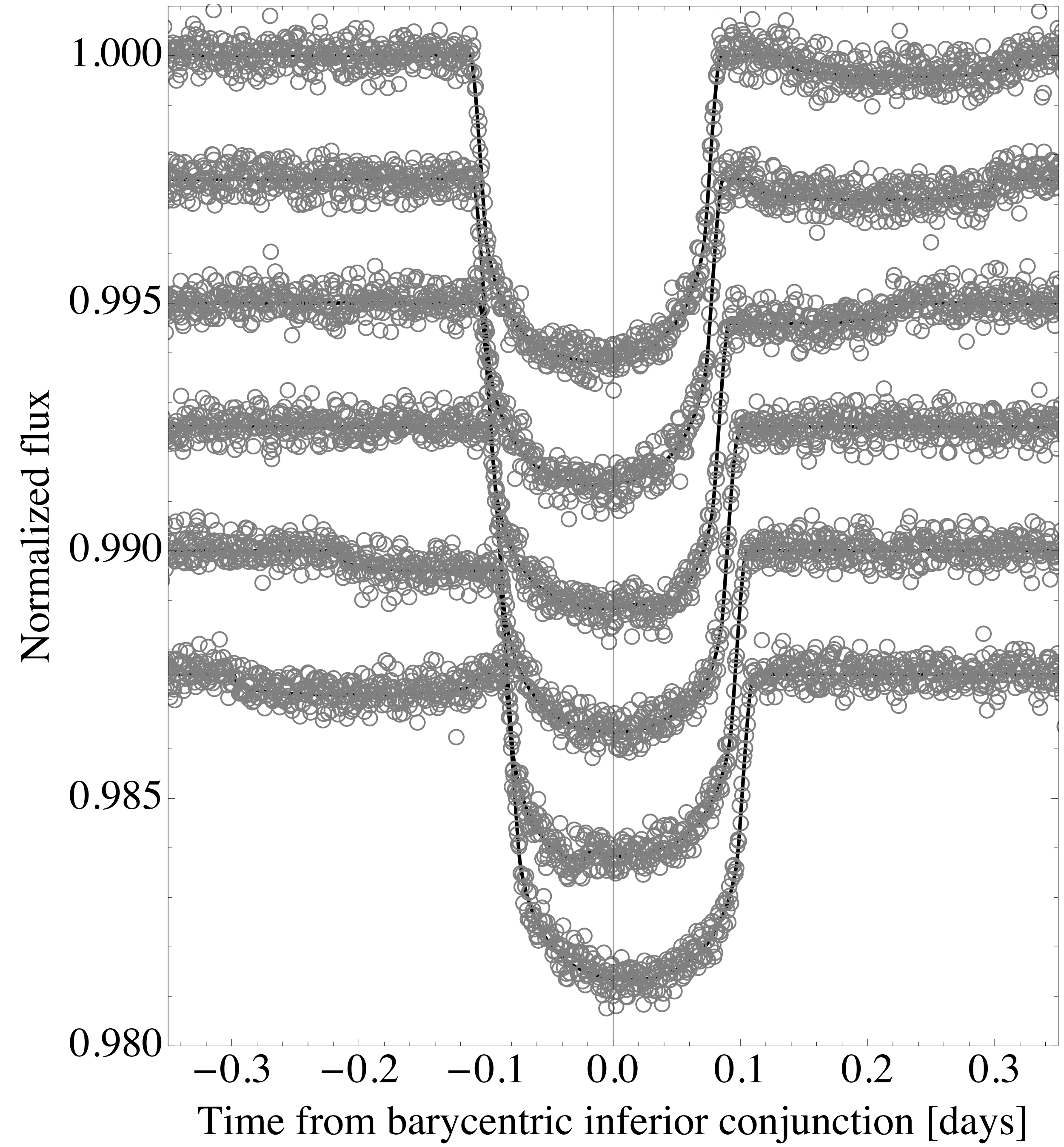
24-sigma detection for typical Kepler noise

Neptune in hab-zone of M2 dwarf with **far-out retrograde** Earth-mass and radius moon



Neptune in hab-zone of M2 dwarf with **far-out retrograde** Earth-mass and radius moon



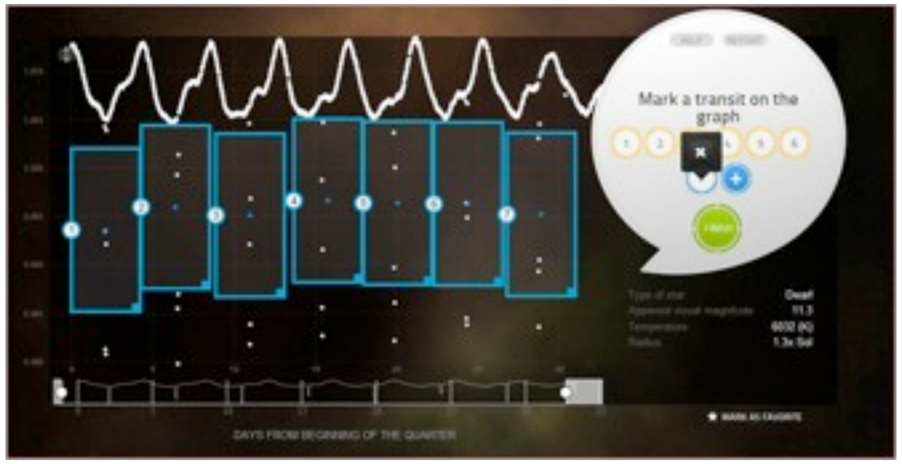


50-sigma detection for typical Kepler noise

Ongoing Searches

- *Hunt for Exomoons with Kepler* (HEK) project using public Kepler data (Kipping et al. 2012) [see www.exomoon.eu]
- PlanetHunters.org (Fischer et al. 2011)
- Kepler Science Team (Borucki et al. 2009)

The Hunt for Exomoons
with Kepler



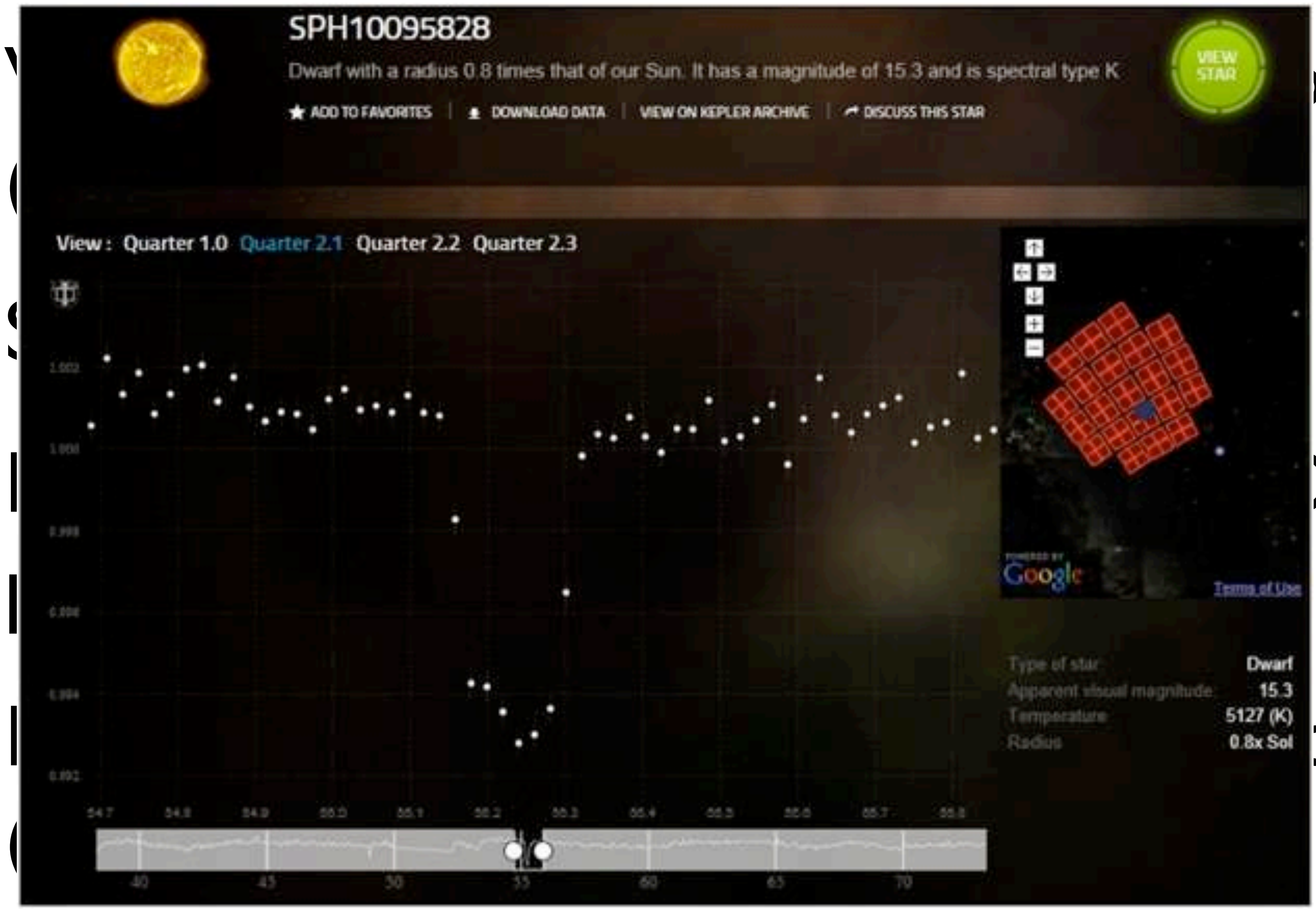
HEK: The Hunt for Exomoons with Kepler

- The first systematic search for transiting exomoons.
- Using public Kepler data
- Utilizing LUNA to identify exomoons
- Primary goal: detect a transiting exomoon(s)
- Secondary goal: obtain upper limits
- Tertiary goal: determine the frequency of large moons around viable planet hosts, $\eta_{☾}$

Search Methods

- Visual inspection e.g. PlanetHunters.org (Fischer et al. 2011)
- Scatter-peak (Simon et al. 2011)
- Epicyclic folding (Parker 2012, see POP presentation by Alex)
- Full model regression e.g. HEK project (Kipping et al. 2012)

