

An introduction to the theory of interferometry

Michelson Summer Workshop

Frontiers of Interferometry: Stars, disks, terrestrial planets

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Preamble

- Learning interferometry is like learning any new skill (e.g. walking):
 - You have to want to learn
 - You start by crawling, then you walk, then you run.
 - Having fancy shoes doesn't help at the start.
 - You don't have to know how shoes are made.
 - At some stage you need to know what direction to head off in.
- This is a school:
 - You should assume nothing.
 - There is a lot to cover – this won't be easy.
 - Knowing what questions to ask is what is important.
 - Please ask, again and again if necessary – especially at the start.
- I am not here to sell interferometry, I'm here to help you understand it.

Outline

- Representing the sky:
 - The diffraction limit.
 - Incoherent imaging.
 - Fourier decomposition.
- Interferometric measurements:
 - What an interferometer measures.
 - Fringes – description and meaning.
 - The van Cittert Zernike theorem.
- Imaging and Sensitivity:
 - Features of the visibility function.
 - Interferometric imaging and sensitivity.

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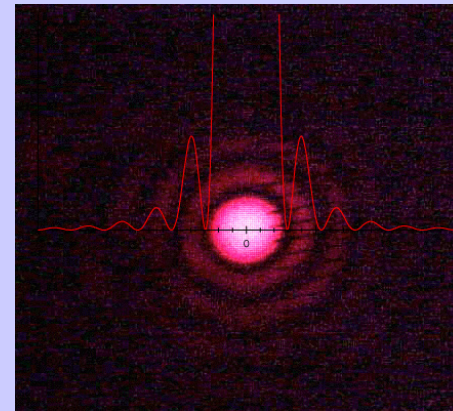
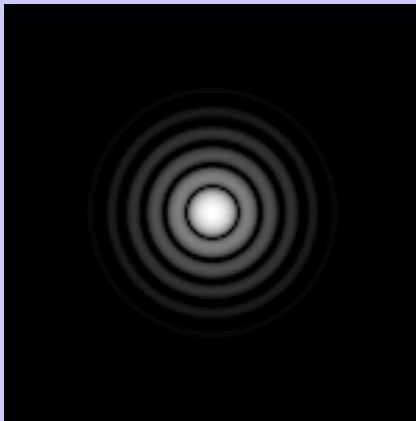
- Imaging and Sensitivity:

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What do imaging “machines” do?

- All imaging instruments have an associated “point spread function”
 - For a conventional telescope this is the familiar Airy pattern.
 - Characteristic width: $\theta = 1.22\lambda/D$.



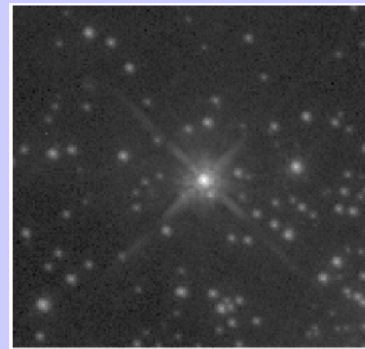
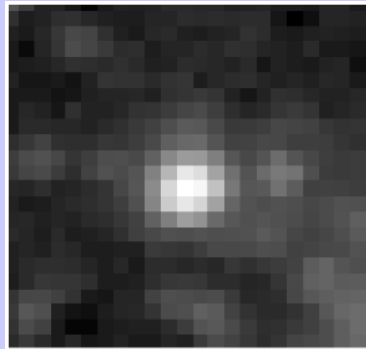
- Image formation (under incoherent & isoplanatic conditions) thus involves:
 - The decomposition of the source brightness distribution into infinitesimal point sources distributed on the sky.
 - The superposition of their (displaced) Airy patterns on the detector plane.

Image formation with conventional telescopes

- The fundamental relationship for incoherent space-invariant imaging is:

$$I(l, m) = \iint P(l-l', m-m') O(l', m') dl' dm' ,$$

i.e. the observed image is the true image convolved with a **point-spread function**, $P(l, m)$. Here l and m are angular coordinates on the sky, measured in radians.



- In this formalism, the description of the sky is characterized by:
 - The locations of the infinitesimal components of the source brightness.
 - The spatial distribution of intensity in the PSF.
 - Note that normally the PSF is spatially compact.

An alternative representation

- This convolutional relationship, which typifies the behaviour of linear space-invariant (isoplanatic) systems, can be written alternatively, by taking the Fourier transform of each side of the imaging equation, to give:

$$I(u, v) = T(u, v) \times O(u, v),$$

where *italic* functions refer to the Fourier transforms of their Roman counterparts, and u and v are now **spatial frequencies** measured in radians⁻¹.

- Importantly, the essential properties of the imaging system are encapsulated in a complex multiplicative **transfer function**, $T(u, v)$ which is just the Fourier transform of the PSF.
- In this formalism, the description of the sky is characterized by:
 - The amplitudes and phases (strengths and locations) of the different sinusoids (Fourier components) that make up the image.
 - Note that, unlike the previous case, the basis functions are not compact.

Key ideas 1

- Decomposition of an image into a series of spatially separated compact PSFs, each with a different amplitude.
- The equivalence of this to a superposition of non-localized sinusoids, each with a different amplitude and phase.
- The description of an image in terms of its Fourier components.
- The action of an incoherent imaging system as a filter for the true spatial Fourier spectrum of the source.
- That if we measure the Fourier components of a source in the sky, we can reconstruct what it looks like by performing an inverse Fourier transform.

Quiz 1

1. Assume you have a representation of the sky in terms of a PSF and a complete list of strengths and locations at which to lay them down. What happens to the “map” if:
 - a) You lose the first quarter of the list of strengths and locations? (*Assume that the list is ordered by decreasing distance from the center of the map, i.e. most distant first.*)
 - b) You lose all information about the PSF between position angles 0° and 90° ?
 - c) You are given a smoothed version of the PSF.

2. Assume you have a representation of the sky in terms of a complete list of Fourier components and a list of strengths and phases for each of these. What happens to the “map” if:
 - a) You lose the first quarter of the list of strengths and phases. (*Assume that this list is ordered by decreasing spatial frequency, i.e. highest spatial frequency first.*)
 - b) The amplitude data in the list is scaled inversely with the modulus of the spatial frequency, so that the amplitudes at higher spatial frequencies are multiplied down?
 - c) The phases of the last quarter of the list are randomized?

