MICROLENSING MAGNIFICATION CALCULATIONS WITH POINT-SOURCE AND FINITE-SOURCE EFFECTS

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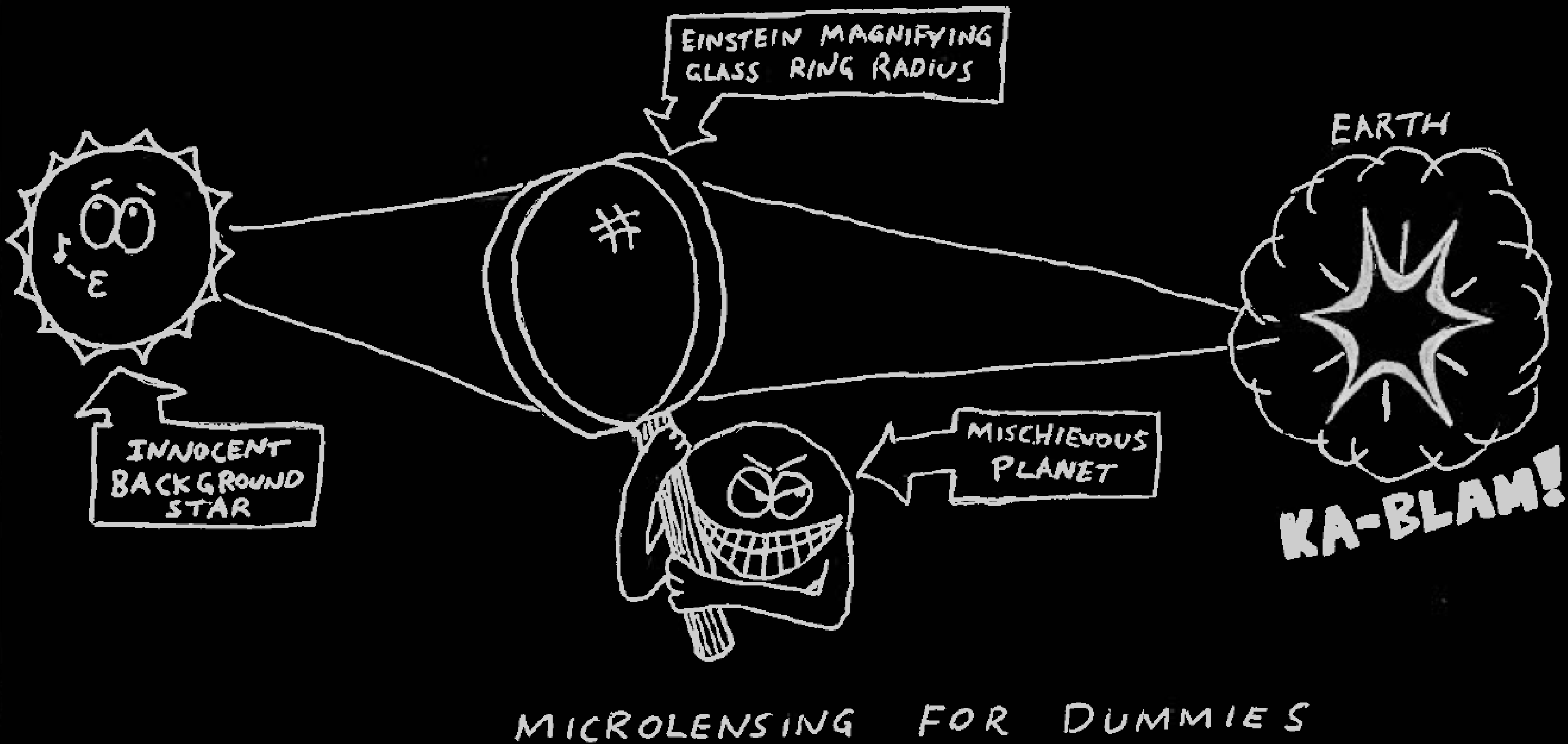