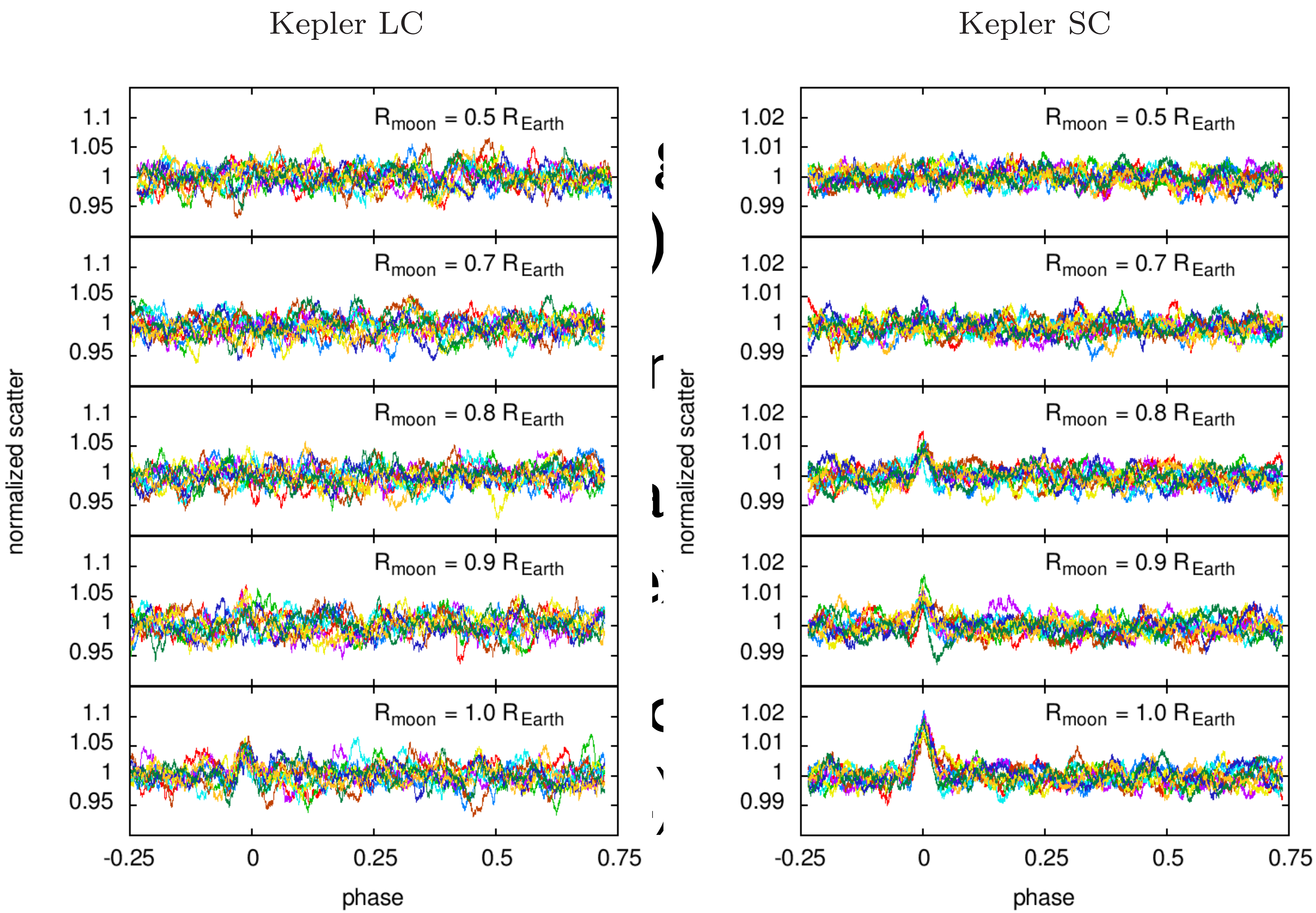
Search Methods



Search Methods

- Visual inspection e.g. PlanetHunters.org (Fischer et al. 2011)
- Scatter-peak (Simon et al. 2011)
- Epicyclic folding (Parker 2012, see POP presentation by Alex)
- Full model regression e.g. HEK project (Kipping et al. 2012)

Search Methods

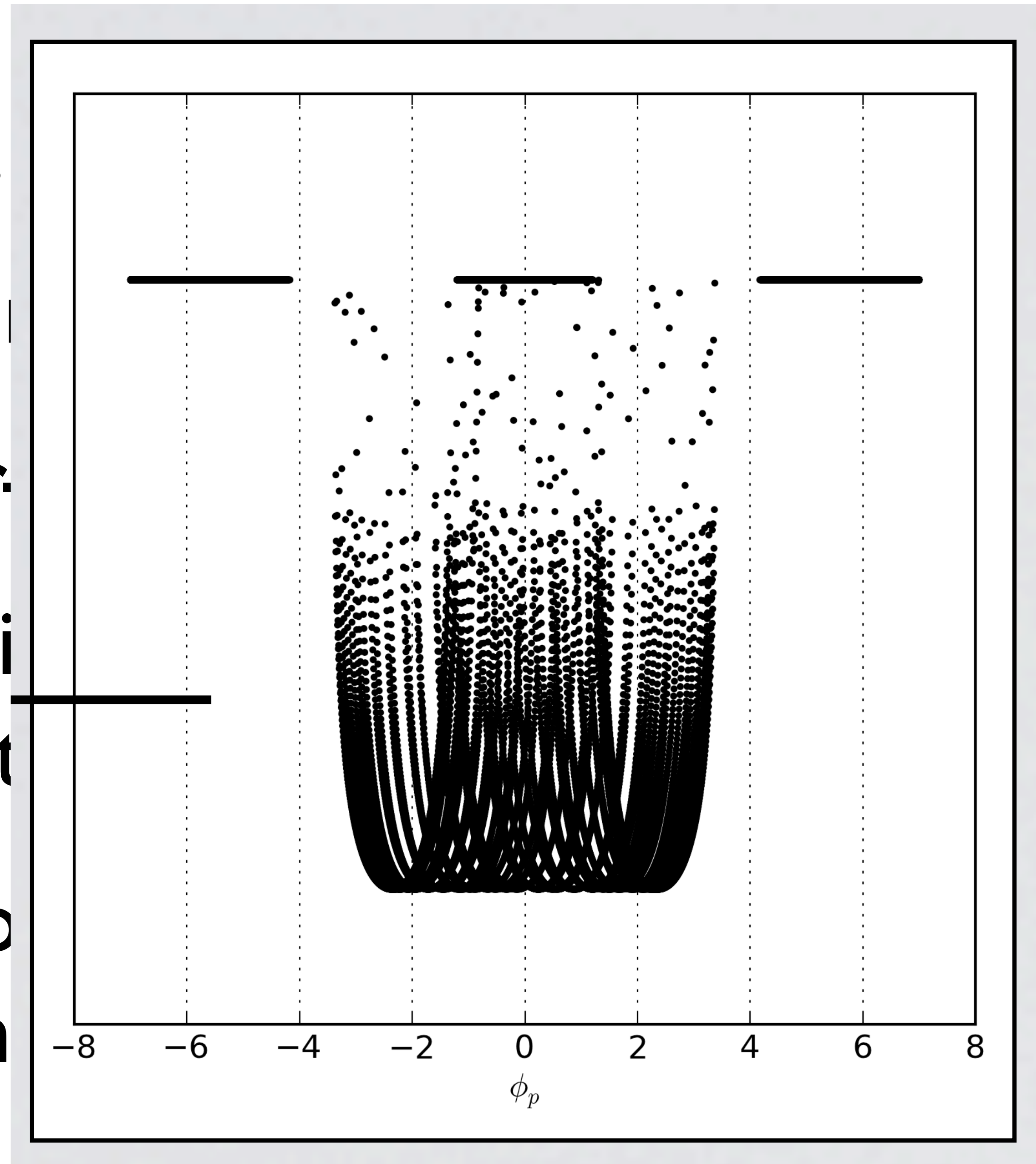


Search Methods

- Visual inspection e.g. PlanetHunters.org (Fischer et al. 2011)
- Scatter-peak (Simon et al. 2011)
- Epicyclic folding (Parker 2012, see POP presentation by Alex)
- Full model regression e.g. HEK project (Kipping et al. 2012)

Search Methods

- Visual i ... rs.org (Fische
- Scatter-
- Epicycli ... POP present
- Full mo ... oject (Kippin

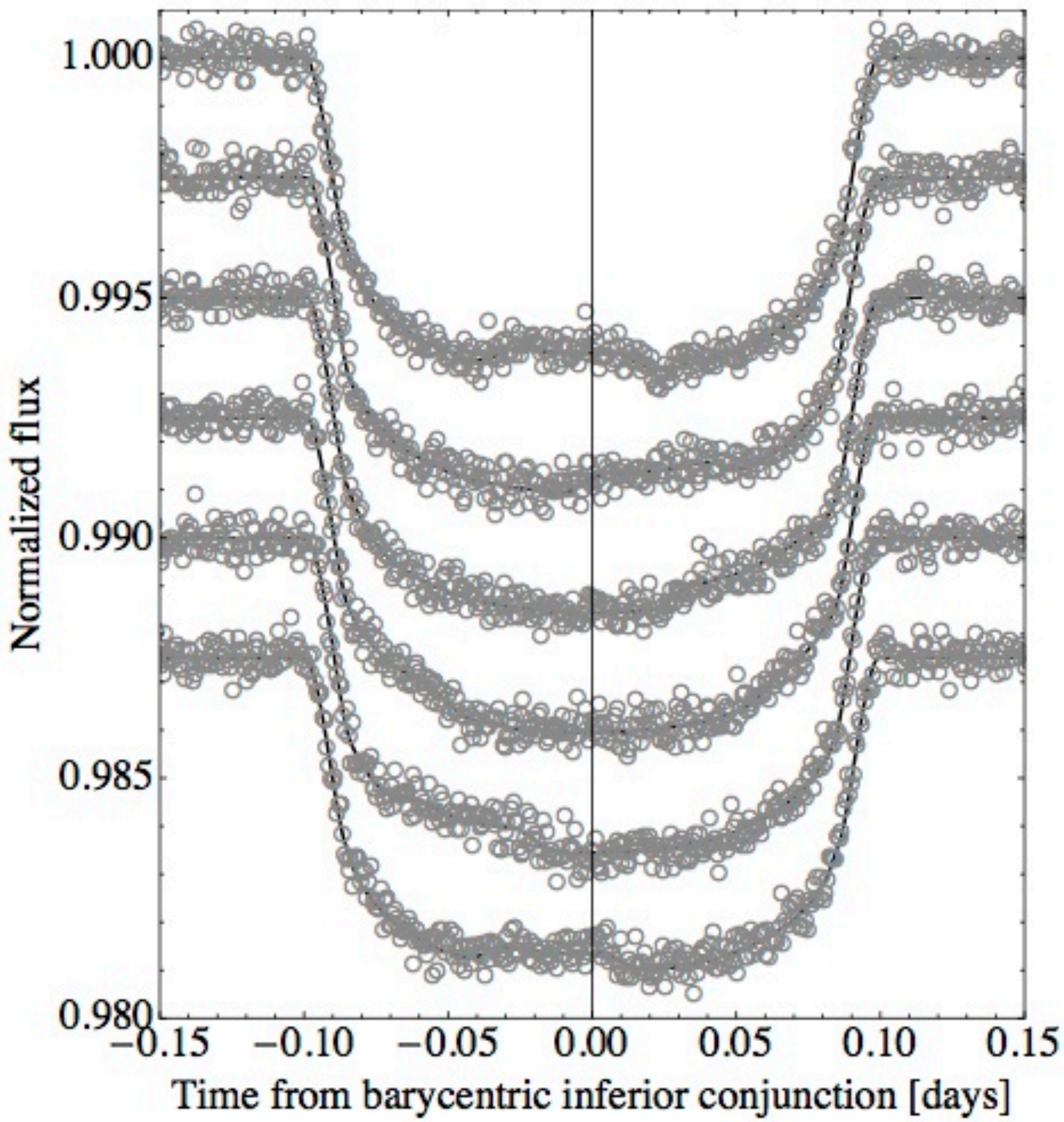


Search Methods

- Visual inspection e.g. PlanetHunters.org (Fischer et al. 2011)
- Scatter-peak (Simon et al. 2011)
- Epicyclic folding (Parker 2012, see POP presentation by Alex)
- Full model regression e.g. HEK project (Kipping et al. 2012)