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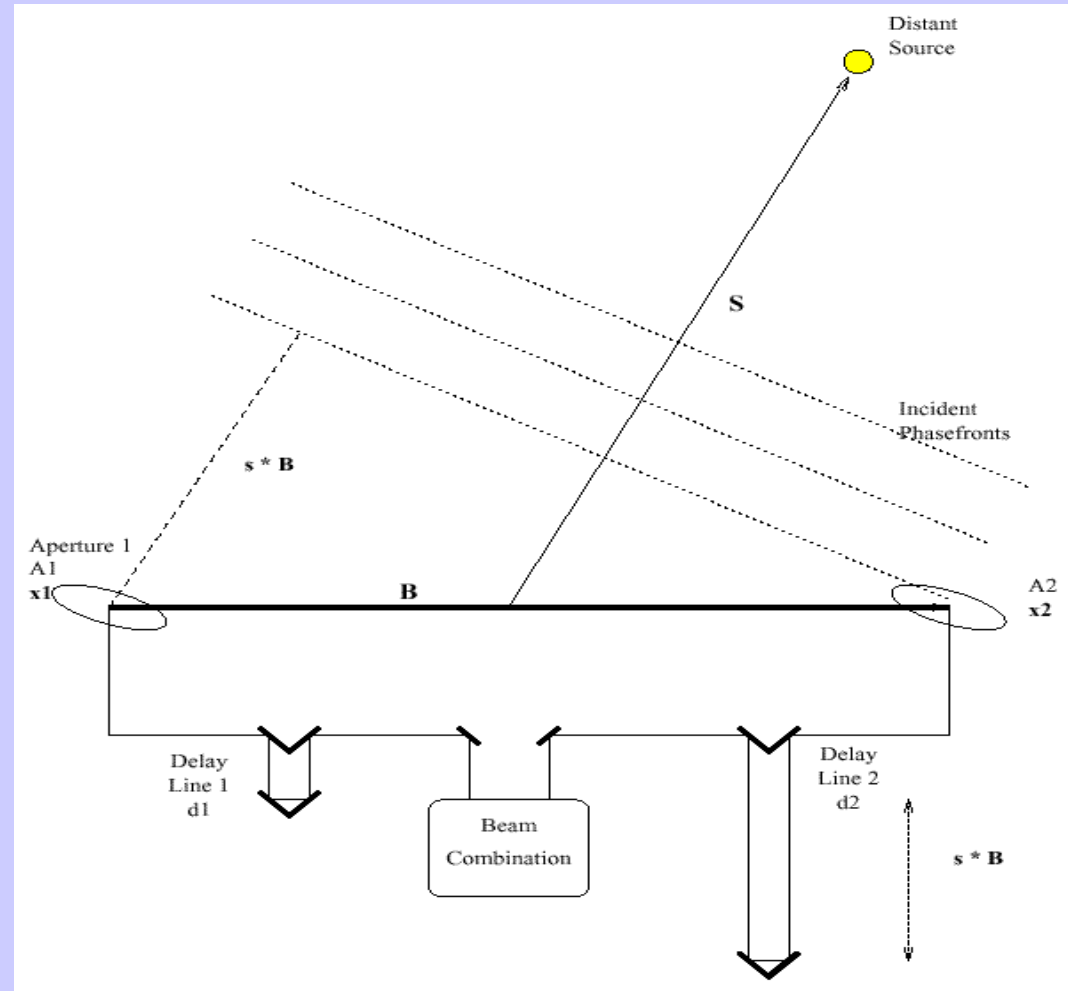
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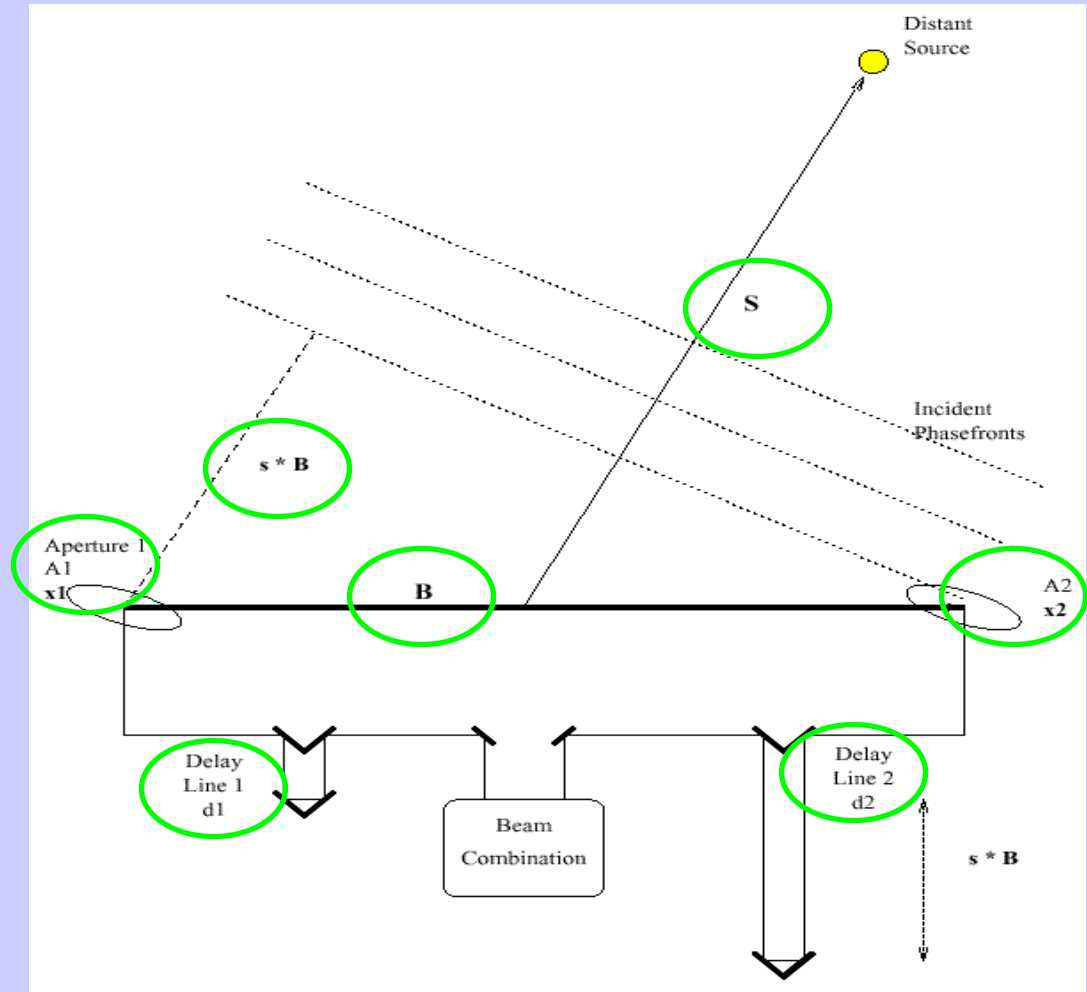
A two element interferometer - function

- Sampling of the radiation (from a distant point source).
- Transport to a common location.
- Compensation for the geometric delay.
- Combination of the beams.
- Detection of the resulting output.



A two element interferometer - nomenclature

- Telescopes located at x_1 & x_2 .
- Baseline $B = (x_1 - x_2)$.
- Pointing direction towards source is S .
- Geometric delay is $\hat{s} \cdot B$, where $\hat{s} = S/|S|$.
- Optical paths along two arms are d_1 and d_2 .