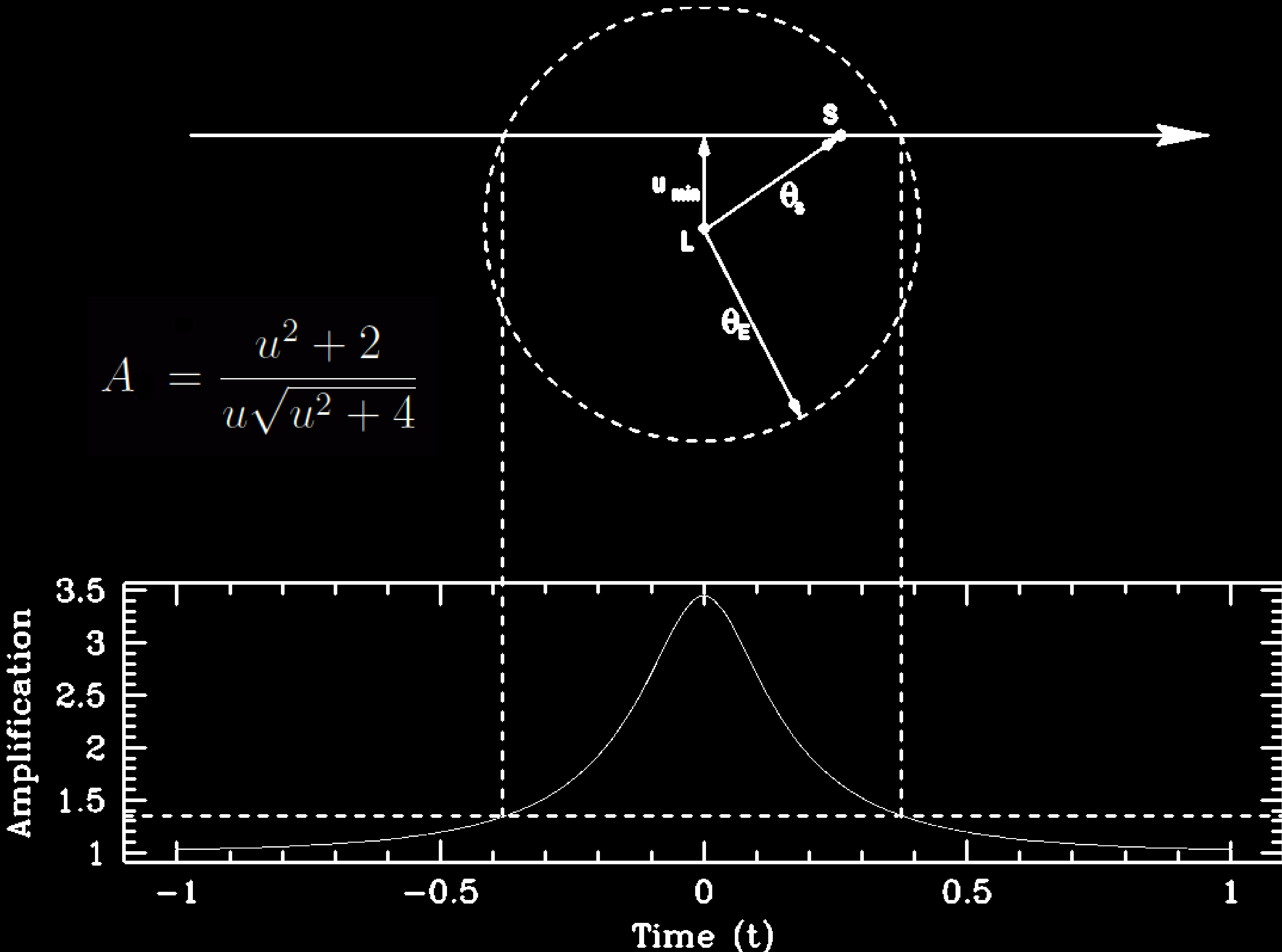

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}}$$

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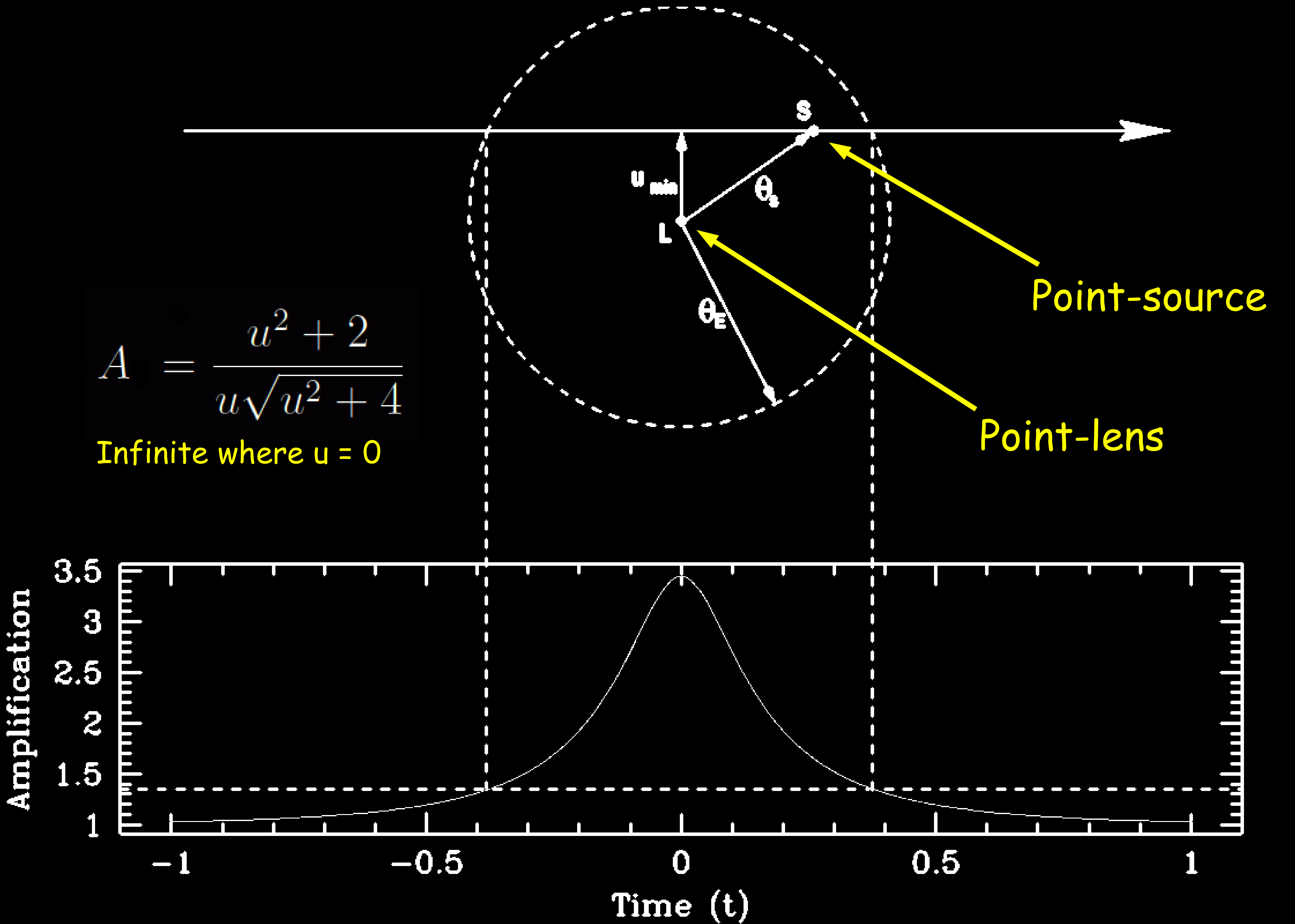
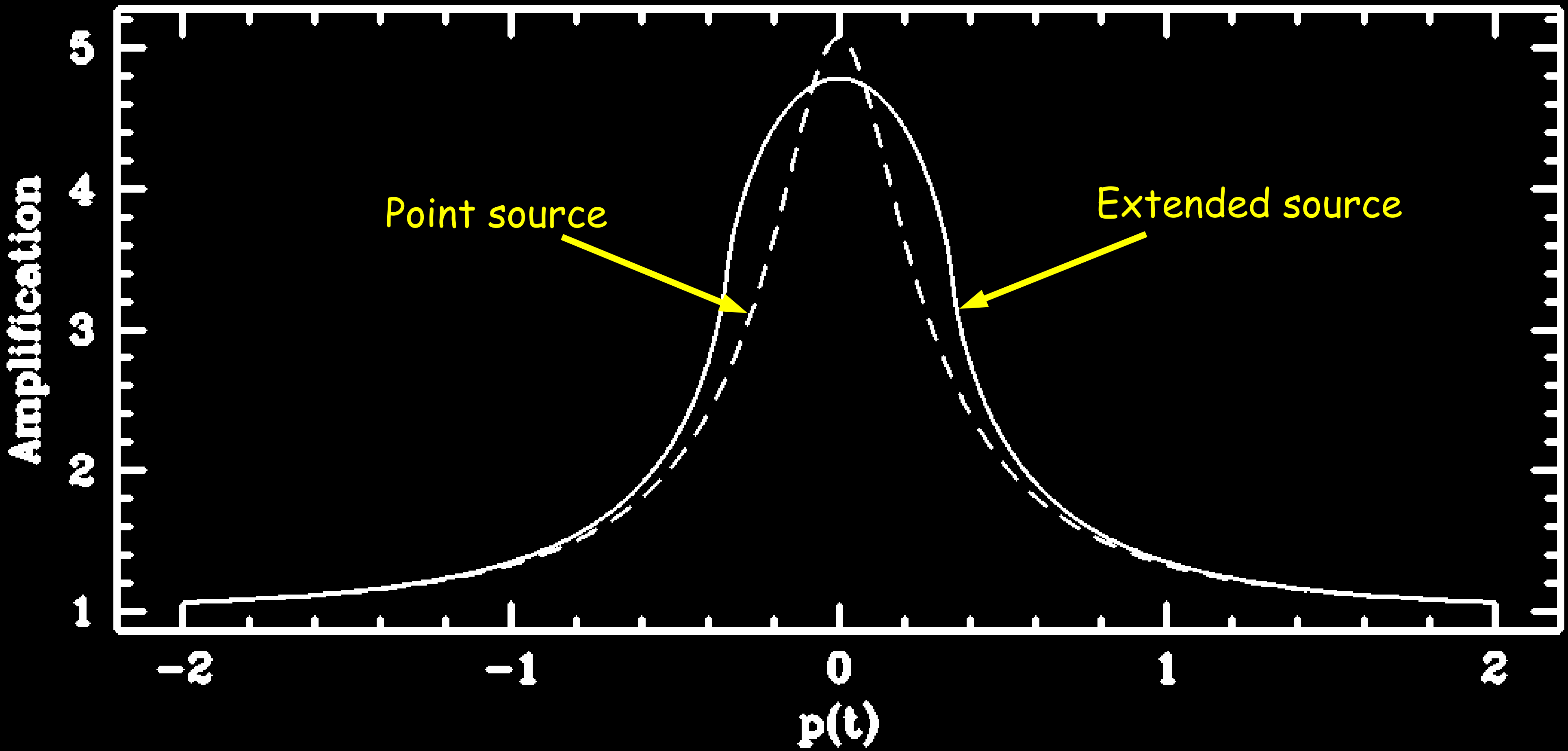
Infinite where u = 0



Can we assume a point-source?

- Point-source is often a reasonable assumption
- Assumption breaks down when the source size becomes comparable to the minimum lens-source separation



Uniform source brightness

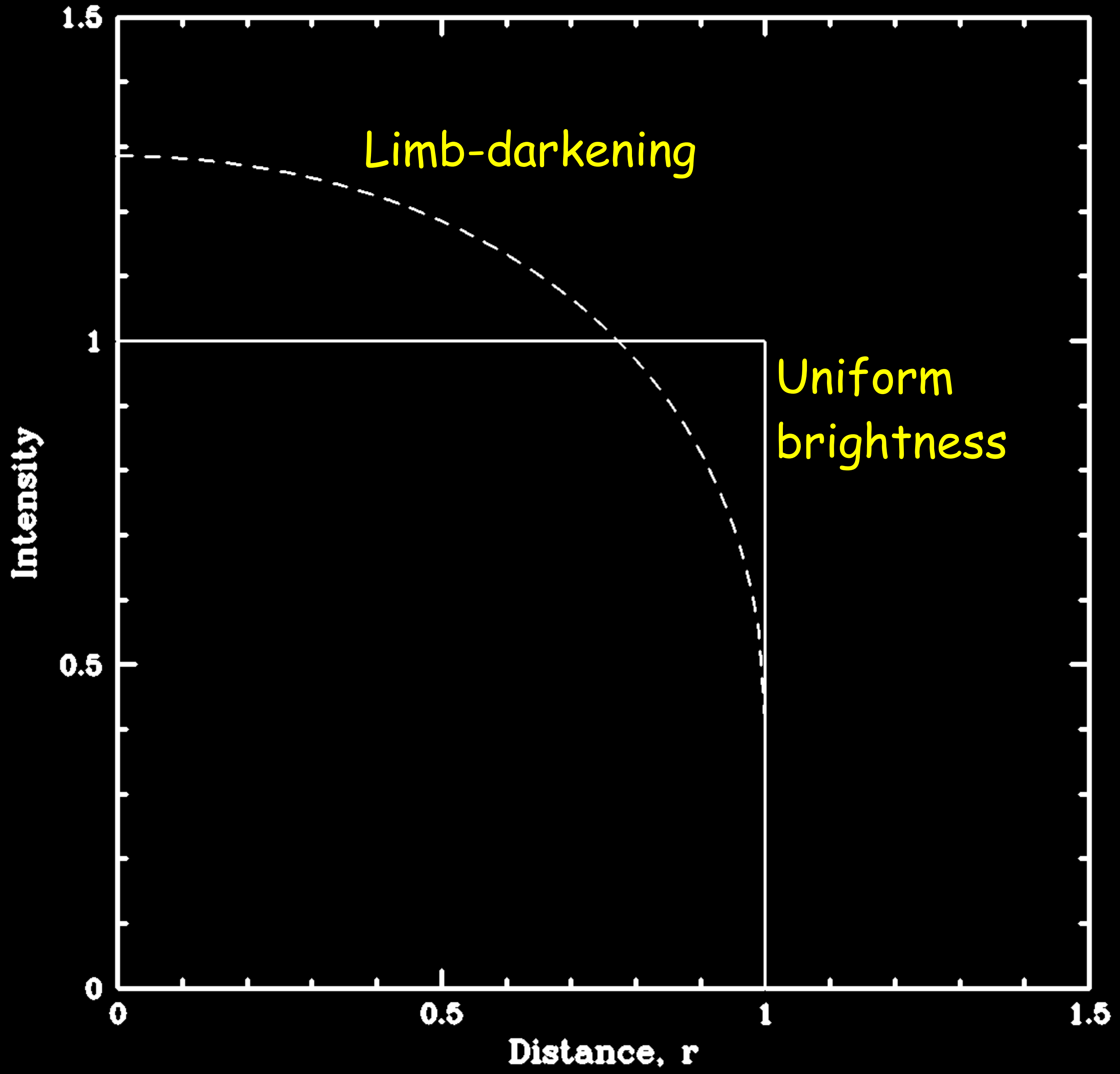
For a source with uniform brightness, the magnification is the ratio of the area of the images to the area of the source.