Search Methods

- Visual i org (Fische
- Scatter-
- Epicycli OP present
- Full mo ect (Kippin



Search Methods

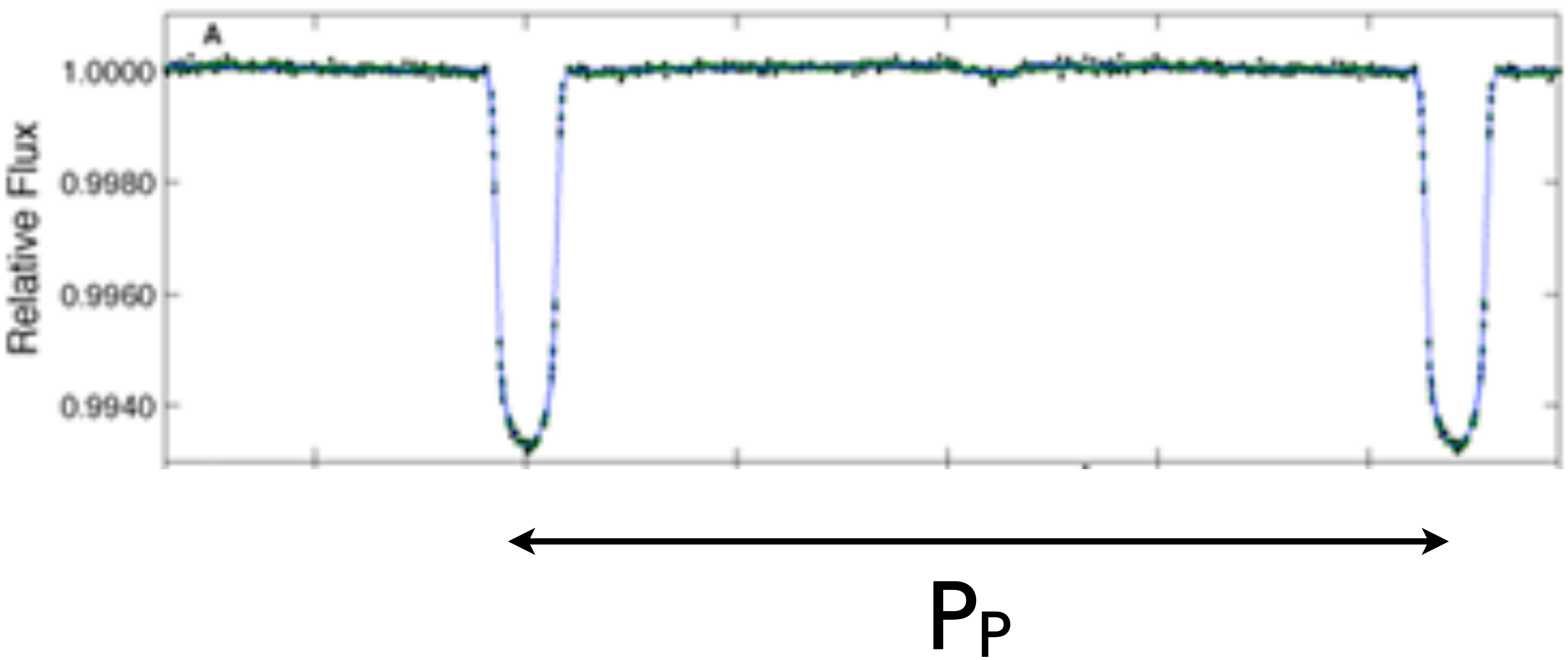
- Visual inspection e.g. PlanetHunters.org (Fischer et al. 2011)
- Scatter-peak (Simon et al. 2011)
- Epicyclic folding (Parker 2012, see POP presentation by Alex)
- Full model regression e.g. HEK project (Kipping et al. 2012)

IV. Challenges

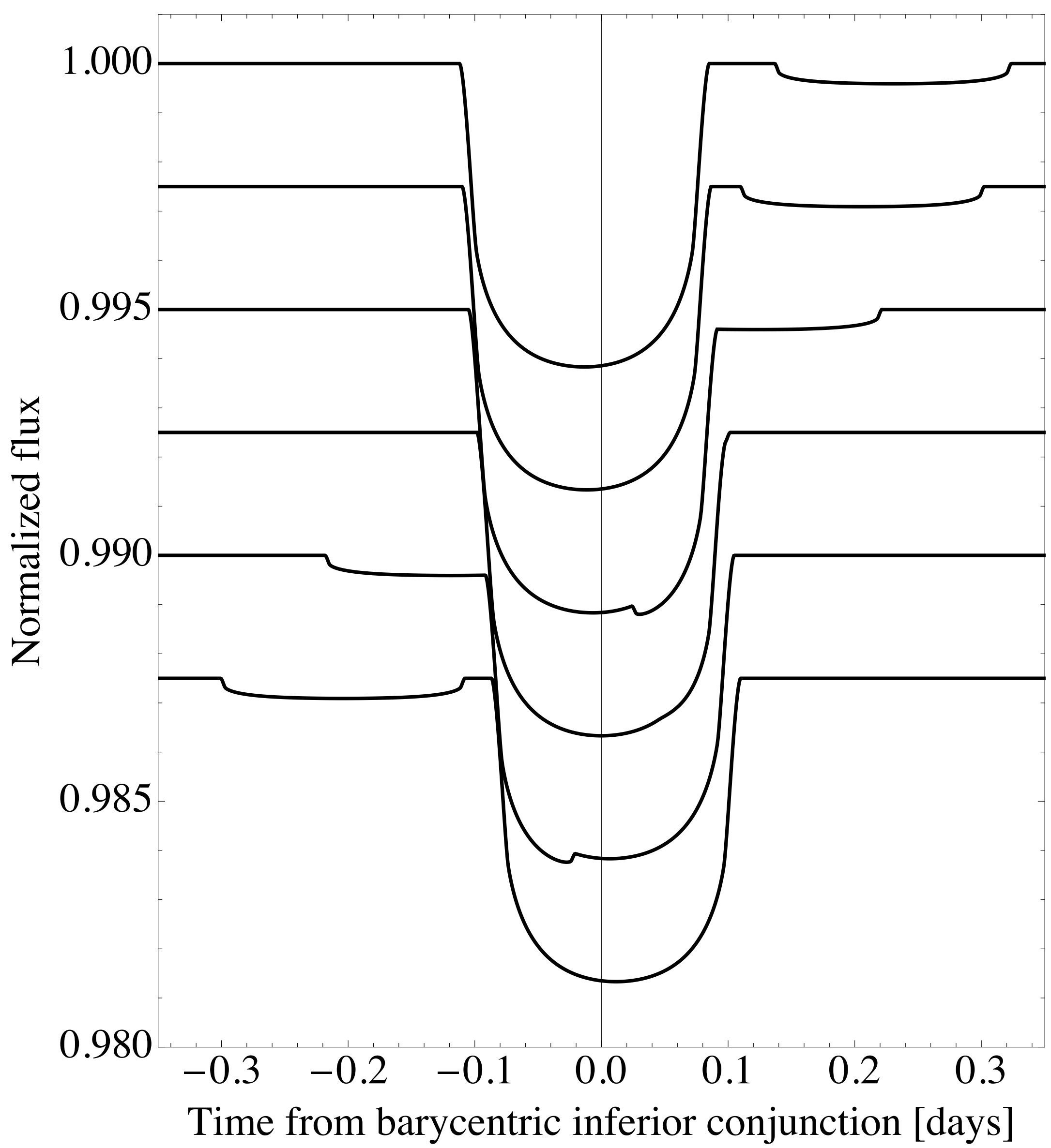
1. Target Selection

- There are >2300 KOIs to choose from.
- Depending on the efficiency of your search, only a fraction of these can be practically analyzed.
- Target selection by dynamics, visual anomalies, bright/quiet stars, etc often required

2. Period Searches Are Tough

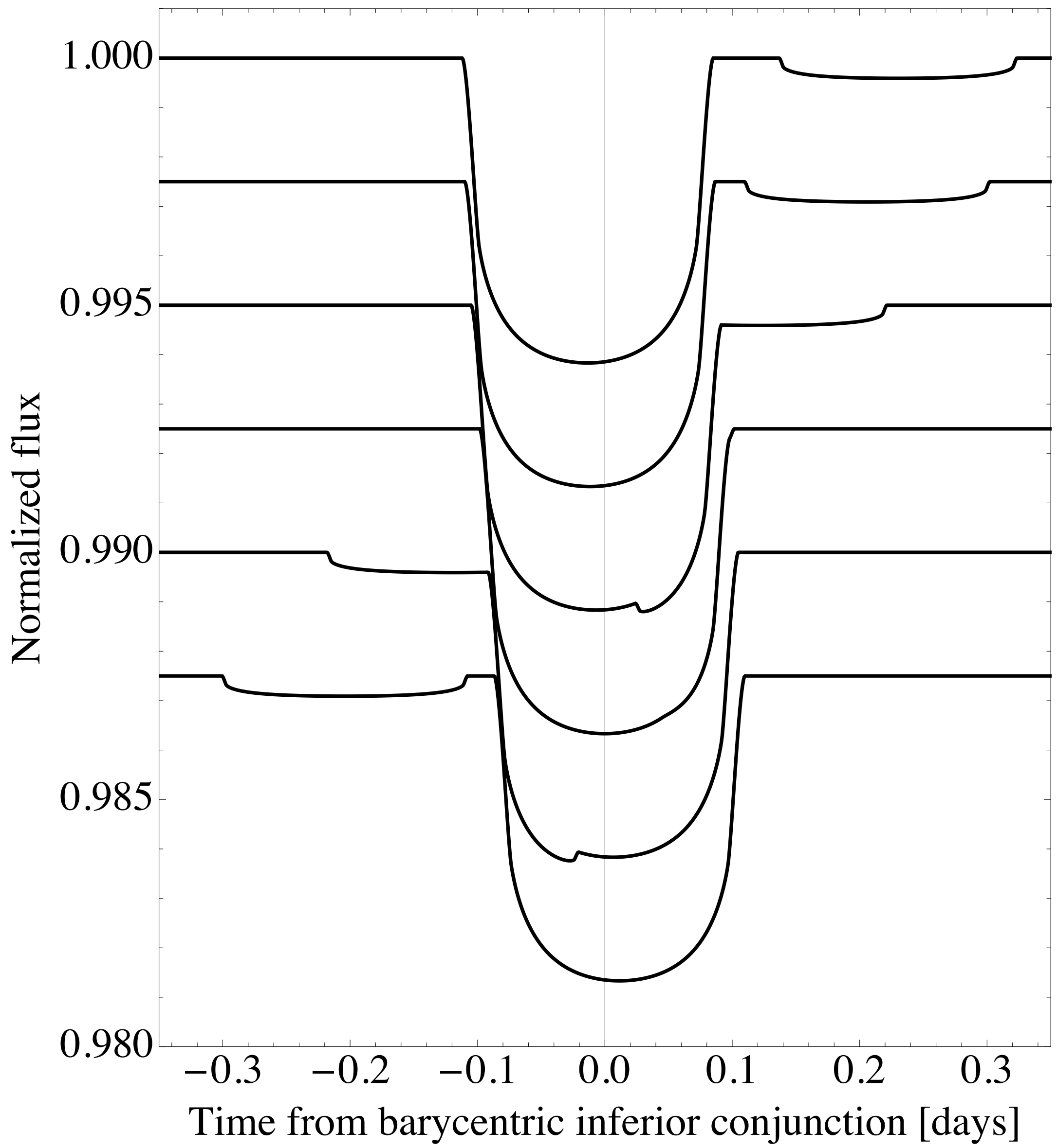


2. Period Searches Are Tough



P_S = ?