Key ideas 2

- Critical functions of an interferometer:
 - Sampling.
 - Optical path matching.
 - Combination.
 - Detection.
- Nomenclature useful for later:
 - Baseline.
 - Pointing direction.
 - Geometric delay.

The output of a 2-element interferometer (i)

- At combination, the E fields from the two collectors can be described as:
 $\psi_1 = A \exp (ik[\hat{s}.B + d_1]) \exp (-i\omega t)$ and $\psi_2 = A \exp (ik[d_2]) \exp (-i\omega t)$.

When these interfere, we obtain their sum:

$$\Psi = \psi_1 + \psi_2 = A \left[\exp (ik[\hat{s}.B + d_1]) + \exp (ik[d_2]) \right] \exp (-i\omega t) .$$

- Hence the time averaged intensity, $\langle \Psi \Psi^* \rangle$, will be given by:

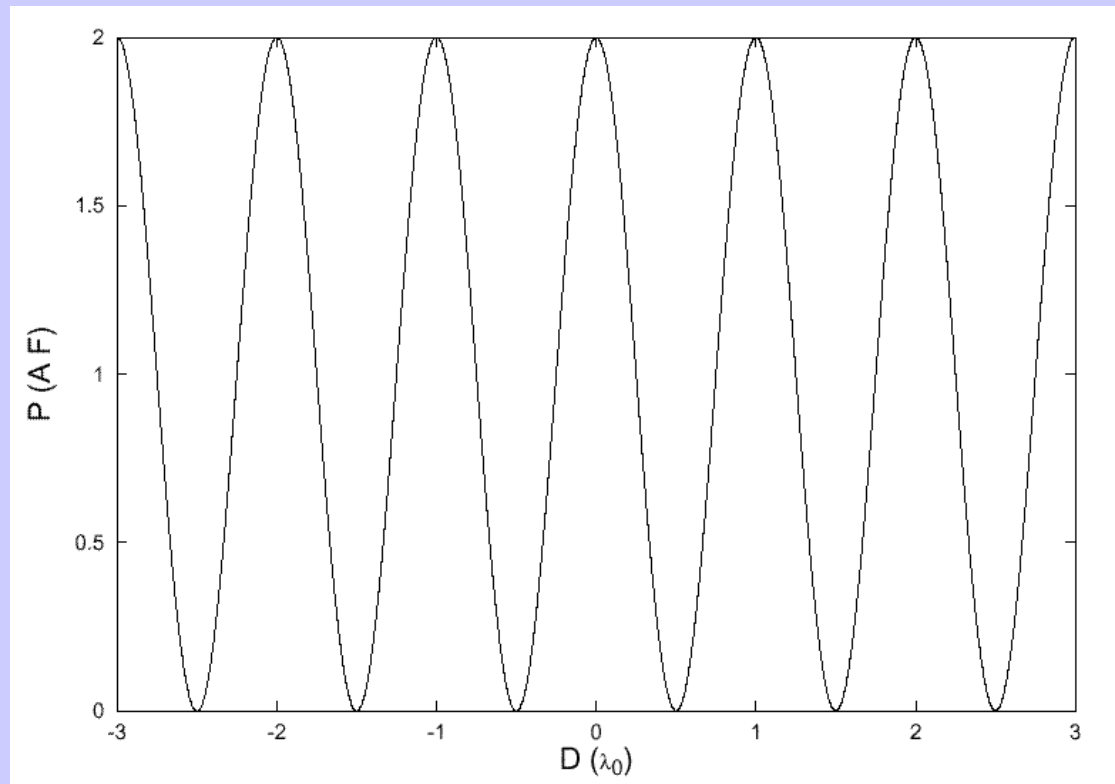
$$\begin{aligned} \langle \Psi \Psi^* \rangle &\propto \langle [\exp(ik[\hat{s}.B + d_1]) + \exp(ik[d_2])] \times [\exp(-ik[\hat{s}.B + d_1]) + \exp(-ik[d_2])] \rangle \\ &\propto 2 + 2 \cos(k[\hat{s}.B + d_1 - d_2]) \\ &\propto 2 + 2 \cos(kD), \end{aligned}$$

where $D = [\hat{s}.B + d_1 - d_2]$.

The output of a 2-element interferometer (ii)

Detected intensity, $P = \langle \Psi \Psi^* \rangle \propto 2 + 2\cos(k[\hat{s} \cdot \mathbf{B} + d_1 - d_2])$
 $\propto 2 + 2\cos(kD)$, where $D = [\hat{s} \cdot \mathbf{B} + d_1 - d_2]$.

- The output varies co-sinusoidally with kD , with $k = 2\pi/\lambda$.
- Adjacent fringe peaks are separated by
 - $\Delta d_{1 \text{ or } 2} = \lambda$ or
 - $\Delta(\hat{s} \cdot \mathbf{B}) = \lambda$ or
 - $\Delta(1/\lambda) = 1/D$.



Key ideas 3

- The output of the interferometer is a time averaged intensity.
- The intensity has a co-sinusoidal variation – these are the “fringes”.
- The fringe varies a function of (kD) , which itself can depend on:
 - The wavevector, $k = 2\pi/\lambda$.
 - The baseline, B .
 - The pointing direction, s .
 - The optical path difference between the two interferometer arms.
- If things are adjusted correctly, the interferometer output is fixed: there are no fringes. This is what most interferometers aim to achieve.