$$A = \frac{\int d^2y \, A_p(\mathbf{y})}{\int d^2y}$$

$$A = \begin{cases} \frac{2}{\pi R_s^2} \int\limits_{u-R_s}^{u+R_s} \frac{r^2+2}{\sqrt{r^2+4}} \arccos \frac{u^2+r^2-R_s^2}{2ur} \, dr & \text{for } u > R_s \\ \frac{2}{\pi R_s^2} \int\limits_{R_s-u}^{R_s+u} \frac{r^2+2}{\sqrt{r^2+4}} \arccos \frac{u^2+r^2-R_s^2}{2ur} + \frac{R_s-u}{R_s^2} \sqrt{(R_s^2-u)^2+4} \, dr & \text{for } u < R_s \\ \frac{2}{\pi} \left[\left(1+\frac{1}{R_s^2}\right) \arcsin \frac{1}{\sqrt{1+\frac{1}{R_s^2}}} + \frac{1}{R_s} \right] & \text{for } u = R_s \end{cases}$$

$$A_{max} = \sqrt{1 + \frac{4}{R_s^2}}$$

Magnification NOT infinite where u = 0



1.5
1
0.5
0
Intensity
0
0.5
1
1.5
Distance, r
Limb-darkening
Uniform
brightness

Limb-darkening

For a source with limb-darkening, different parts of the source will be magnified differently.

$$A = \frac{\int d^2y \, A_p(\mathbf{y})}{\int d^2y}$$

$$A = \frac{\int d^2y \, I(\mathbf{y}) A_p(\mathbf{y})}{\int d^2y \, I(\mathbf{y})}$$

$$I(r) = I(0)[1 - \kappa_1 Y - \kappa_2 Y^2]$$

$$Y = 1 - \sqrt{1 - r^2}$$

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This is not trivially solved for most binary lens events!

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- **Well it depends ...**
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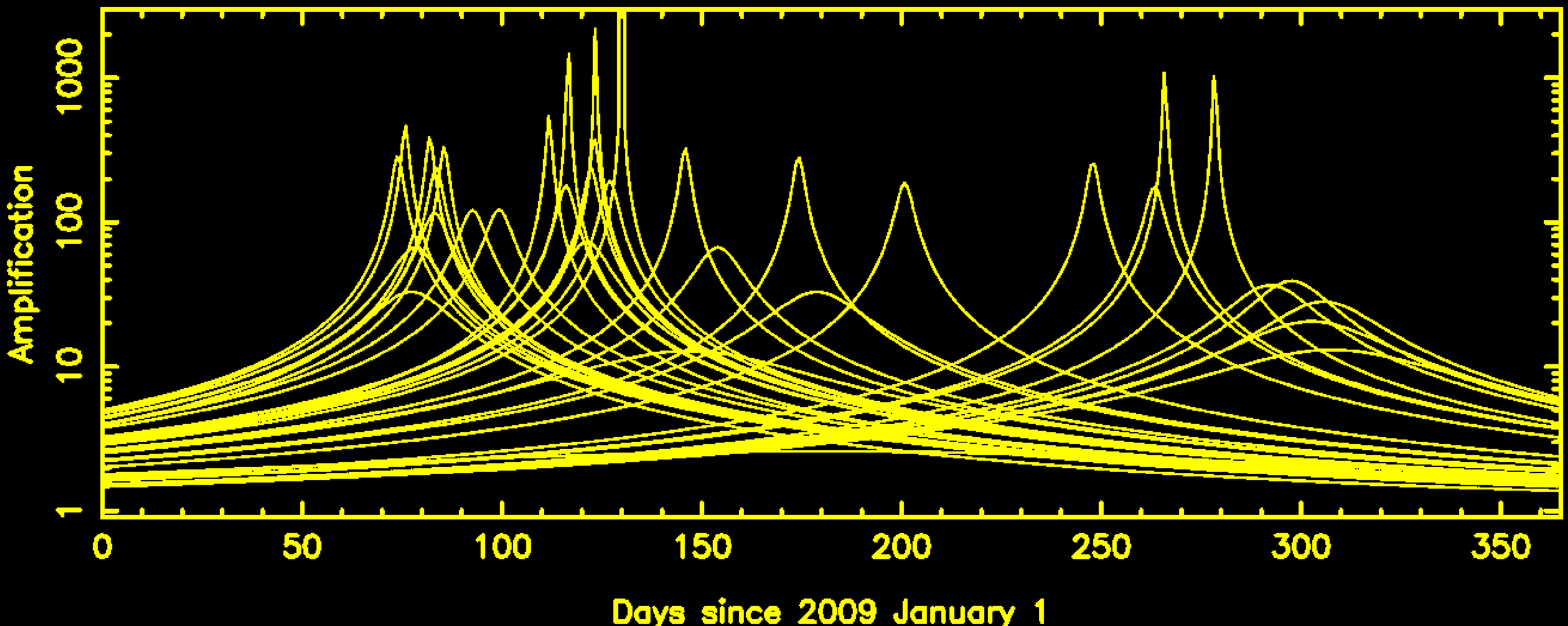
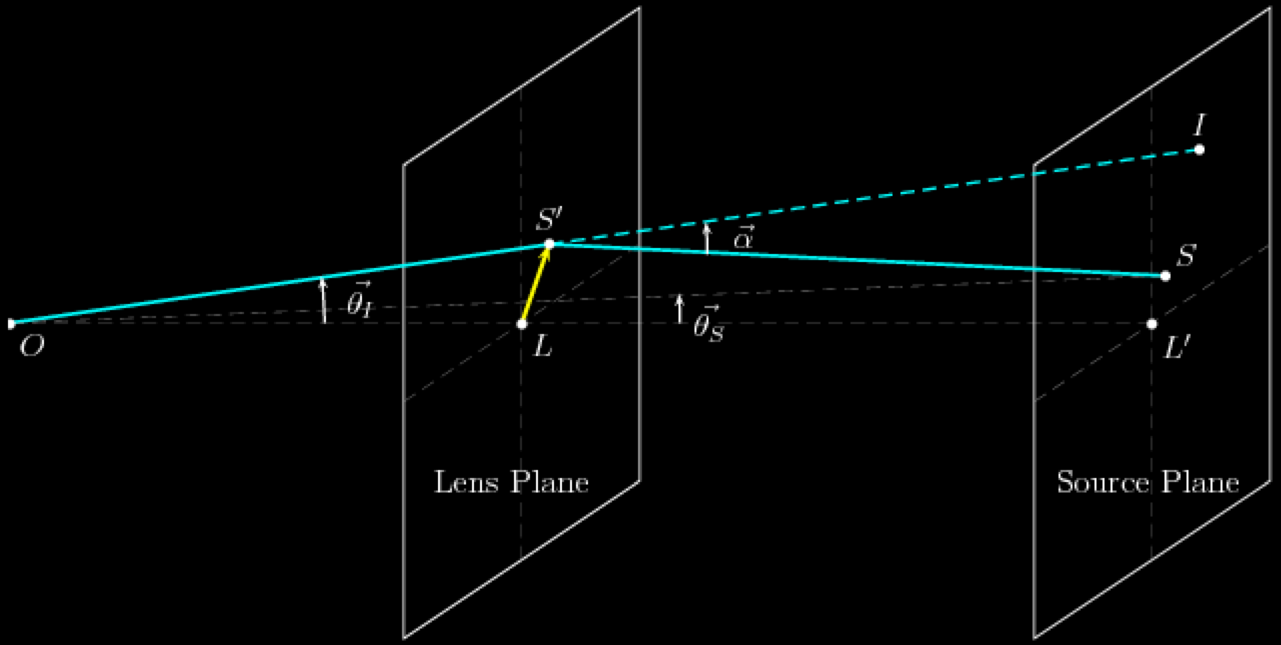
Can we assume a point-source?