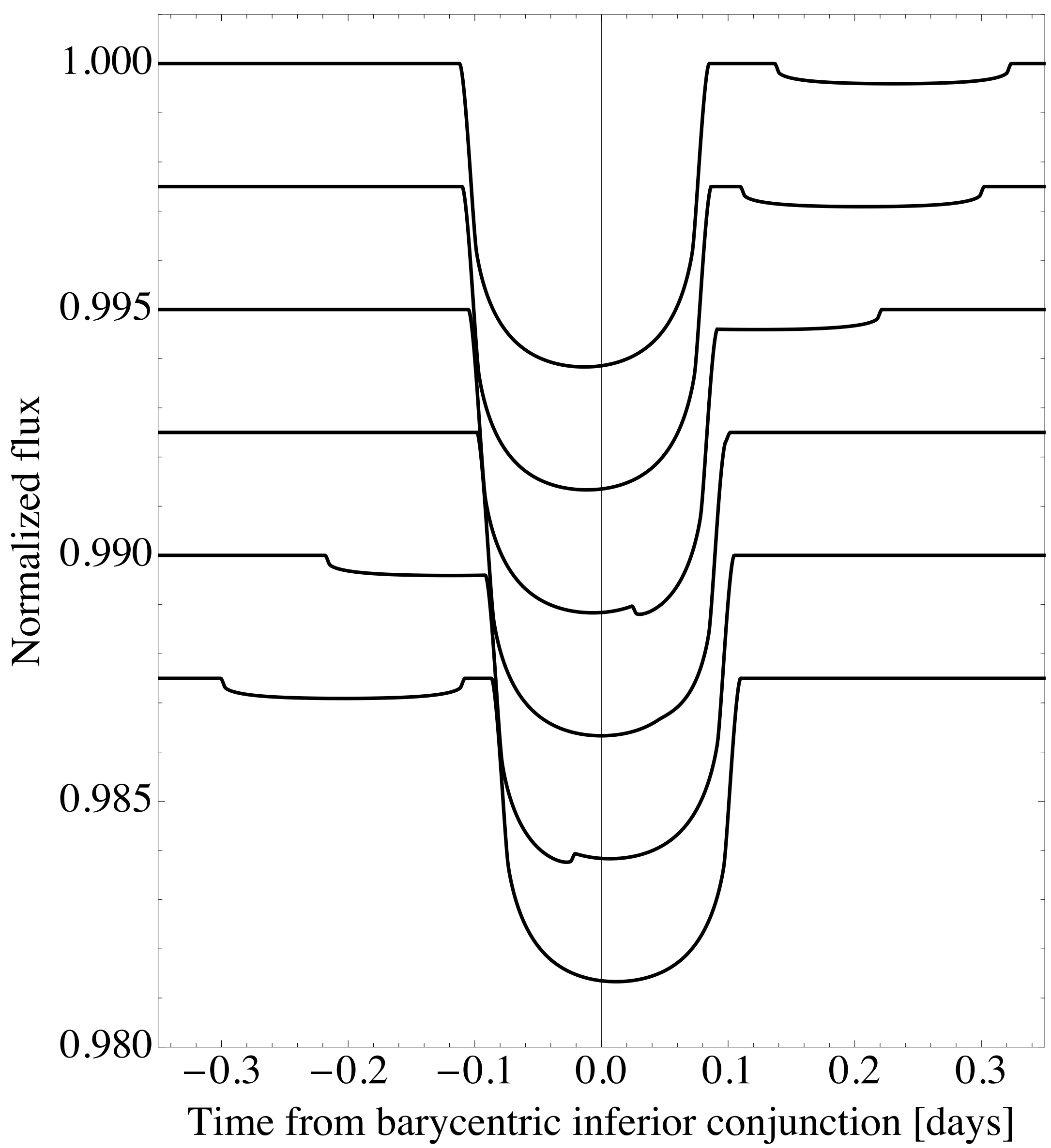
2. Period Searches Are Tough



- Multiple modes present

$P_S = ?$

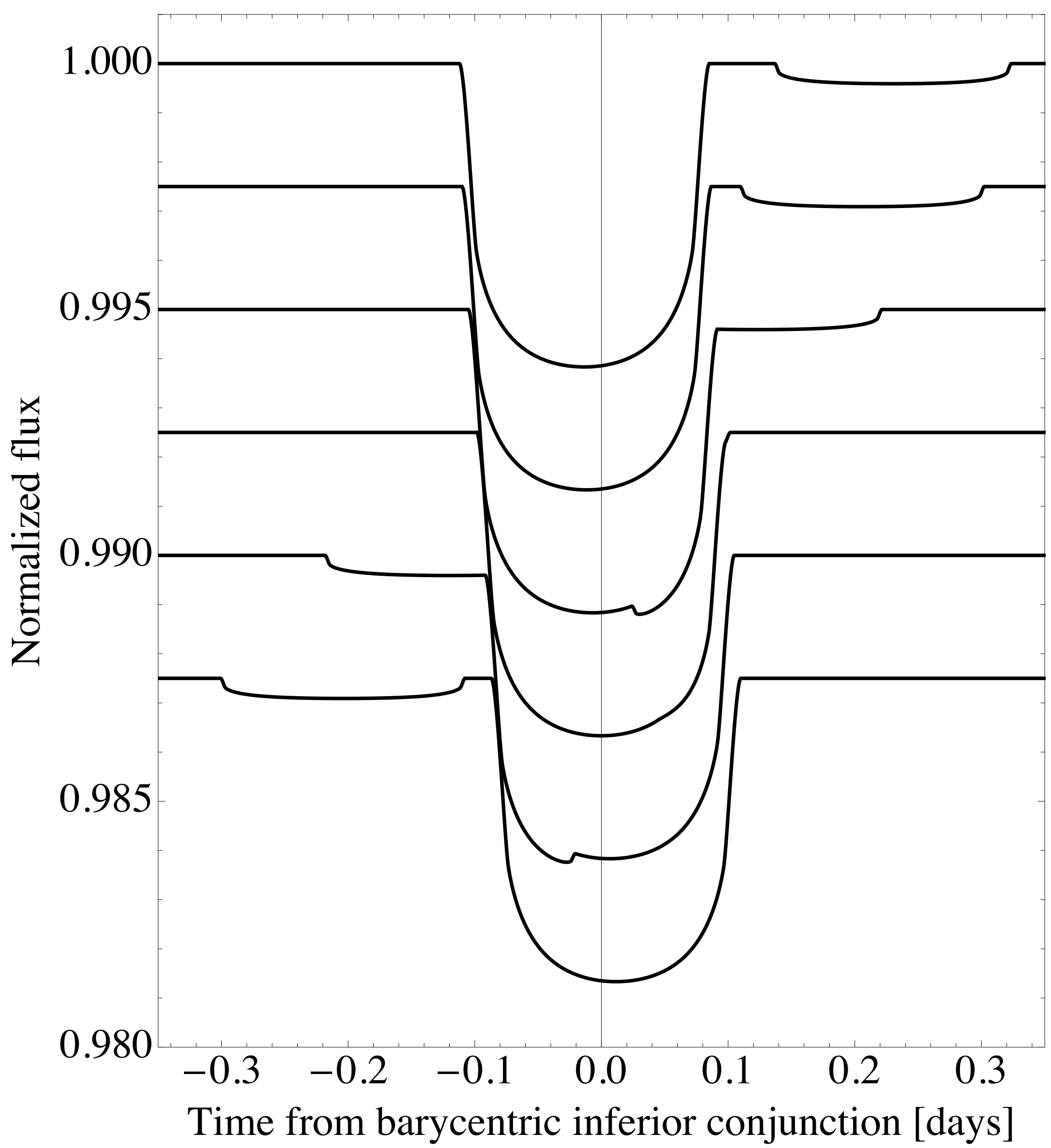
2. Period Searches Are Tough



- Multiple modes present
- Very challenging to visually guess where these modes will occur

$P_S = ?$

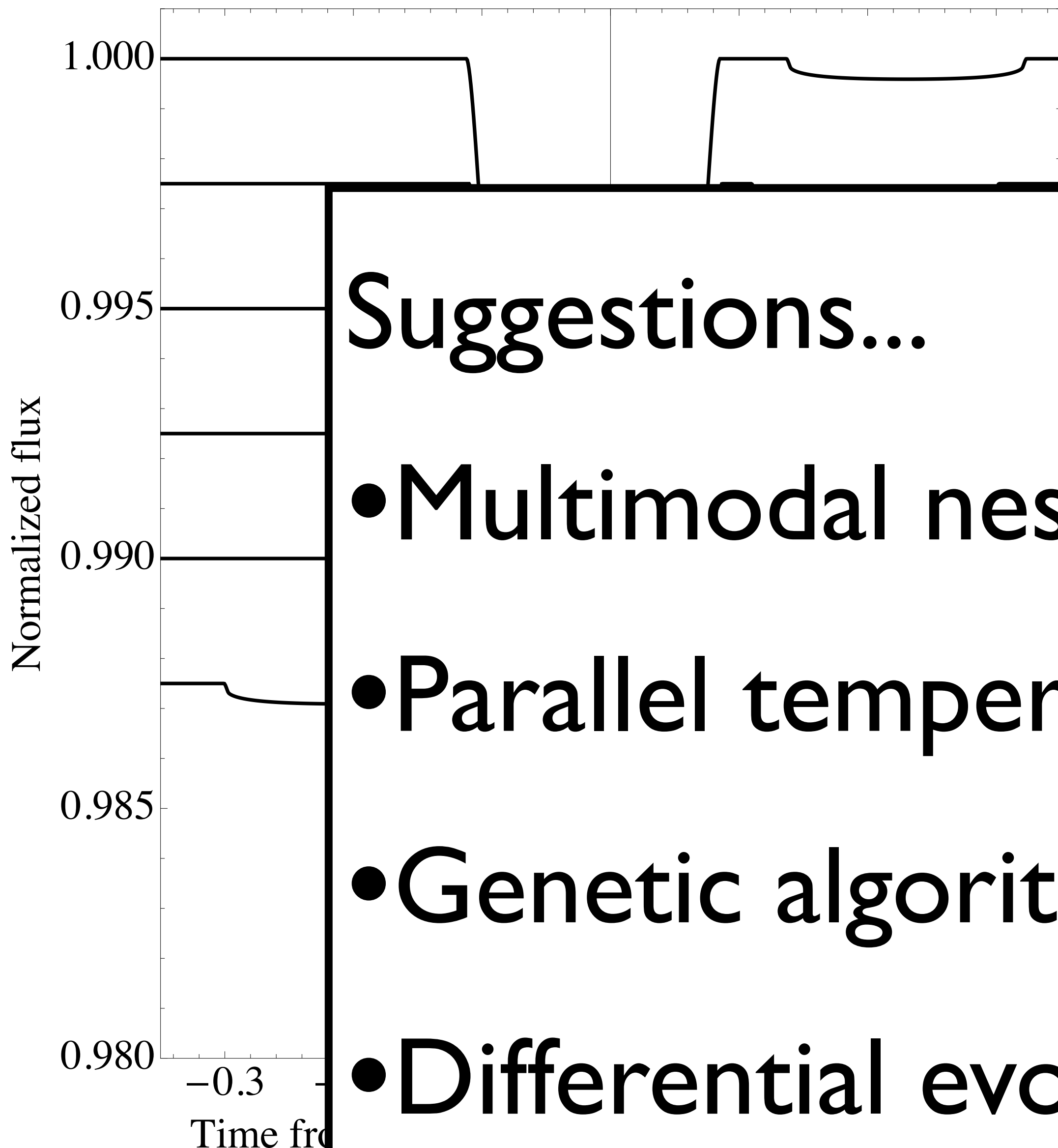
2. Period Searches Are Tough



- Multiple modes present
- Very challenging to visually guess where these modes will occur
- Requires a **comprehensive** and **multimodal** parameter search

$P_S = ?$

2. Period Searches Are Tough



Suggestions...