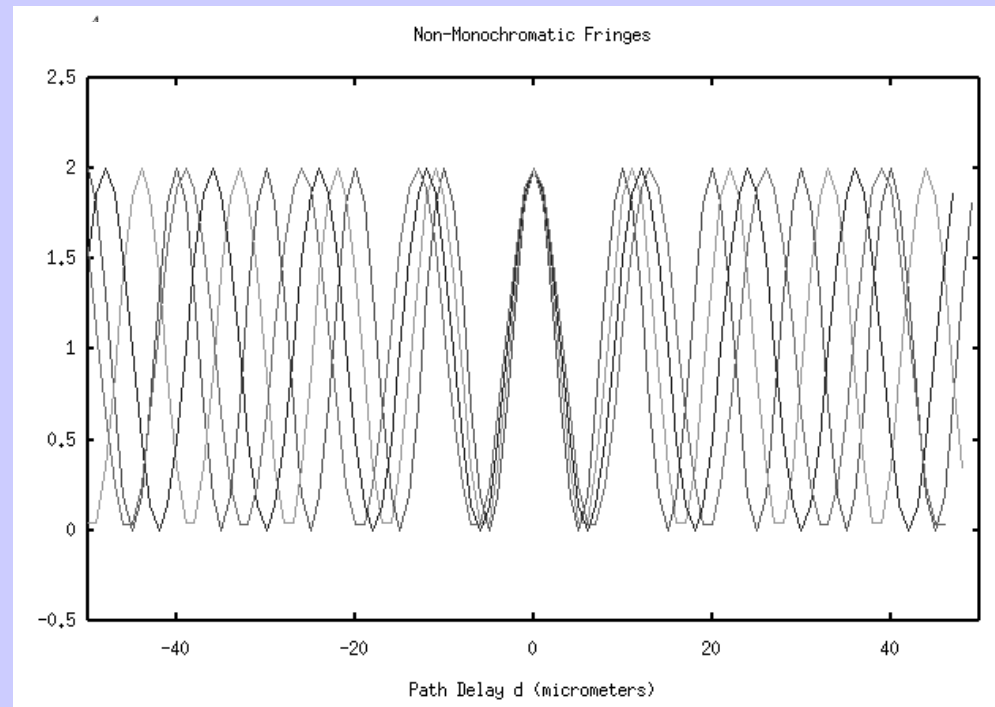
Quiz 2

1. Check that you understand the “periodicity” of the interferometer output as a function of:
 - a) The wavevector, $k = 2\pi/\lambda$.
 - b) The baseline, B .
 - c) The pointing direction, s .
 - d) The optical path difference between the two interferometer arms.
2. How rapidly does the geometric delay change during the night? (*Only an approximate answer is needed – assume a 100m baseline interferometer observing a source at 45° elevation*).
3. Imagine an interferometer with a 100m baseline is observing a target at the zenith at a wavelength of 1 micron. If the source could be moved by some small distance in the sky, how far would you have to move it to see the interferometer response change from a maximum to a minimum and then back to a maximum again?

Extension to polychromatic light

- We can integrate the previous result over a range of wavelengths:
 - E.g for a uniform bandpass of $\lambda_0 \pm \Delta\lambda/2$ (i.e. $\nu_0 \pm \Delta\nu/2$) we obtain:

$$\begin{aligned}
 P &\propto \int_{\lambda_0 - \Delta\lambda/2}^{\lambda_0 + \Delta\lambda/2} [2 + 2 \cos(kD)] d\lambda \\
 &= \int_{\lambda_0 - \Delta\lambda/2}^{\lambda_0 + \Delta\lambda/2} 2 [1 + \cos(2\pi D / \lambda)] d\lambda \\
 &= \Delta\lambda \left[1 + \frac{\sin \pi D \Delta\lambda / \lambda_0^2}{\pi D \Delta\lambda / \lambda_0^2} \cos k_0 D \right] \\
 &= \Delta\lambda \left[1 + \frac{\sin \pi D / \Lambda_{coh}}{\pi D / \Lambda_{coh}} \cos k_0 D \right].
 \end{aligned}$$



So, the fringes are modulated with an envelope with a characteristic width equal to the coherence length, $\Lambda_{coh} = \lambda_0^2 / \Delta\lambda$.

Key ideas 4

- The response to a polychromatic source is given by integrating the intensity response for each color.
- This alters the interferometric response and can lead to “washing out” of the fringe modulation completely:
 - The desired response is only achieved when $k [\hat{s}.B + d_1 - d_2] = 0$.
 - This is the so called white-light condition.

This is the primary motivation for matching the optical paths in an interferometer and correcting for the geometric delay.

- The narrower the range of wavelengths detected, the smaller is the effect of this modulation:
 - This is usually quantified via the coherence length, $\Lambda_{\text{coh}} = \lambda^2_0 / \Delta\lambda$.
 - But narrower bandpasses mean less light.

Quiz 3

1. You are observing with a 100m baseline interferometer at a wavelength of 1 micron. Due to unknown causes, the optical paths in the interferometer are unmatched by 5 microns. By how much are the maximum and minimum outputs of the interferometer altered if the fractional bandwidth of the light being collected is:
 - a) 1 percent?
 - b) 5 percent?
 - c) 10 percent?
2. What are the coherence lengths of light passed by the standard J, H and K band near-infrared filters? (*Only approximate answers are needed.*)
3. Assume you are observing with a 100m baseline interferometer at a wavelength of 1 micron with a 10 percent fractional bandwidth. If you are looking at a target at an elevation of 45° , how far away can another target be such that the geometric delay for the secondary target is no more than a coherence length different from that of the primary target?

What does this tell you about the maximum field of view of an interferometer given its baseline and its fractional bandwidth?

Measurements of fringes

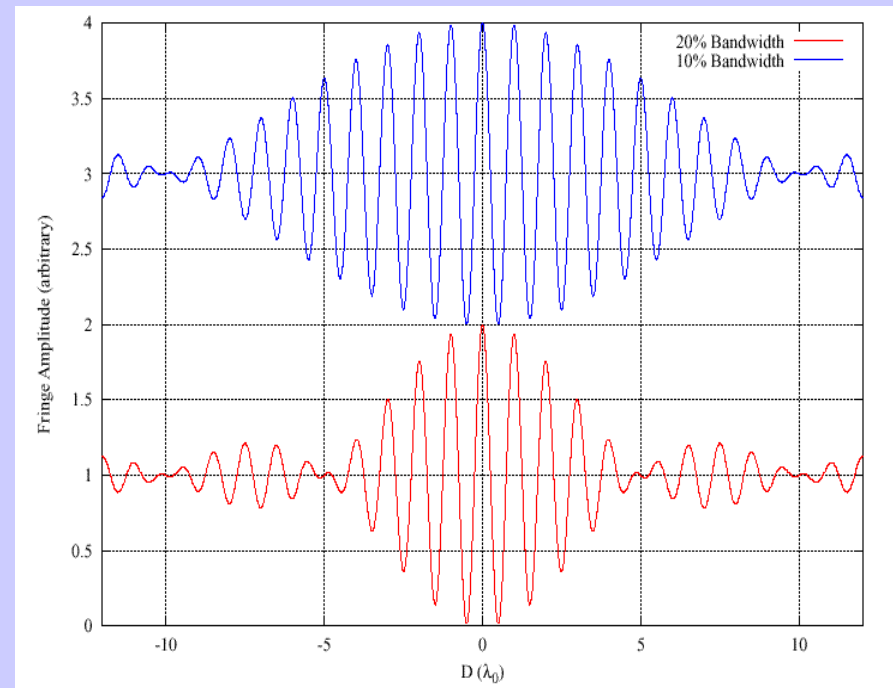
- From an interferometric point of view the key features of any interference fringe are its modulation and its location with respect to some reference point.
- In particular we can characterise these as follows:

- The fringe **visibility**:

$$V = \frac{[I_{\max} - I_{\min}]}{[I_{\max} + I_{\min}]}$$

- The fringe **phase**:

- The location of the white-light fringe as measured from some reference (radians).