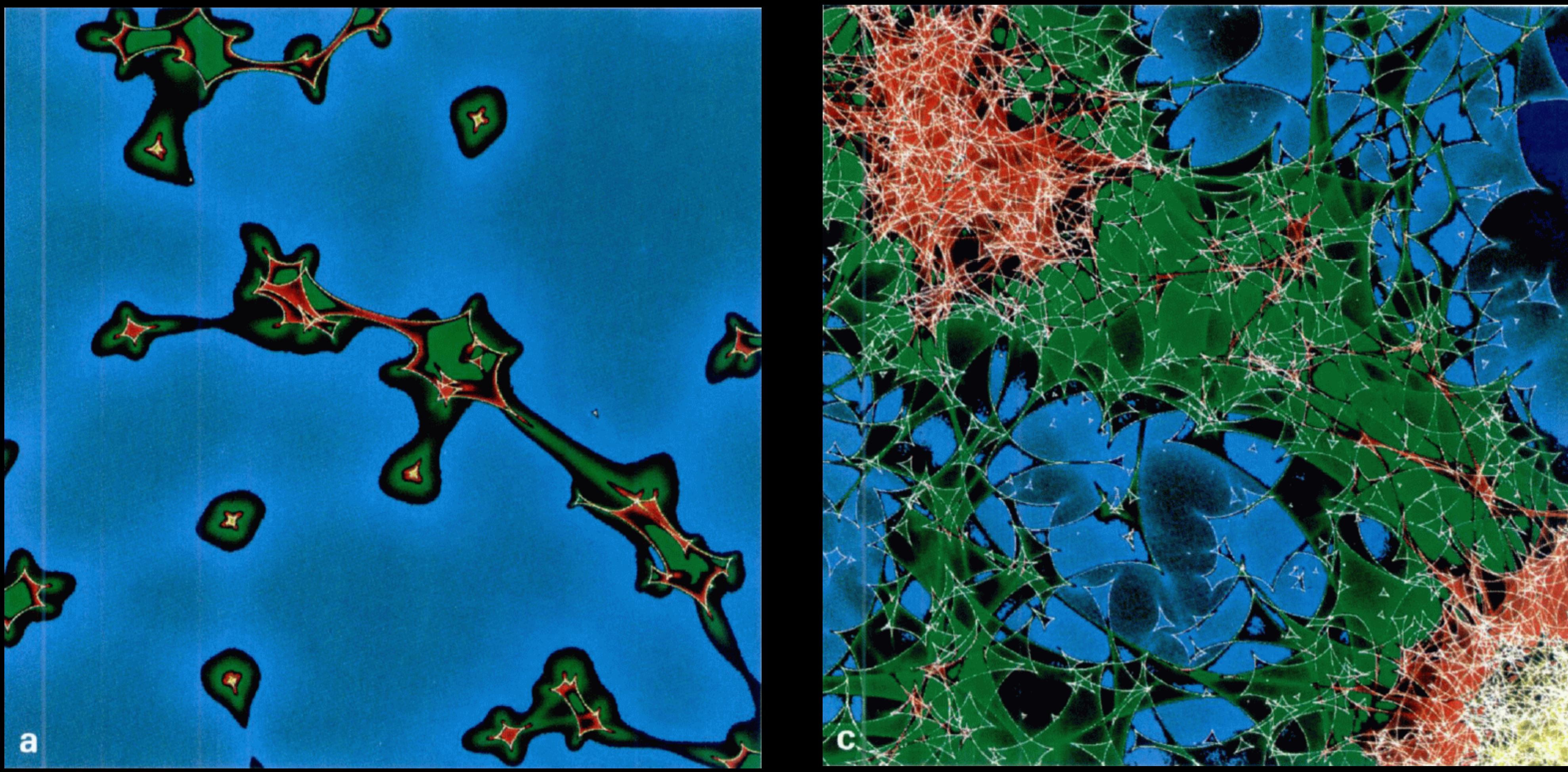
- Well it depends ...
- Assumption breaks down when the source size becomes comparable to the minimum lens-source separation (or planetary Einstein ring)
- The event-rate is increasing, including that of high-magnification events!
- Need to find an optimal path for efficient computational handling of so many events with such a large parameter space



Magnification Map (ray-shooting)



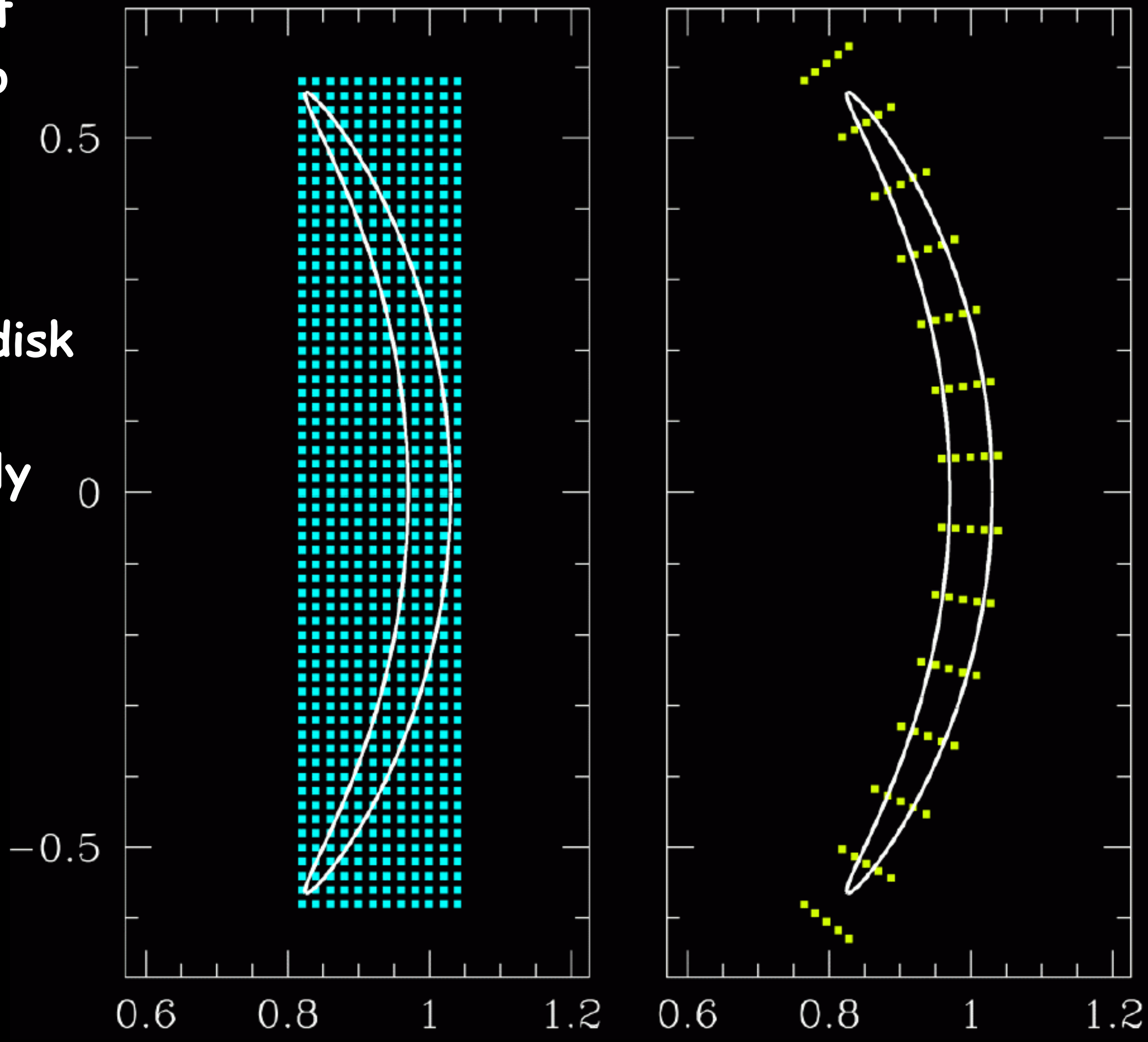
- Brute-force approach
- Handles complicated including multiple lens systems very well (frequently used in quasar lensing modeling)
- Good usage example is Wambsganss, 1997, MNRAS, 284, 172
- Can be computationally expensive – particularly where orbital motion is included!