- Multimodal nested sampling
- Parallel tempering
- Genetic algorithms
- Differential evolution

es present

ng to

where

will occur

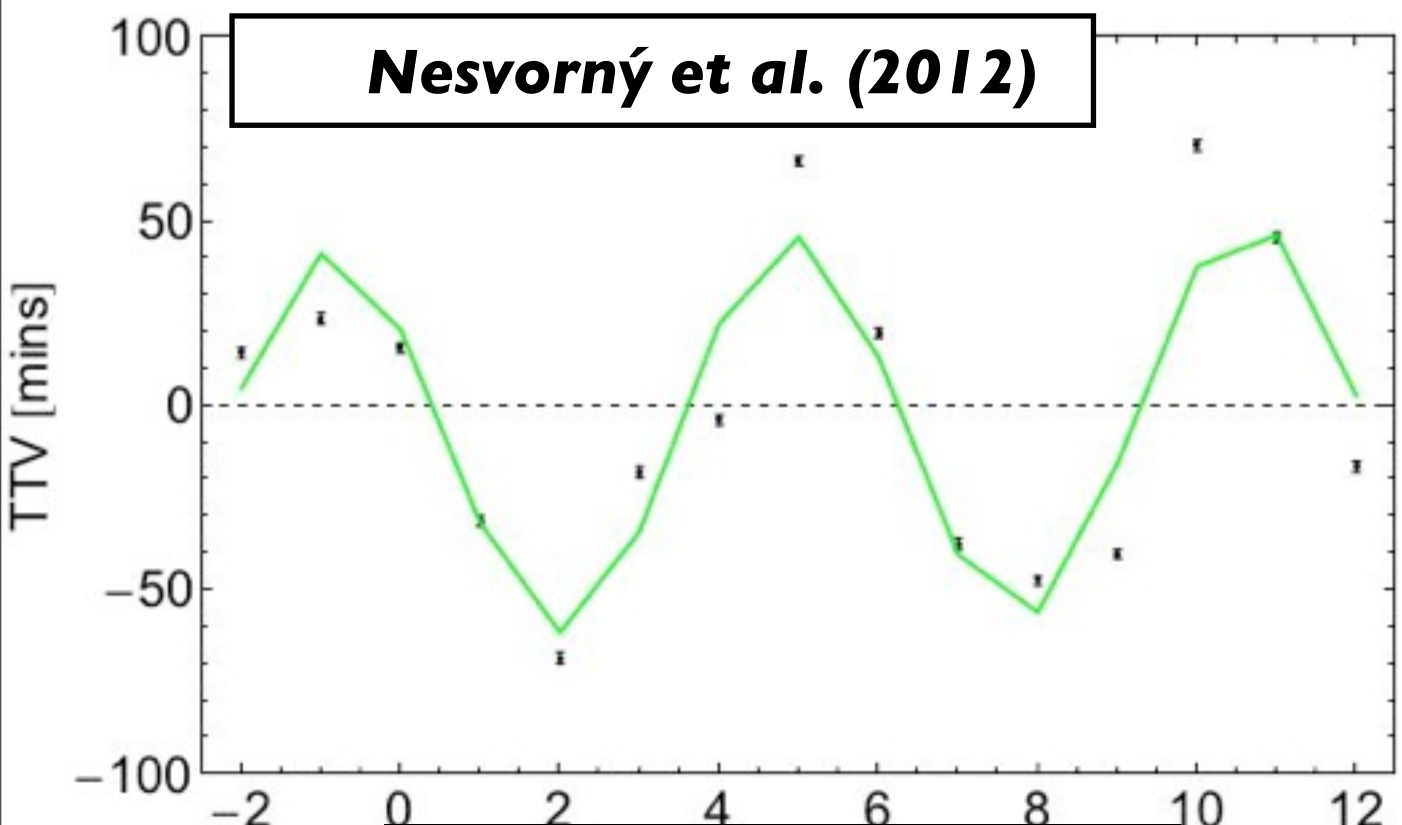
nsive and

parameter search

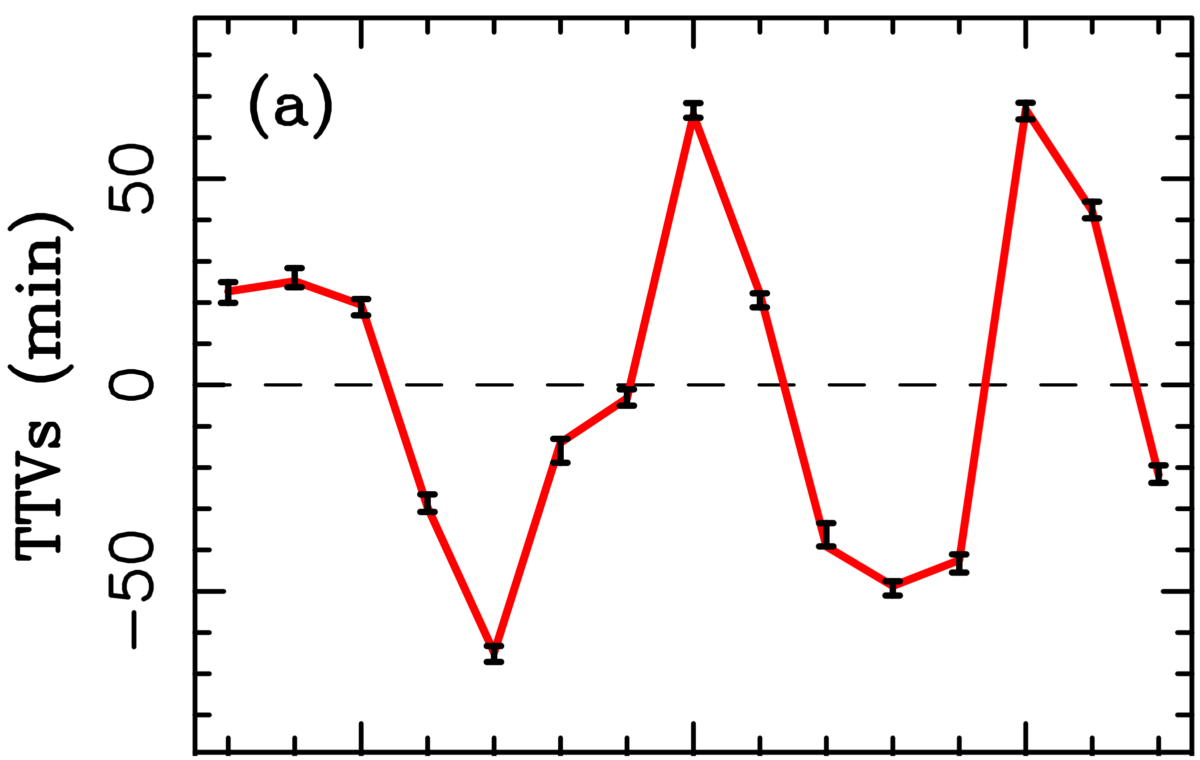
P_S = ?

3. Confounding Effects

- A second planet in the system may also induce TTVs.
- If TDVs exist, one can use the phase-trick.
- If not, one must compare whether a moon or a second planet explain the data better:



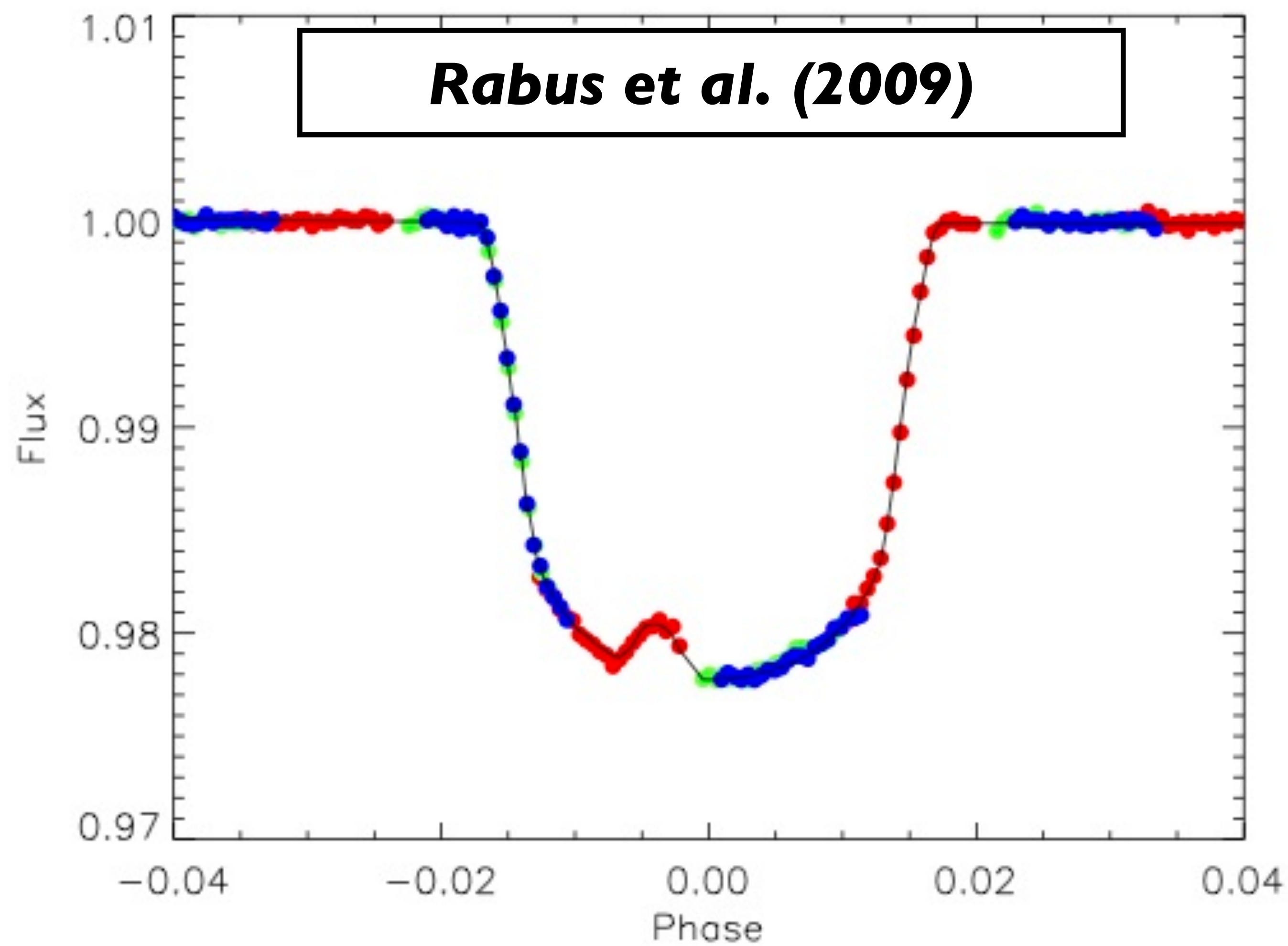
Moon-fit of KOI-872b



2nd planet-fit of KOI-872b

3. Confounding Effects

- Starspot crossings morphologically resemble an exomoon mutual transit.
- Spots can reveal spin-orbit alignments (see talk by Roberto)...
- ...but for moon-hunters they are a pain!



3. Confounding Effects

3. Confounding Effects

- Spots should exhibit out-of-transit modulation which can be tracked.

3. Confounding Effects

- Spots should exhibit out-of-transit modulation which can be tracked.
- An absence of any such behaviour would detract form the spot hypothesis.

3. Confounding Effects

- Spots should exhibit out-of-transit modulation which can be tracked.
- An absence of any such behaviour would detract form the spot hypothesis.
- Spots are chromatic, moons are achromatic.