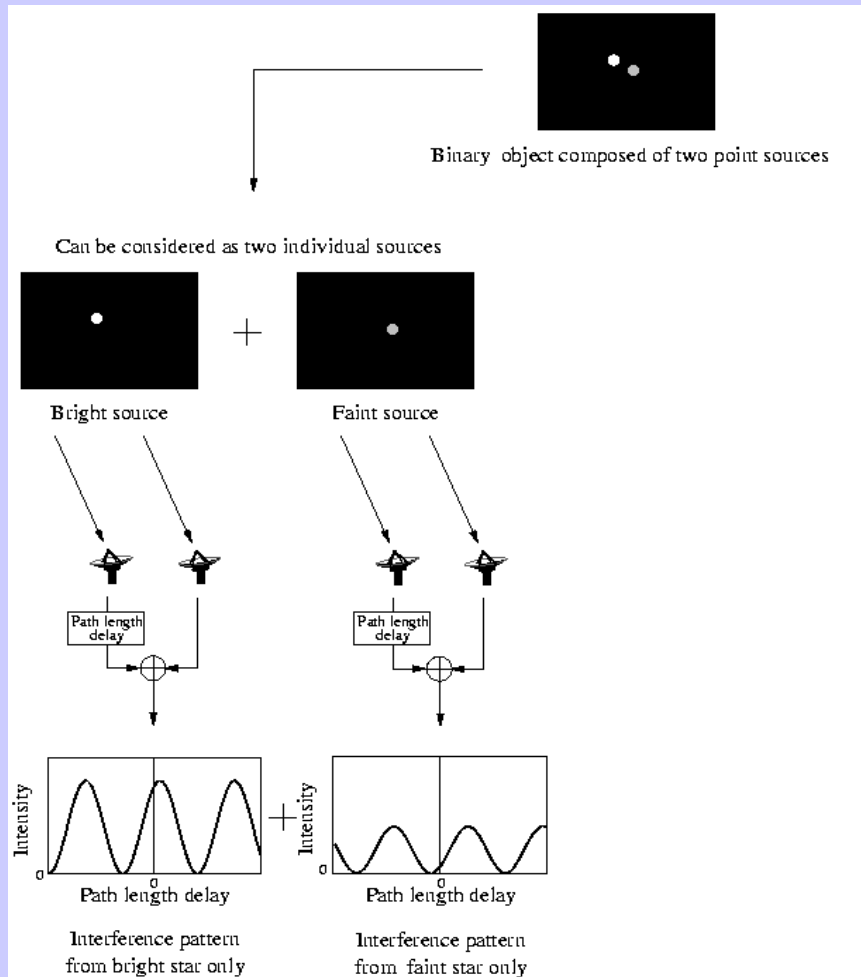


But what do these quantities tell us?

Key ideas 5

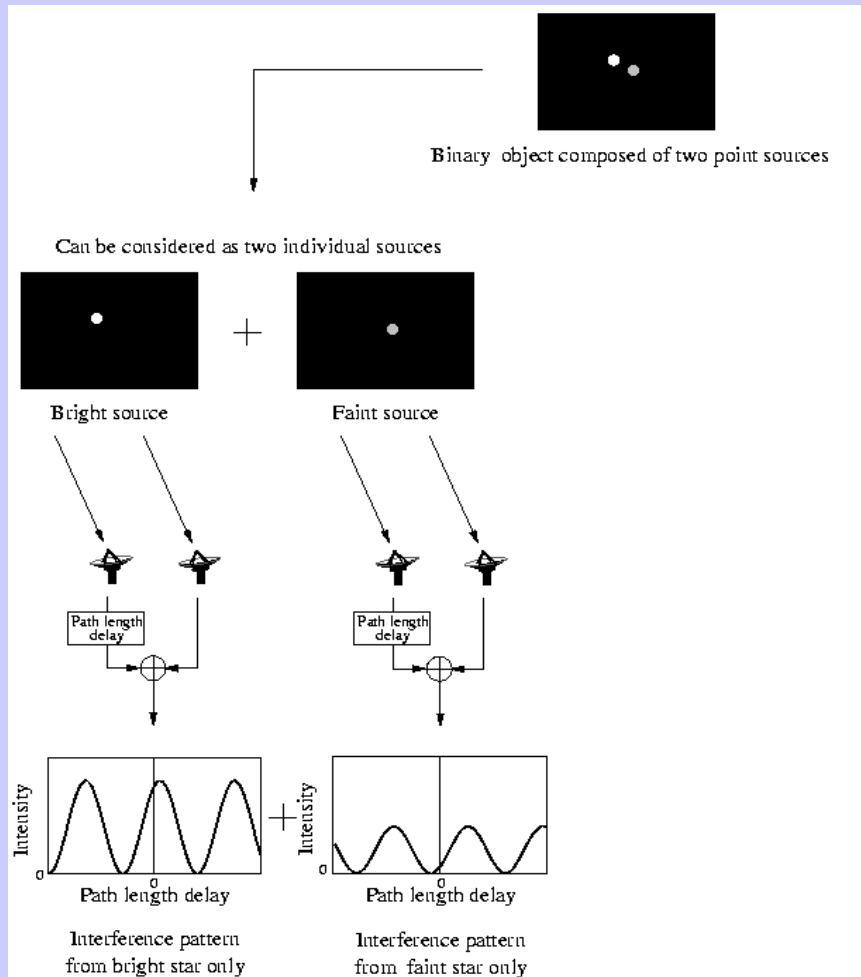
- The parameters of interference fringes that we are usually interested in are:
 - The fringe contrast (excluding any finite bandwidth effects).
 - The fringe phase.
- We are usually not interested in:
 - The fringe period.

Heuristic operation of an interferometer



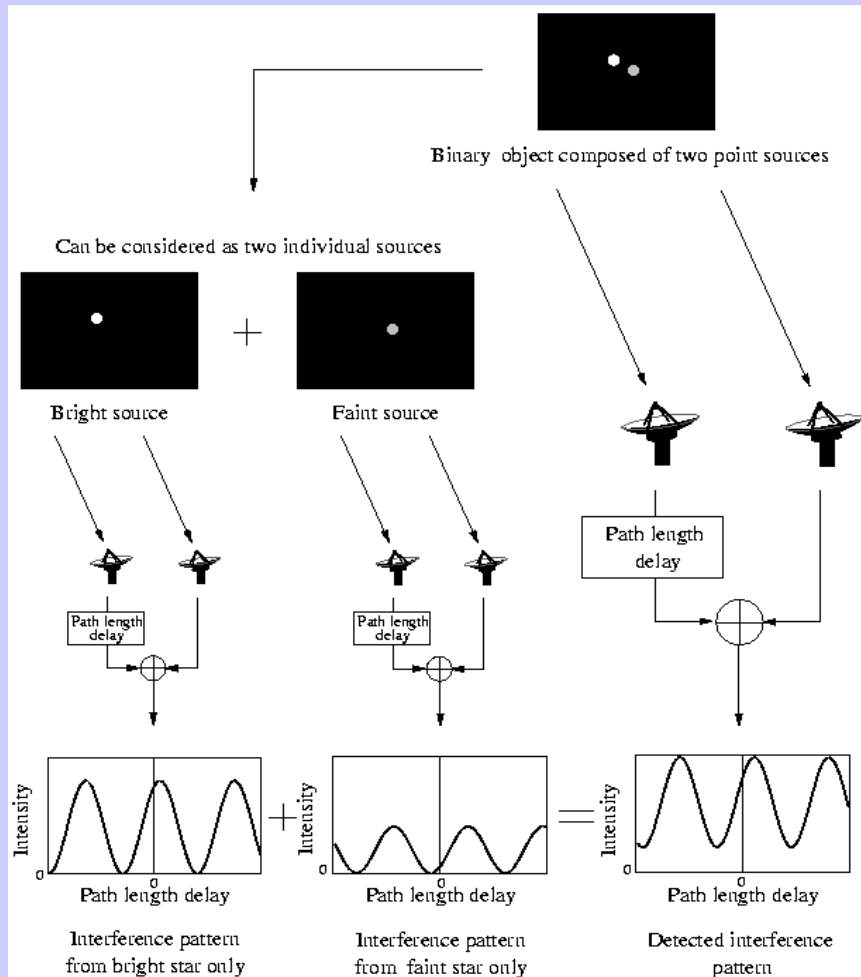
- Each unresolved element of the source produces **its own fringe pattern**.
- These have **unit visibilities** and phases that are associated with the **location** of that source element in the sky:
 - This is the basis for astrometric measurements with interferometers.