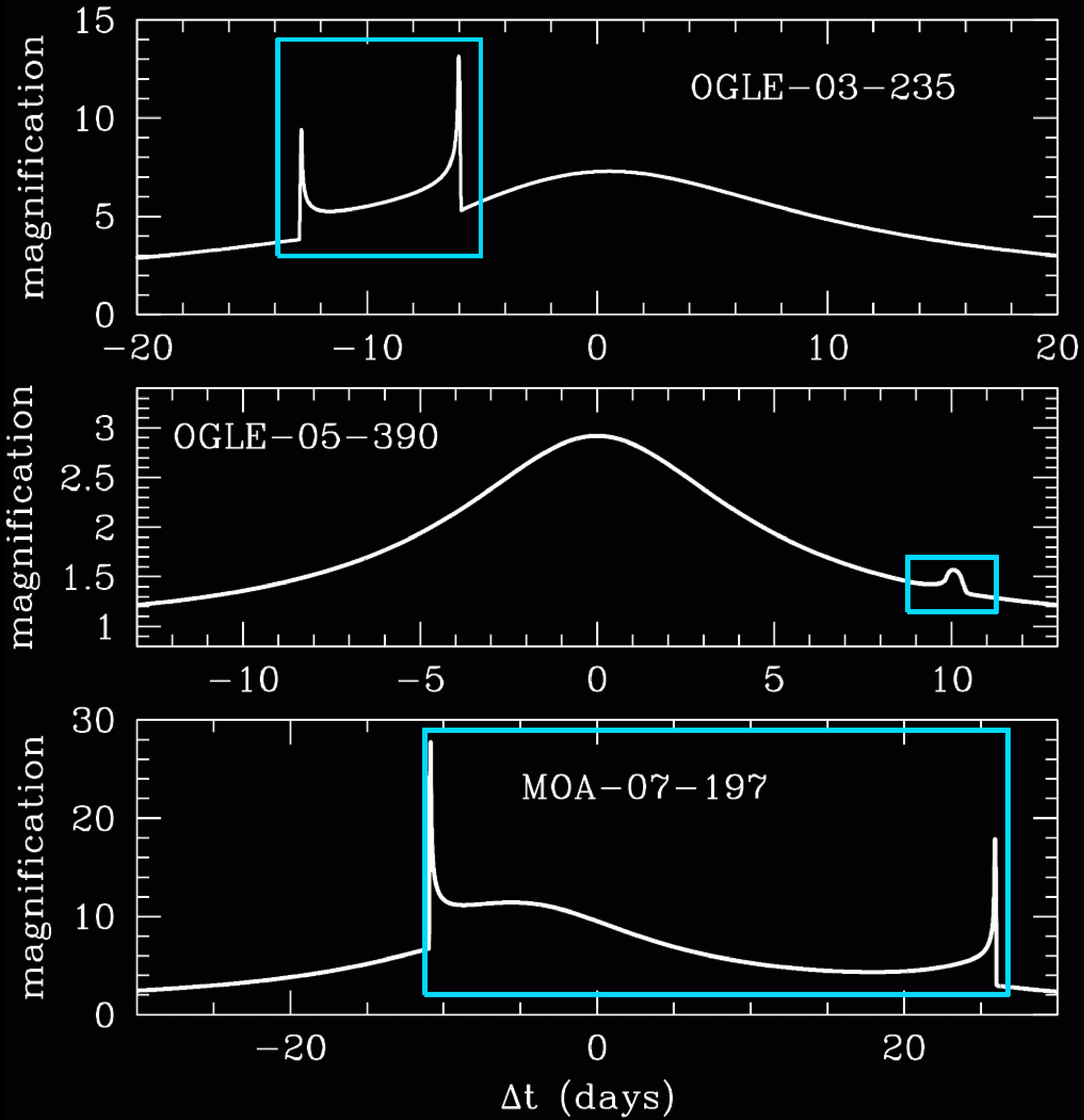


Application to complicated combinations of stellar densities
Wambsganss, Witt, & Schneider, 1992, A&A, 258, 591

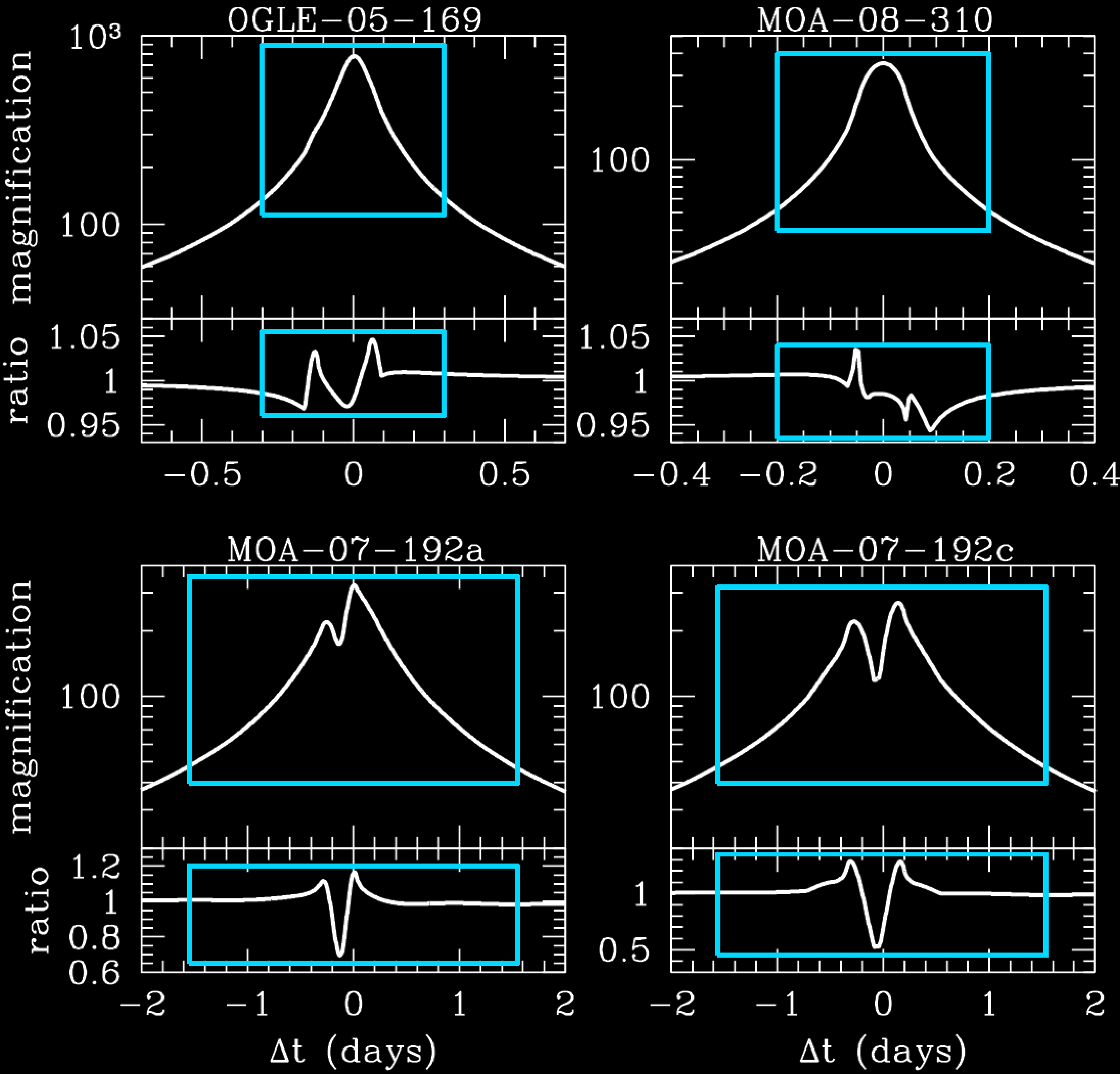
Magnification Map (ray-shooting)

- The image-centered approach
- Bennett & Rhie, 1996, ApJ, 472, 660; Bennett, 2010, ApJ, 716, 1408
- Use point-source model except when source or image is close to caustic
- Shoot rays from point-source image centers
- Include partial images where disk crosses a caustic
- Polar coordinate system greatly reduces the number of needed grid points compared with Cartesian system





Light curve calculation tests: low magnification



Light curve calculation tests: high magnification

Stokes/Green Theorem

- Perform contour integration in the image plane
- Stokes' theorem generalizes integration theorems in vector calculus

$$\oint_{\partial I} \mathbf{L} \cdot \mathbf{dx} = \iint_{I} dS(\nabla \wedge \mathbf{L}) \cdot \mathbf{n}$$

- Green's theorem is a special case in two dimensions

$$\oint_{\partial I} (L_1 dx_1 + L_2 dx_2) = \iint_{I} dx_1 dx_2 \left(\frac{\partial L_2}{dx_1} - \frac{\partial L_1}{dx_2} \right)$$

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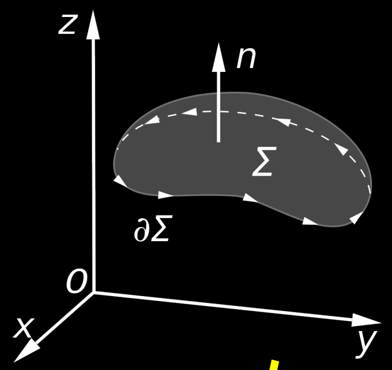