3. Confounding Effects

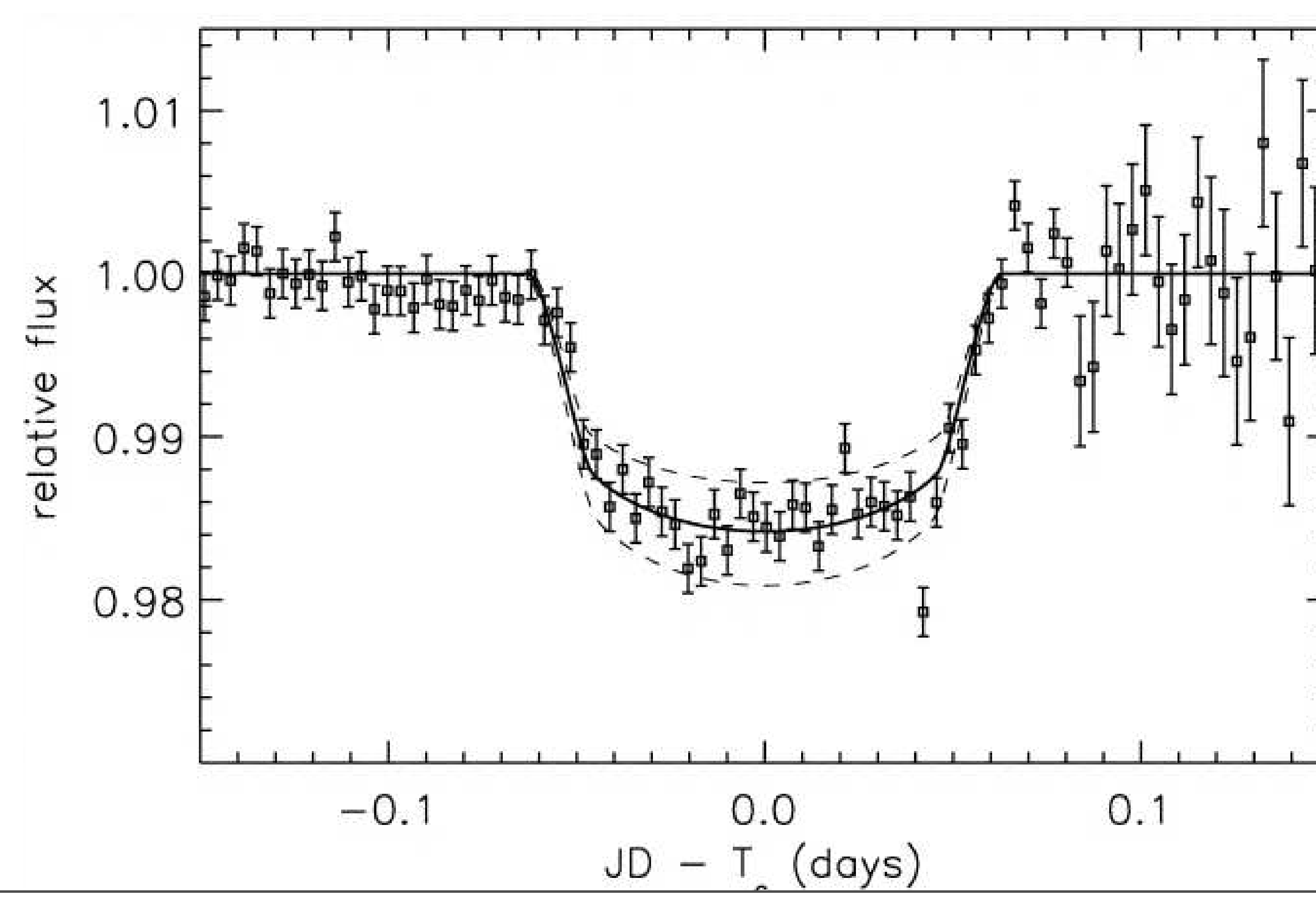
- Spots should exhibit out-of-transit modulation which can be tracked.
- An absence of any such behaviour would detract form the spot hypothesis.
- Spots are chromatic, moons are achromatic.
- Moons allow us to determine the density of the planet, which can be used in vetting.

3. Confounding Effects

- Spots should exhibit out-of-transit modulation which can be tracked.
- An absence of any such behaviour would detract form the spot hypothesis.
- Spots are chromatic, moons are achromatic.
- Moons allow us to determine the density of the planet, which can be used in vetting.
- Ultimately it would be advantageous to have a full starspot model for comparison

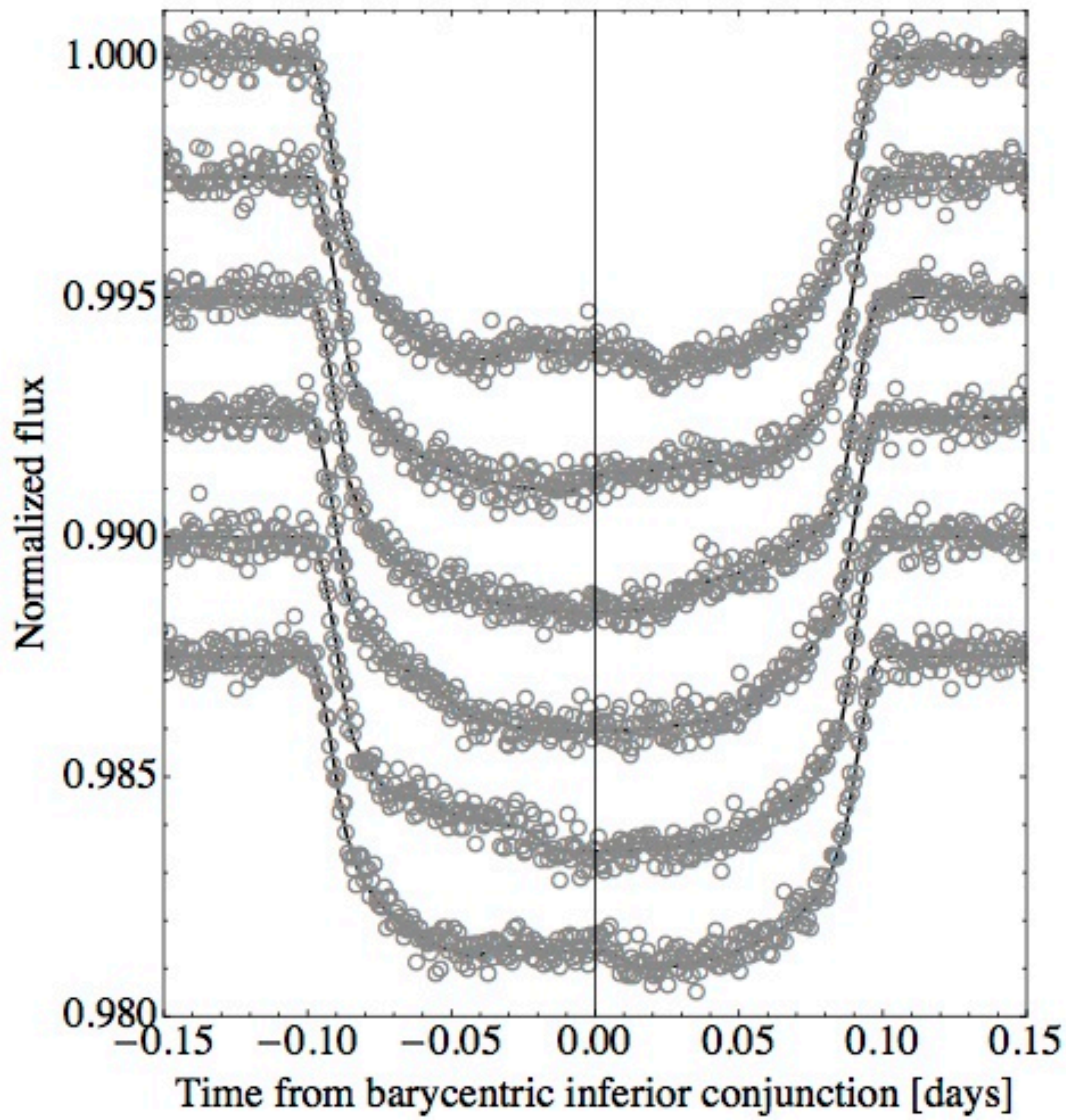
4. What is a “detection”?

- First transiting planet detection, HD 209458b (Charbonneau et al. 2000)
- Simple signal, high SNR, few alternative explanations



4. What is a “detection”?

- Exomoons induce complex signals
- Low SNR
- Several alternative explanations



4. What is a "detection"?

- Data could be due to a planet-with-moon, a planet with correlated noise, a planet and starspots, a planet + perturbing planet, etc...
- We need to perform **model selection** between these options.

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)} \frac{\Pr(\mathcal{M}_1)}{\Pr(\mathcal{M}_2)}$$

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

We want to calculate this

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)} \frac{\Pr(\mathcal{M}_1)}{\Pr(\mathcal{M}_2)}$$

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

We want to calculate this

Prior probability of model 1

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)} \frac{\Pr(\mathcal{M}_1)}{\Pr(\mathcal{M}_2)}$$

Prior probability of model 2

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

We want to calculate this

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)} \frac{\Pr(\mathcal{M}_1)}{\Pr(\mathcal{M}_2)}$$

Prior probability of model 1

Prior probability of model 2

Normally we assume no prior preference between models 1 and 2:

$$\Pr(\mathcal{M}_1) = \Pr(\mathcal{M}_2)$$

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

We want to calculate this

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)}$$

Normally we assume no prior preference between models 1 and 2:

$$\Pr(\mathcal{M}_1) = \Pr(\mathcal{M}_2)$$

What is model selection?

- Bayesian model selection compares the probability of model 1, given data D, versus the probability of model 2, given data D.

We want to calculate this

$$\frac{\Pr(\mathcal{M}_1|\mathbf{D})}{\Pr(\mathcal{M}_2|\mathbf{D})} = \frac{\Pr(\mathbf{D}|\mathcal{M}_1)}{\Pr(\mathbf{D}|\mathcal{M}_2)}$$

=> need to calculate these aka the ***Bayesian evidence***

Normally we assume no prior preference between models 1 and 2:

$$\Pr(\mathcal{M}_1) = \Pr(\mathcal{M}_2)$$

What is the Bayesian evidence?

- We need calculate the ***Bayesian evidence***.
- You may be asking yourself - *Why have I not heard of this before?*

$$\mathrm{Pr}(\mathbf{\Theta}|\mathbf{D},\mathcal{M}) = \frac{\mathrm{Pr}(\mathbf{D}|\mathbf{\Theta},\mathcal{M})\mathrm{Pr}(\mathbf{\Theta}|\mathcal{M})}{\mathrm{Pr}(\mathbf{D}|\mathcal{M})}$$

What is the Bayesian evidence?

- We need calculate the ***Bayesian evidence***.
- You may be asking yourself - *Why have I not heard of this before?*

$$\Pr(\mathbf{\Theta}|\mathbf{D},\mathcal{M}) = \frac{\Pr(\mathbf{D}|\mathbf{\Theta},\mathcal{M})\Pr(\mathbf{\Theta}|\mathcal{M})}{\Pr(\mathbf{D}|\mathcal{M})}$$

Posterior probability distribution of the parameters

Likelihood

Prior

Bayesian evidence

What is the Bayesian evidence?

- We need calculate the ***Bayesian evidence***.
- You may be asking yourself - *Why have I not heard of this before?*

Posterior probability distribution of the parameters

Likelihood

Prior

$$\Pr(\mathbf{\Theta}|\mathbf{D},\mathcal{M}) = \frac{\Pr(\mathbf{D}|\mathbf{\Theta},\mathcal{M})\Pr(\mathbf{\Theta}|\mathcal{M})}{\Pr(\mathbf{D}|\mathcal{M})}$$

Bayesian evidence

- The Bayesian evidence is a normalization factor, and so it is frequently ignored

Why is the Bayesian evidence usually ignored?