Heuristic operation of an interferometer



- The observed fringe pattern from a distributed source is just the **intensity superposition** of these individual fringe pattern.
- This relies upon the individual elements of the source being “**spatially incoherent**”.

Heuristic operation of an interferometer



- The resulting fringe pattern has a **modulation depth** that is reduced with respect to that from each source individually.
- The positions of the sources are encoded (in a scrambled manner) in the resulting **fringe phase**.

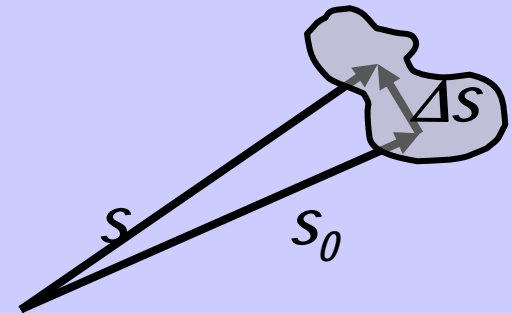
Key ideas 6

- Any general source can be described as a superposition of point sources.
- Each of these produces its own interference pattern.
- The superposition of these is what is actually measured.
 - Technically this is known as the “spatially-incoherent source” approximation.
- The modulation and phase of the resulting fringe pattern encode the source structure (albeit in an apparently complicated way).

Response to a distributed source

- Consider looking at an incoherent source whose brightness on the sky is described by $I(\hat{s})$. This can be written as $I(\hat{s}_0 + \Delta s)$, where \hat{s}_0 is a vector in the pointing direction, and Δs is a vector perpendicular to this.
- The detected intensity will be given by:

$$\begin{aligned}
 P(s_0, B) &\propto \int I(s) [1 + \cos kD] d\Omega \\
 &\propto \int I(s) [1 + \cos k(s \cdot B + d_1 - d_2)] d\Omega \\
 &\propto \int I(s) [1 + \cos k([s_0 + \Delta s] \cdot B + d_1 - d_2)] d\Omega \\
 &\propto \int I(s) [1 + \cos k(s_0 \cdot B + \Delta s \cdot B + d_1 - d_2)] d\Omega \\
 &\propto \int I(\Delta s) [1 + \cos k(\Delta s \cdot B)] d\Omega'.
 \end{aligned}$$



Heading towards the van Cittert-Zernike theorem

- Consider now adding a small path delay, δ , to one arm of the interferometer. The detected intensity will become:

$$\begin{aligned} P(s_0, B, \delta) &\propto \int I(\Delta s) [1 + \cos k(\Delta s \cdot B + \delta)] d\Omega' \\ &\propto \int I(\Delta s) d\Omega' + \cos k\delta \cdot \int I(\Delta s) \cos k(\Delta s \cdot B) d\Omega' \\ &\quad - \sin k\delta \cdot \int I(\Delta s) \sin k(\Delta s \cdot B) d\Omega'. \end{aligned}$$

- We now define something called the complex visibility $V(k, B)$:

$$V(k, B) = \int I(\Delta s) \exp[-ik\Delta s \cdot B] d\Omega',$$

so that we can simplify the equation above to:

$$P(s_0, B, \delta) \propto \int I(\Delta s) d\Omega' + \cos k\delta \operatorname{Re}[V] + \sin k\delta \operatorname{Im}[V].$$

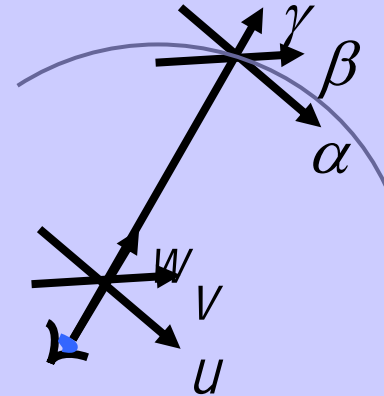
$$P(s_0, B, \delta) = I_{total} + \operatorname{Re}[V \exp[-ik\delta]].$$

What is this V that we have introduced?

$$V(k, B) = \int I(\Delta s) \exp[-ik\Delta s \cdot B] d\Omega'$$

- Lets assume $\hat{s}_0 = (0,0,1)$ and Δs is $\approx (\alpha, \beta, 0)$, with α and β small angles measured in radians.

$$\begin{aligned} V(k, B) &= \int I(\alpha, \beta) \exp[-ik(\alpha B_x + \beta B_y)] d\alpha d\beta \\ &= \int I(\alpha, \beta) \exp[-i2\pi(\alpha u + \beta v)] d\alpha d\beta. \end{aligned}$$



- Here, $u (= B_x/\lambda)$ and $v (= B_y/\lambda)$ are the **projections** of the baseline onto a plane perpendicular to the pointing direction.
- These co-ordinates have units of rad^{-1} and are the **spatial frequencies** associated with the physical baselines.
- These are the same u and v we saw earlier when we introduced the Fourier representation of the incoherent imaging equation.

So, the complex quantity V we introduced was the Fourier Transform of the source brightness distribution.

The bottom line

- So, we can put this all together as follows:
 - Our interferometer measures: $P(s_0, B, \delta) = I_{total} + \text{Re}[V \exp[-ik\delta]]$.
 - So, if we make measurements with, say, two values of δ ($= 0$ and $\lambda/4$), and we know the total intensity from the target, we can recover the real and imaginary parts of the complex visibility, V .
- Since the complex visibility is nothing more than the Fourier transform of the brightness distribution, we have our final result:

The output of an interferometer measures the Fourier transform of the source brightness distribution at a frequency determined by the projected interferometer baseline.

(Note that the amplitude and phase of the interferometer fringes are the amplitude and phase of the FT of the brightness distribution.)

Key ideas 7

- The complex visibility is also known as the “spatial coherence” function.
- Since the FT is a linear transform, if we know the complex visibility we can recover the source brightness distribution.
- Since the visibility function is complex, it has an amplitude and a phase.
- The amplitude and phase of the interference fringes we spoke of earlier, are actually the amplitude and phase of the complex visibility.
- To measure these quantities we have to make measurements for different values of D . (In the example just now we changed δ .)
- A measurement from a single interferometer baseline gives a measurement of one value of the FT of the source brightness distribution.
- Long interferometer baselines measure small structures on the sky, and short baselines, large structures.