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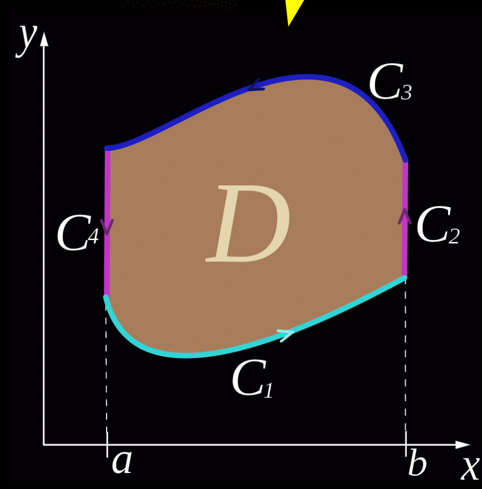
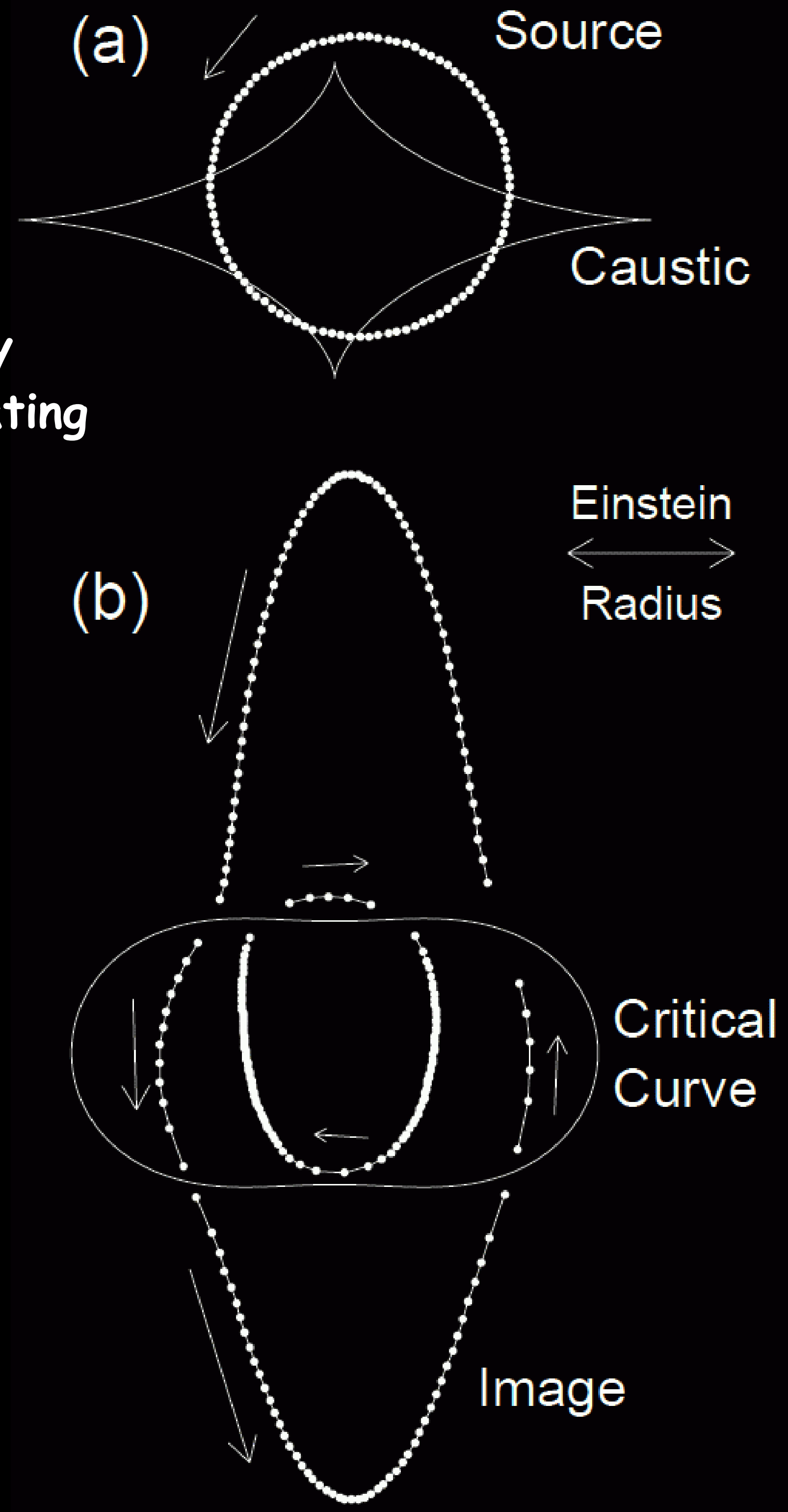


- Fast for uniform sources

Stokes/Green Theorem

- Calculate area of a microlensing image by a contour integral on the image boundary
- Sample the source boundary by approximating as a polygon
- Invert the lens equation to find image boundaries
- Re-order the points in the sample
- Approximate the contour integral using the sample
- Model limb-darkening by using rings of constant brightness



Stokes/Green Theorem

- Gould & Gaucherel, 1997, ApJ, 477, 580
- Dominik, 1998, A&A, 333, L79

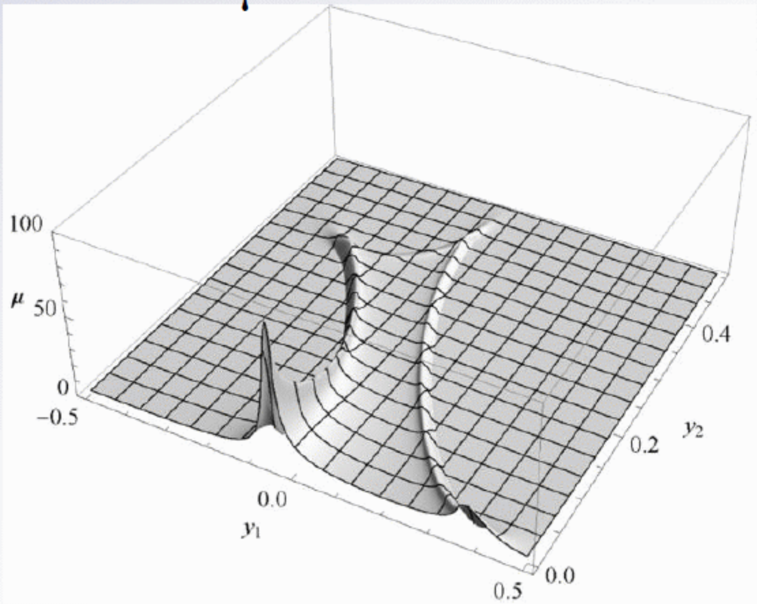
(Application to microlensing)
- Dong et al., 2006, ApJ, 642, 842

(Hybridization with inverse ray shooting)
- Dominik, 2007, MNRAS, 377, 1679

(Adaptive grid search)
- Bozza, 2010, MNRAS, 408, 2188

(Advanced contour integration)

$\delta\mu = 10^{-2}$



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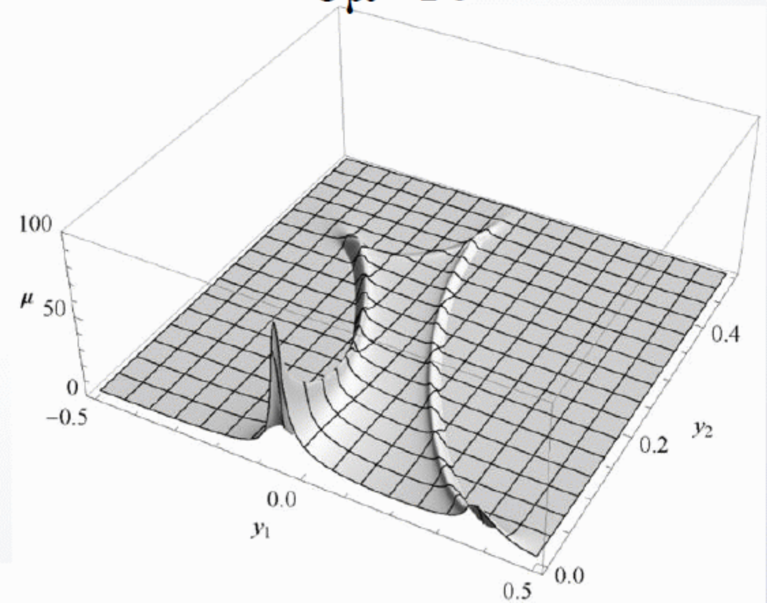
$\delta\mu = 10^{-5}$