Why is the Bayesian evidence usually ignored?

- The most common parameter search algorithm for exoplanetary scientists is MCMC.

Why is the Bayesian evidence usually ignored?

- The most common parameter search algorithm for exoplanetary scientists is MCMC.
- In almost all planet-related applications, the main objective derive the posteriors probability distribution of the parameters, □.

Why is the Bayesian evidence usually ignored?

- The most common parameter search algorithm for exoplanetary scientists is MCMC.
- In almost all planet-related applications, the main objective derive the posteriors probability distribution of the parameters, □.
- Pr(D|M) is a normalization factor and thus does not affect the parameter posteriors, therefore it is not needed.

Why is the Bayesian evidence usually ignored?

- The most common parameter search algorithm for exoplanetary scientists is MCMC.
- In almost all planet-related applications, the main objective derive the posteriors probability distribution of the parameters, □
- Pr(D|M) is a normalization factor and thus does not affect the parameter posteriors, therefore it is not needed.
- Further, it is highly computational expensive to evaluate Pr (D|M).

Why is the Bayesian evidence usually ignored?

- The most common parameter search algorithm for exoplanetary scientists is MCMC.
- In almost all planet-related applications, the main objective derive the posteriors probability distribution of the parameters, □.
- Pr(D|M) is a normalization factor and thus does not affect the parameter posteriors, therefore it is not needed.
- Further, it is highly computational expensive to evaluate Pr (D|M).
- Thus MCMC techniques ignore the normalization and do not yield the Bayesian evidence.

Why is the Bayesian evidence usually ignored?

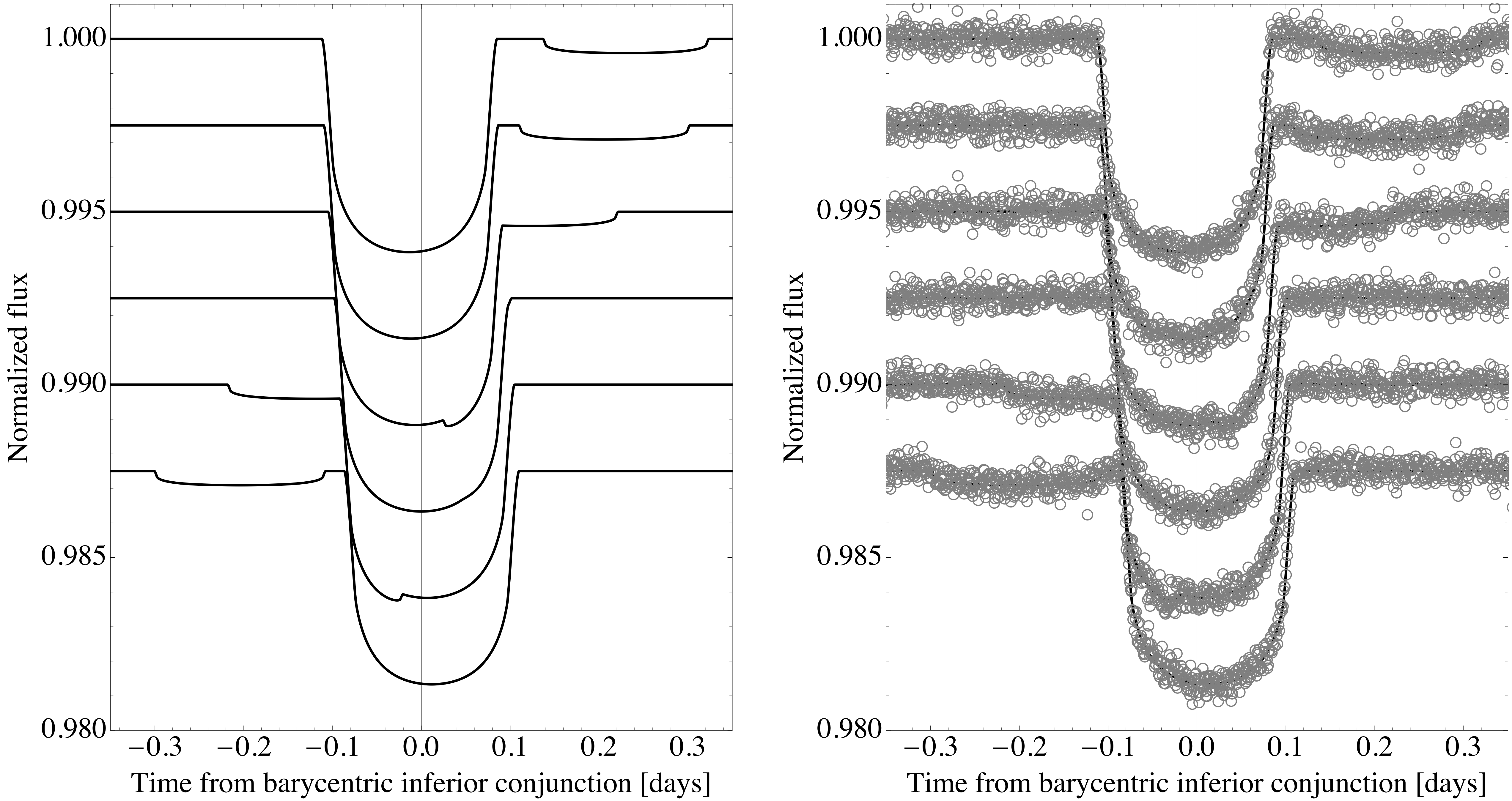
- The most common parameter search algorithm for exoplanetary scientists is MCMC.
- In almos ... objective derive t ... the paramet
- Pr(D|M) ... not affect the para ... eded.
- Further ... evaluate Pr (D|M).
- Thus MCMC techniques ignore the normalization and do not yield the Bayesian evidence.

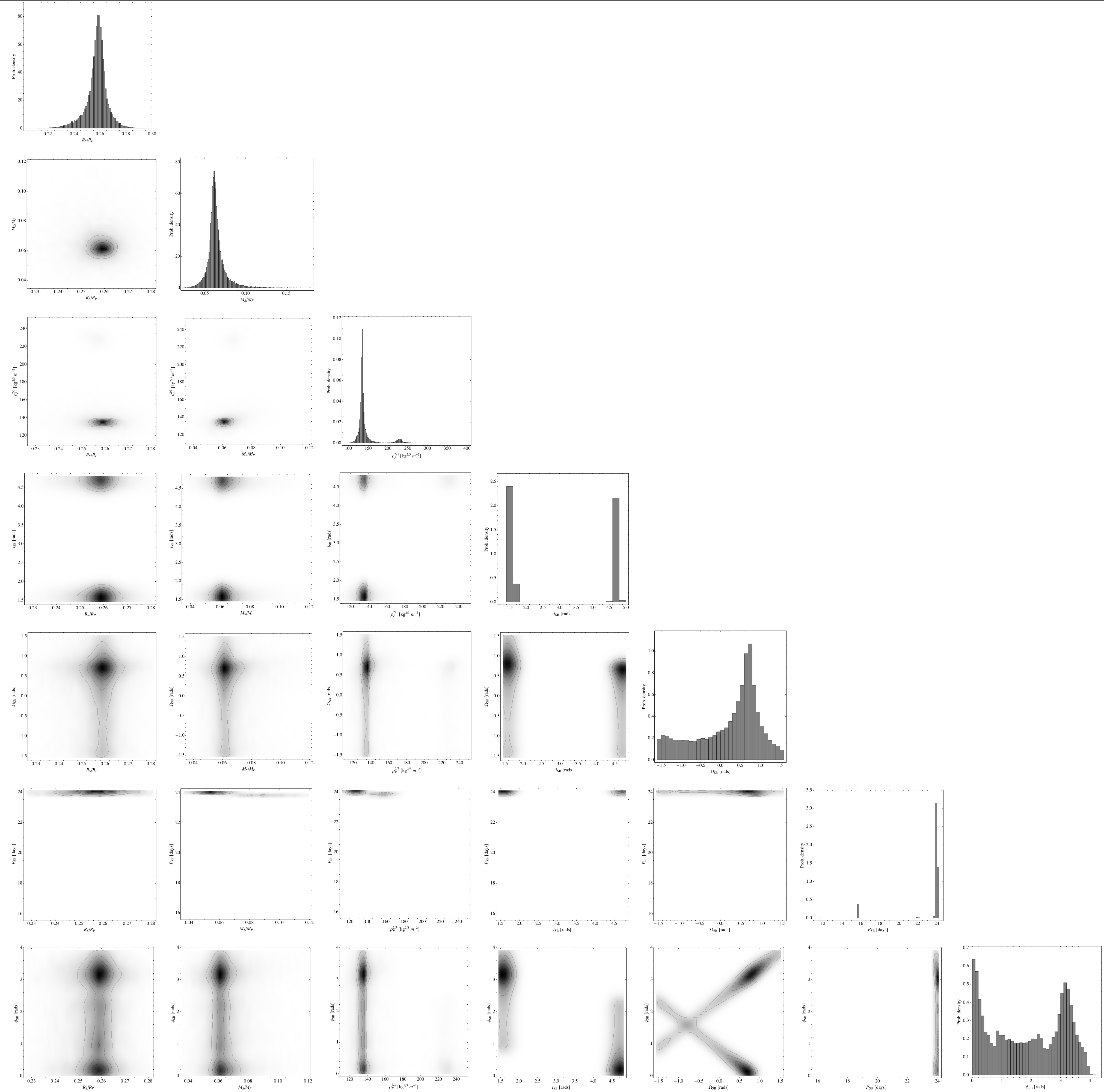
Suggestions...

- Multimodal nested sampling
- Thermodynamic integration (very slow)

V. Synthetic Example

Neptune in hab-zone of M2 dwarf with **far-out retrograde** Earth-mass and radius moon





Three Modes

Parameter	Truth	Mode 1	Mode 2	Mode 3	Global
$\log \mathcal{Z}$	-	23552.49 ± 0.36	23523.81 ± 0.99	23551.27 ± 0.45	23552.75 ± 0.27
Moon params.					
R_S/R_P	0.2570	$0.2587^{+0.0053}_{-0.0069}$	$0.2559^{+0.0051}_{-0.0065}$	$0.2587^{+0.0052}_{-0.0070}$	$0.2585^{+0.0053}_{-0.0069}$
M_S/M_P	0.0583	$0.0620^{+0.0086}_{-0.0058}$	$0.0672^{+0.0072}_{-0.0073}$	$0.0622^{+0.0096}_{-0.055}$	$0.0624^{+0.0092}_{-0.0059}$
$[\rho_P]^{2/3}$ [$\mathrm{kg}^{2/3}\,\mathrm{m}^{-2}$]	139.0	$134.5^{+5.8}_{-4.7}$	$229.4^{+10.2}_{-8.9}$	$134.7^{+6.6}_{-4.4}$	$134.9^{+11.0}_{-4.5}$
i_{SB} [°]	267.06	$90.1^{+1.4}_{-1.5}$	$270.20^{+1.20}_{-0.70}$	$270.1^{+1.3}_{-1.2}$	90^{+180}_{-3}
Ω_{SB} [°]	5	37^{+24}_{-72}	25^{+18}_{-71}	16^{+24}_{-69}	28^{+23}_{-73}
P_{SB} [days]	23.995	$23.990^{+0.020}_{-0.047}$	$15.755^{+0.010}_{-0.014}$	$23.989^{+0.019}_{-0.053}$	$23.987^{+0.021}_{-0.081}$
ϕ_{SB} [°]	40	173^{+25}_{-72}	23^{+70}_{-18}	31^{+69}_{-24}	112^{+75}_{-95}