Quiz 4

1. Check that you understand how introducing two values of δ ($= 0$ and $\lambda/4$), allows one to recover the Real and Imaginary parts of the complex visibility.
2. What happens to the interferometer output, $P(s_0, B, \delta)$ if δ is changed smoothly from zero to 2λ ? Sketch it. (*You will need to assume values for I_{total} and V – let I_{total} equal 1.0 and let V (don't forget it is complex) have an amplitude of 0.5 and a phase of 45° .*)
3. Assume you are using an interferometer with a baseline of 100m, oriented in an E-W direction at a latitude of 45° , and observing a target at declination 15° . If the source is observed at ± 3 hours around transit, what values do u and v (the projected baseline components) take? Sketch them on a plot of the uv plane.

Outline

- Representing the sky:

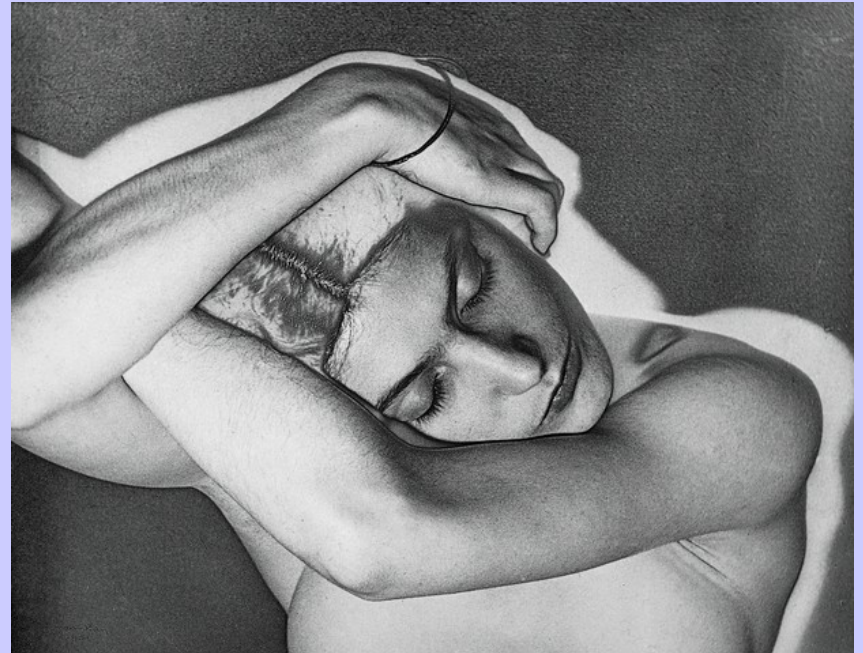
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Review of where we are

- Physical basis is the van Cittert-Zernike theorem:
 - The amplitude and phase of the interferometric fringes measure the Fourier transform of the brightness distribution.
 - This function, the coherence or visibility function, $V(u, v) = V(B_x/\lambda, B_y/\lambda)$, is measured at locations determined by the projected interferometer baselines.

- Some questions we might wish to ask ourselves are:
 - How easy is it to measure $V(u, v)$?
 - How easy is it to interpret measurements of $V(u, v)$?
 - What do we do with measurements of $V(u, v)$ if we wish to make a map of the sky?

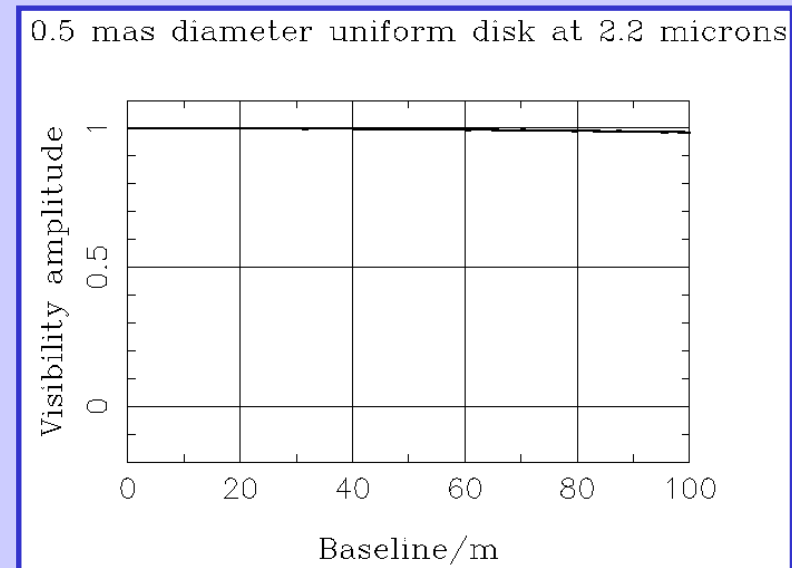
Visibility functions of simple 1-d sources (i)

$$V(u) = \int I(l) e^{-i2\pi(ul)} dl \div \int I(l) dl.$$

Point source of strength A_1 and located at angle l_1 relative to the optical axis.

$$\begin{aligned} V(u) &= \int A_1 \delta(l-l_1) e^{-i2\pi(ul)} dl \div \int A_1 \delta(l-l_1) dl \\ &= e^{-i2\pi(ul_1)}. \end{aligned}$$

- The **visibility amplitude** is unity $\forall u$.
- The **visibility phase** varies linearly with u ($= B/\lambda$).
- Sources such as this are easy to observe since the interferometer output gives fringes with high contrast.

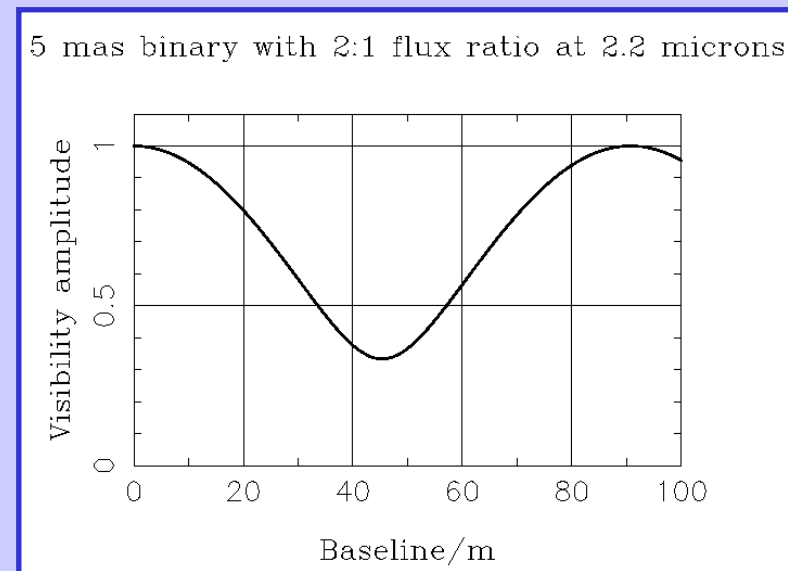


Visibility functions of simple sources (ii)

A double source comprising point sources of strength A_1 and A_2 located at angles 0 and l_2 relative to the optical axis.

$$\begin{aligned} V(u) &= \int [A_1\delta(l) + A_2\delta(l-l_2)] e^{-i2\pi(ul)} dl \div \int [A_1\delta(l) + A_2\delta(l-l_2)] dl \\ &= [A_1 + A_2 e^{-i2\pi(ul_2)}] \div [A_1 + A_2]. \end{aligned}$$

- The visibility amplitude and phase **oscillate** as functions of u .
- To identify this as a binary, baselines from $0 \rightarrow \lambda/l_2$ are required.
- If the ratio of component fluxes is large the modulation of the visibility becomes increasingly difficult to measure, i.e. the interferometer output looks the same for all baselines.