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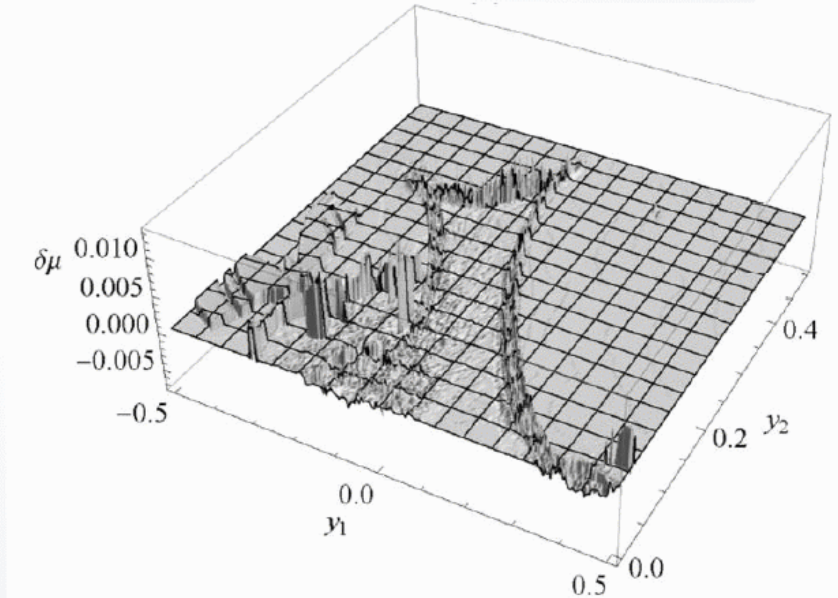
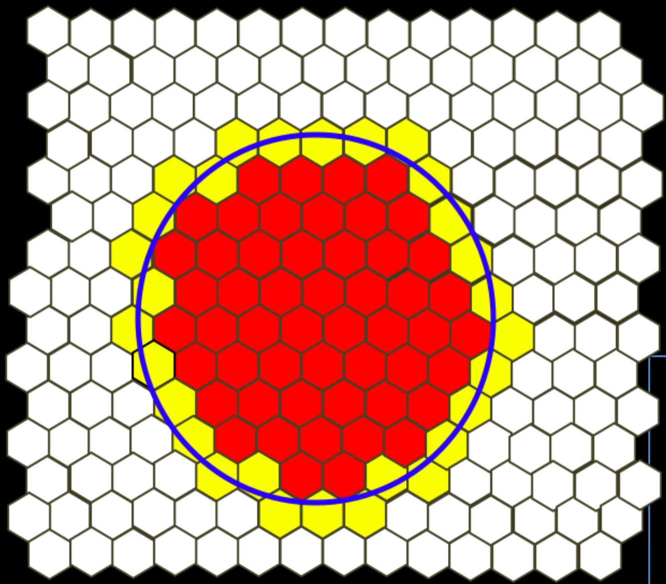


- **Bozza, 2010, MNRAS, 408, 2188**

(Advanced contour integration)

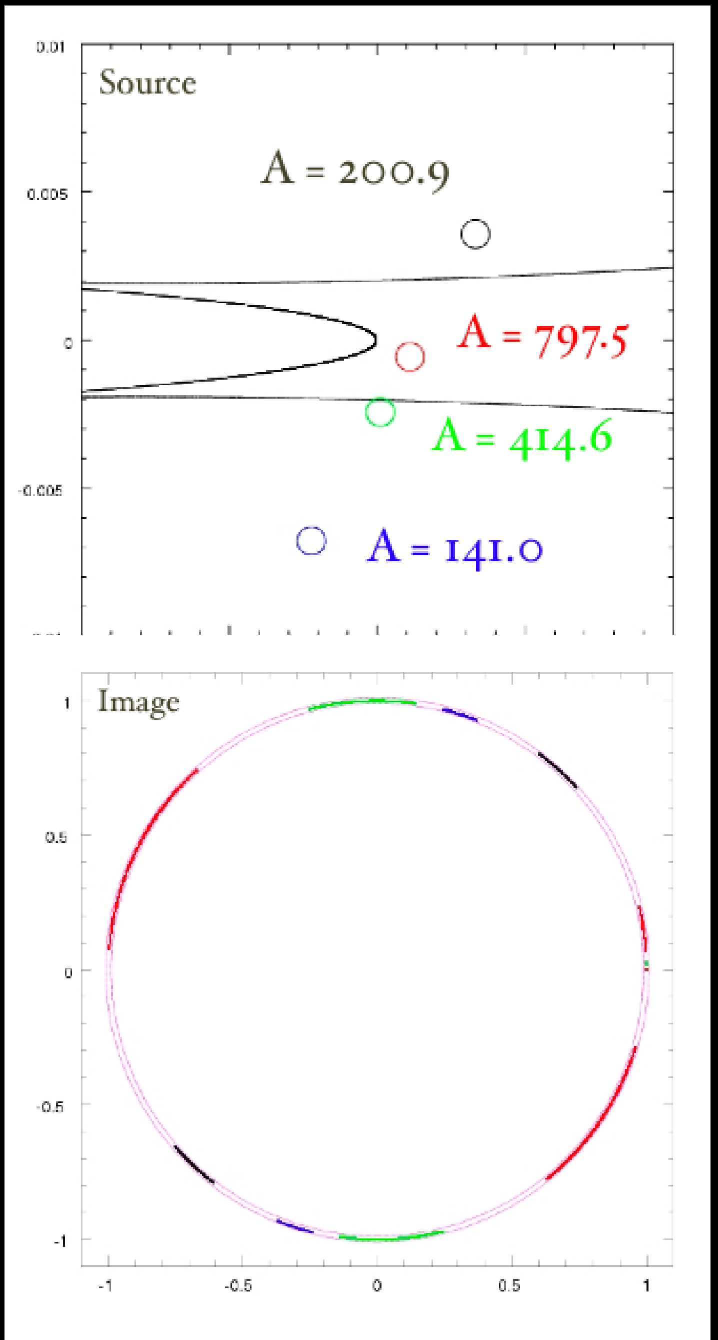
An Efficient and Robust Algorithm



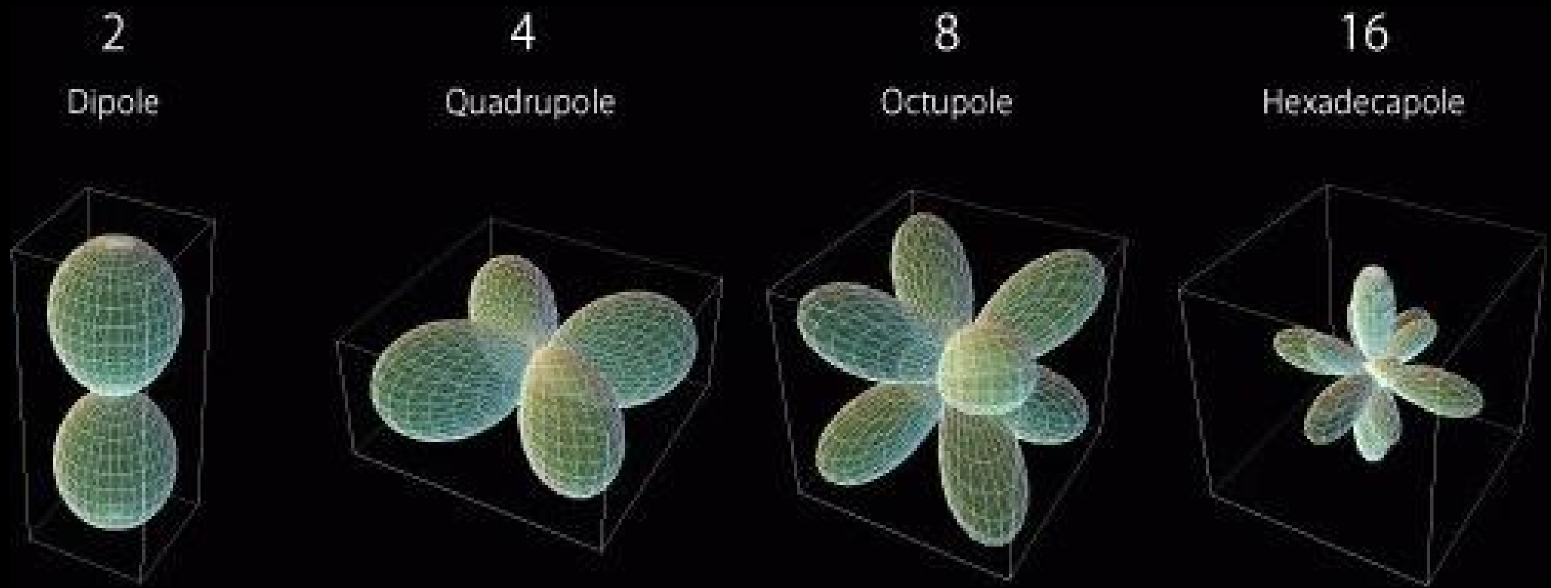
Dong, S., et al., 2006, ApJ, 642, 842 (Appendix A)
Dong, S. et al., 2009, ApJ, 698, 1826

- (Hexagon-cell Magnification) Map-Making (workhorse):
- Shoot rays from a narrow annulus on the image plane - reduce the overhead by orders of magnitude
- On the source plane, a combination of pixels and rays, with enhanced speed while preserving accuracy

Loop-Linking (backup) combines contour integration and ray-shooting



Hexadecapole



- Uses 13 point “grid” in the source plane
- Several times slower than point-source calculations but orders of magnitude faster than finite-source calculations
- Requires more than two source radii away from caustic
- Best when combined with other methods
- Particularly useful where planetary orbital motion is included
- Gould, 2008, ApJ, 681, 1593