Three Modes

These two gives best evidence (indistinguishable)

Parameter	**Truth**	**Mode 1**	**Mode 2**	**Mode 3**	**Global**
$\log \mathcal{Z}$	-	23552.49 ± 0.36	23523.81 ± 0.99	23551.27 ± 0.45	23552.75 ± 0.27
Moon params.					
R_S/R_P	0.2570	$0.2587^{+0.0053}_{-0.0069}$	$0.2559^{+0.0051}_{-0.0065}$	$0.2587^{+0.0052}_{-0.0070}$	$0.2585^{+0.0053}_{-0.0069}$
M_S/M_P	0.0583	$0.0620^{+0.0086}_{-0.0058}$	$0.0672^{+0.0072}_{-0.0073}$	$0.0622^{+0.0096}_{-0.055}$	$0.0624^{+0.0092}_{-0.0059}$
$[\rho_P]^{2/3}$ [$\mathrm{kg}^{2/3}\ \mathrm{m}^{-2}$]	139.0	$134.5^{+5.8}_{-4.7}$	$229.4^{+10.2}_{-8.9}$	$134.7^{+6.6}_{-4.4}$	$134.9^{+11.0}_{-4.5}$
i_{SB} [°]	267.06	$90.1^{+1.4}_{-1.5}$	$270.20^{+1.20}_{-0.70}$	$270.1^{+1.3}_{-1.2}$	90^{+180}_{-3}
Ω_{SB} [°]	5	37^{+24}_{-72}	25^{+18}_{-71}	16^{+24}_{-69}	28^{+23}_{-73}
P_{SB} [days]	23.995	$23.990^{+0.020}_{-0.047}$	$15.755^{+0.010}_{-0.014}$	$23.989^{+0.019}_{-0.053}$	$23.987^{+0.021}_{-0.081}$
ϕ_{SB} [°]	40	173^{+25}_{-72}	23^{+70}_{-18}	31^{+69}_{-24}	112^{+75}_{-95}

Three Modes

These two gives best evidence (indistinguishable)

Parameter	Truth	Mode 1	Mode 2	Mode 3	Global
$\log \mathcal{Z}$	-	23552.49 ± 0.36	23523.81 ± 0.99	23551.27 ± 0.45	23552.75 ± 0.27
Moon params.					
R_S/R_P	0.2570	$0.2587^{+0.0053}_{-0.0069}$	$0.2559^{+0.0051}_{-0.0065}$	$0.2587^{+0.0052}_{-0.0070}$	$0.2585^{+0.0053}_{-0.0069}$
M_S/M_P	0.0583	$0.0620^{+0.0086}_{-0.0058}$	$0.0672^{+0.0072}_{-0.0073}$	$0.0622^{+0.0096}_{-0.055}$	$0.0624^{+0.0092}_{-0.0059}$
$[\rho_P]^{2/3}$ [$\mathrm{kg}^{2/3}\,\mathrm{m}^{-2}$]	139.0	$134.5^{+5.8}_{-4.7}$	$229.4^{+10.2}_{-8.9}$	$134.7^{+6.6}_{-4.4}$	$134.9^{+11.0}_{-4.5}$
i_{SB} [°]	267.06	$90.1^{+1.4}_{-1.5}$	$270.20^{+1.20}_{-0.70}$	$270.1^{+1.3}_{-1.2}$	90^{+180}_{-3}
Ω_{SB} [°]	5	37^{+24}_{-72}	25^{+18}_{-71}	16^{+24}_{-69}	28^{+23}_{-73}
P_{SB} [days]	23.995	$23.990^{+0.020}_{-0.047}$	$15.755^{+0.010}_{-0.014}$	$23.989^{+0.019}_{-0.053}$	$23.987^{+0.021}_{-0.081}$
ϕ_{SB} [°]	40	173^{+25}_{-72}	23^{+70}_{-18}	31^{+69}_{-24}	112^{+75}_{-95}

Three Modes

These two gives best evidence (indistinguishable)

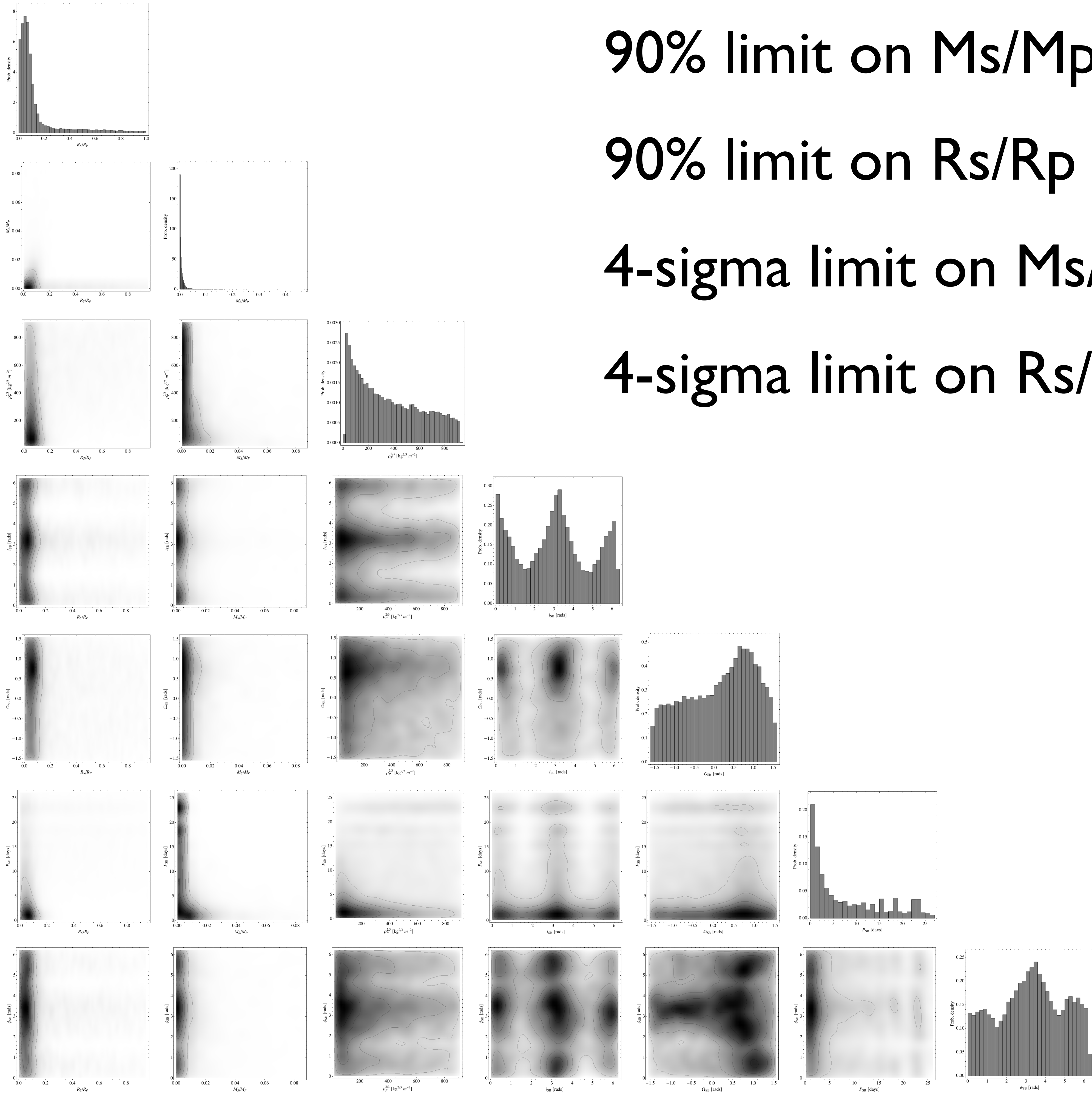
Parameter	Truth	Mode 1	Mode 2	Mode 3	Global
$\log \mathcal{Z}$	-	23552.49 ± 0.36	23523.81 ± 0.99	23551.27 ± 0.45	23552.75 ± 0.27
Moon params.					
R_S/R_P	0.2570	$0.2587^{+0.0053}_{-0.0069}$	$0.2559^{+0.0051}_{-0.0065}$	$0.2587^{+0.0052}_{-0.0070}$	$0.2585^{+0.0053}_{-0.0069}$
M_S/M_P	0.0583	$0.0620^{+0.0086}_{-0.0058}$	$0.0672^{+0.0072}_{-0.0073}$	$0.0622^{+0.0096}_{-0.055}$	$0.0624^{+0.0092}_{-0.0059}$
$[\rho_P]^{2/3}$ [kg$^{2/3}$ m^{-2}]	139.0	$134.5^{+5.8}_{-4.7}$	$229.4^{+10.2}_{-8.9}$	$134.7^{+6.6}_{-4.4}$	$134.9^{+11.0}_{-4.5}$
i_{SB} [°]	267.06	$90.1^{+1.4}_{-1.5}$	$270.20^{+1.20}_{-0.70}$	$270.1^{+1.3}_{-1.2}$	90^{+180}_{-3}
Ω_{SB} [°]	5	37^{+24}_{-72}	25^{+18}_{-71}	16^{+24}_{-69}	28^{+23}_{-73}
P_{SB} [days]	23.995	$23.990^{+0.020}_{-0.047}$	$15.755^{+0.010}_{-0.014}$	$23.989^{+0.019}_{-0.053}$	$23.987^{+0.021}_{-0.081}$
ϕ_{SB} [°]	40	173^{+25}_{-72}	23^{+70}_{-18}	31^{+69}_{-24}	112^{+75}_{-95}

Three Modes

These two gives best evidence (indistinguishable)

Parameter	Truth	Mode 1	Mode 2	Mode 3	Global
$\log \mathcal{Z}$	-	23552.49 ± 0.36	23523.81 ± 0.99	23551.27 ± 0.45	23552.75 ± 0.27
Moon params.					
R_S/R_P	0.2570	$0.2587^{+0.0053}_{-0.0069}$	$0.2559^{+0.0051}_{-0.0065}$	$0.2587^{+0.0052}_{-0.0070}$	$0.2585^{+0.0053}_{-0.0069}$
M_S/M_P	0.0583	$0.0620^{+0.0086}_{-0.0058}$	$0.0672^{+0.0072}_{-0.0073}$	$0.0622^{+0.0096}_{-0.055}$	$0.0624^{+0.0092}_{-0.0059}$
$[\rho_P]^{2/3}$ [kg$^{2/3}$ m^{-2}]	139.0	$134.5^{+5.8}_{-4.7}$	$229.4^{+10.2}_{-8.9}$	$134.7^{+6.6}_{-4.4}$	$134.9^{+11.0}_{-4.5}$
i_{SB} [°]	267.06	$90.1^{+1.4}_{-1.5}$	$270.20^{+1.20}_{-0.70}$	$270.1^{+1.3}_{-1.2}$	90^{+180}_{-3}
Ω_{SB} [°]	5	37^{+24}_{-72}	25^{+18}_{-71}	16^{+24}_{-69}	28^{+23}_{-73}
P_{SB} [days]	23.995	$23.990^{+0.020}_{-0.047}$	$15.755^{+0.010}_{-0.014}$	$23.989^{+0.019}_{-0.053}$	$23.987^{+0.021}_{-0.081}$
ϕ_{SB} [°]	40	173^{+25}_{-72}	23^{+70}_{-18}	31^{+69}_{-24}	112^{+75}_{-95}

This is the real mode, but we could only get the correct period blindly, not the correct sense of orbital motion



90% limit on Ms/Mp < 0.018

90% limit on Rs/Rp < 0.44

4-sigma limit on Ms/Mp<0.369

4-sigma limit on Rs/Rp<0.999

Questions?