Hexadecapole

$$A_{\text{finite}}(\rho, x_0, y_0) \equiv \frac{\int_0^\rho dw \ \int_0^{2\pi} d\eta \, A(w, \eta)}{\pi \rho^2}$$

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Hexadecapole

$$A_{w,+} \equiv \frac{1}{4}\sum_{j=0}^{3} A\left[w\cos\left(\phi + j\frac{\pi}{2}\right), w\sin\left(\phi + j\frac{\pi}{2}\right)\right] - A_0$$

$$= A_2 w^2 + \frac{(A_{40} + A_{44})(1 + \cos^2 2\phi) + A_{42}\sin^2 2\phi}{4} w^4,$$

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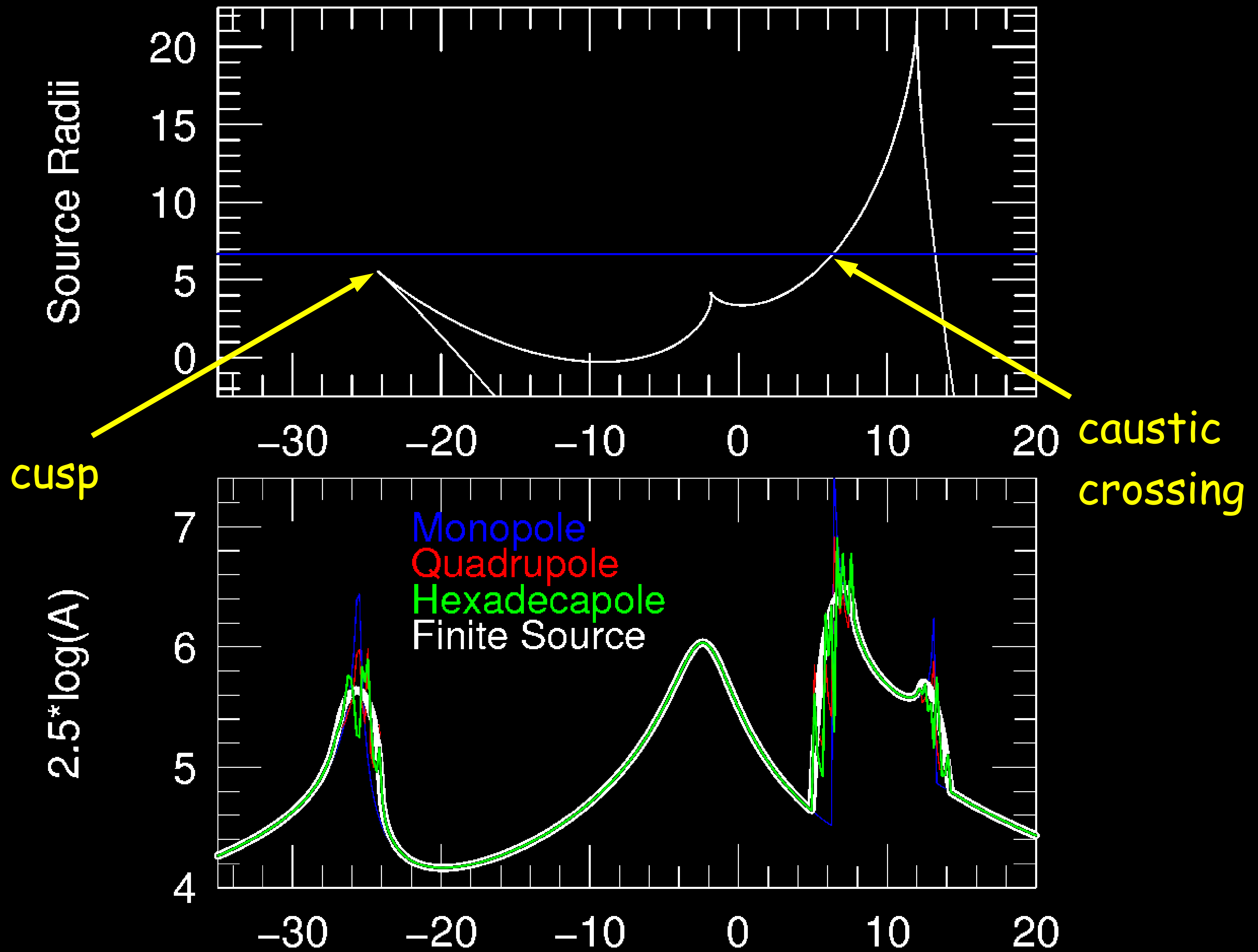
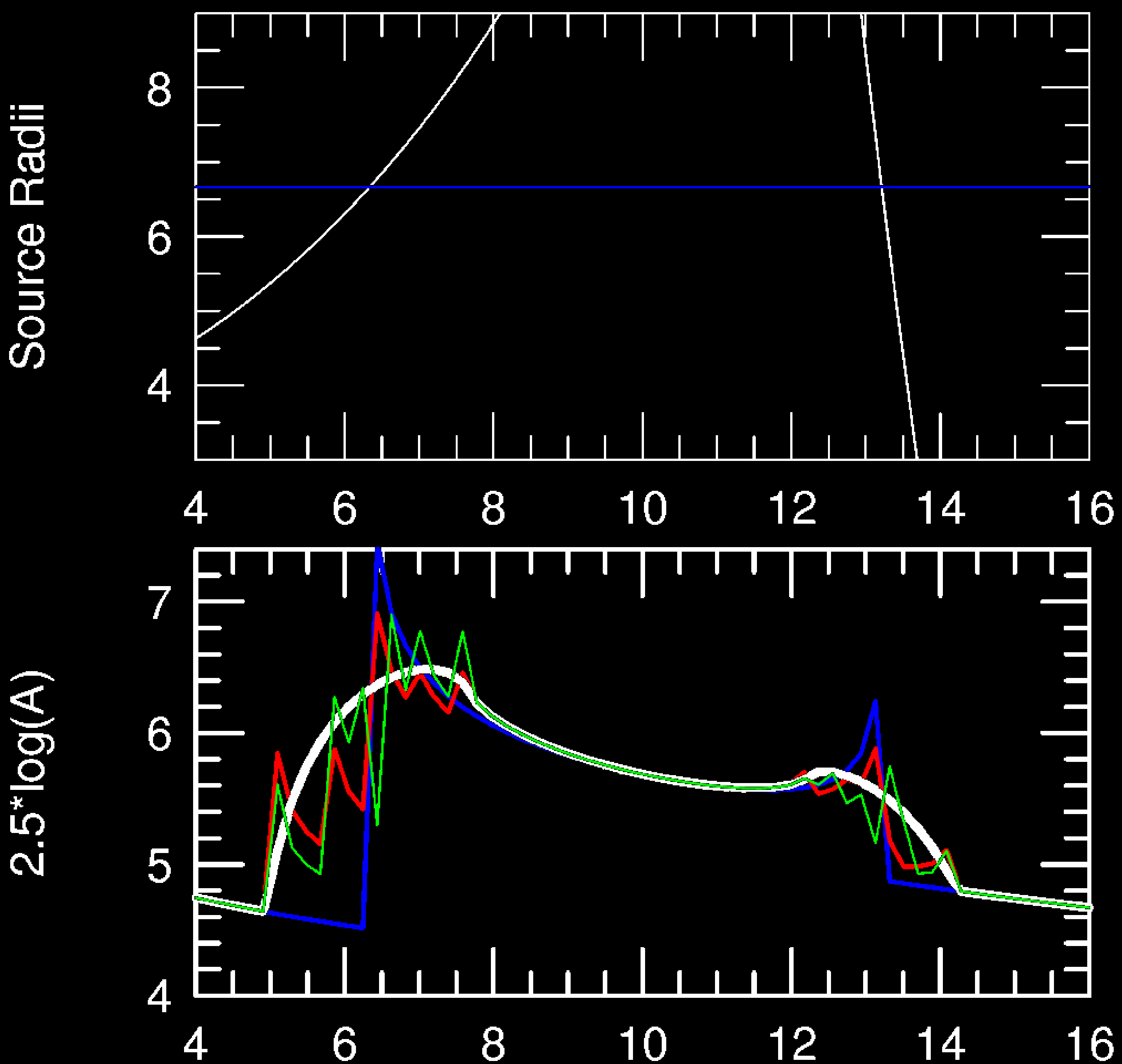


Source Radii
20
15
10
5
0
-30
-20
-10
0
10
20
cusp
caustic crossing
2.5*log(A)
7
6
5
4
Monopole
Quadrupole
Hexadecapole
Finite Source
-30
-20
-10
0
10
20



Source Radii
8
6
4
4
6
8
10
12
14
16
2.5*log(A)
7
6
5
4
4
6
8
10
12
14
16

MICROLENSING MAGNIFICATION CALCULATIONS WITH POINT-SOURCE AND FINITE-SOURCE EFFECTS

Practical implementation in Hands-on session after lunch

(Subo Dong & Jan Skowron)