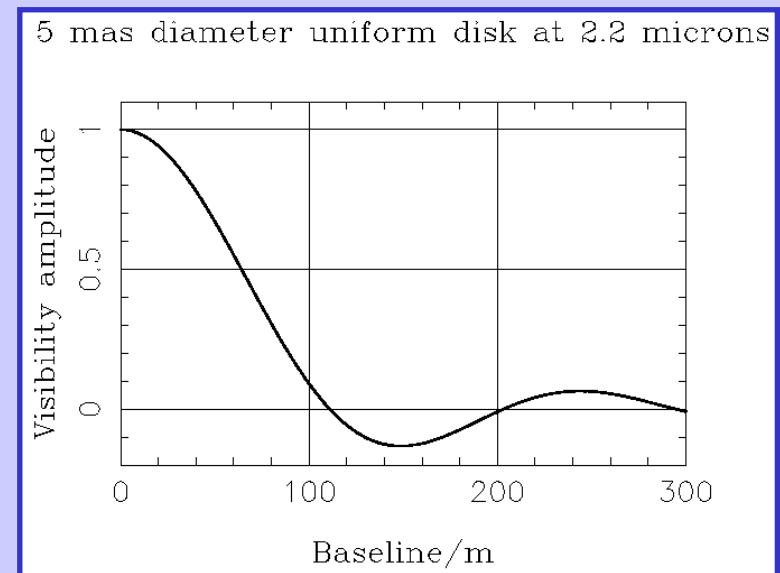


Visibility functions of simple sources (iii)

A uniform on-axis disc source of diameter θ .

$$V(u_r) \propto \int_0^{\theta/2} \rho J_0(2\pi\rho u_r) d\rho$$
$$= 2J_1(\pi\theta u_r) \div (\pi\theta u_r) .$$

- The visibility amplitude **falls rapidly** as u_r increases.
- To identify this as a disc requires baselines from $0 \rightarrow \lambda/\theta$ at least.
- Information on scales smaller than the disc diameter correspond to values of u_r where $V \ll 1$, and is hence difficult to measure. This is because the interferometer output gives fringes with very small modulation.



Key ideas 8

- Unresolved, sources have visibility functions that remain high whatever the baseline, and produce high contrast fringes all the time.
- Resolved sources have visibility functions that fall to low values at long baselines, giving fringes with very low contrast.
 - ⇒ Fringe parameters for resolved sources will be difficult to measure.
- To usefully constrain a source, the visibility function must be measured adequately. Measurements on a single, or small number of, baselines are normally not enough for unambiguous interpretation.
- Imaging – which necessarily requires information on both small and large scale features in a target, will generally need measurements where the fringe contrast is both high and low.

Quiz 5

1. Plot the visibility amplitude and phase for a 5mas separation binary (see slide 36) as a function of projected baseline length up to 100m assuming you are taking observations at 1 micron and that the binary has:
 - a) a flux ratio of 2:1,
 - b) a flux ratio of 100:1,
 - c) a flux ratio of 1000:1.

(Remember, $V = V(u)$ and $u = B/\lambda$).
2. Assuming observations at 1 micron, what baseline lengths are needed to resolve main sequence stars of spectral types O, A, G, and M, if their apparent magnitudes are:
 - a) 0,
 - b) 6,
 - c) 12.

What do these results tell you about observing faint targets with an optical interferometer?

(You will need to look up the absolute magnitudes and physical radii of these stars in a textbook.)

Introduction to interferometric imaging

- The visibility function, $V(u, v)$ is the Fourier transform of the source brightness distribution:

$$V(u, v) = \int I(\alpha, \beta) \exp[-i2\pi(\alpha u + \beta v)] d\alpha d\beta.$$

- So, if we can measure V for as many values of u and v as possible & perform an inverse FT, we should recover the source:

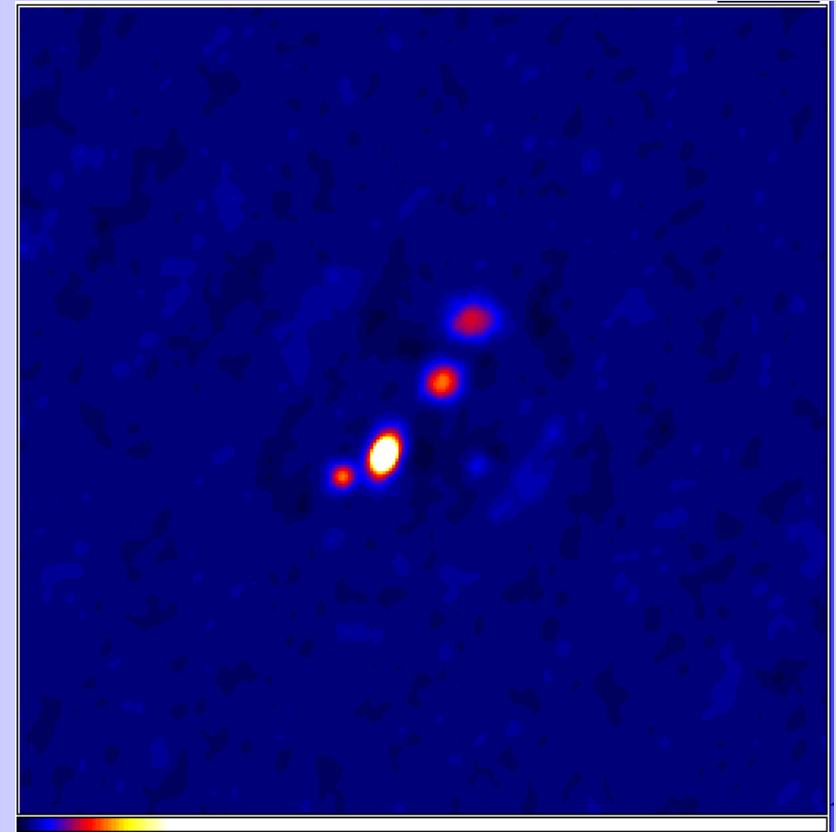
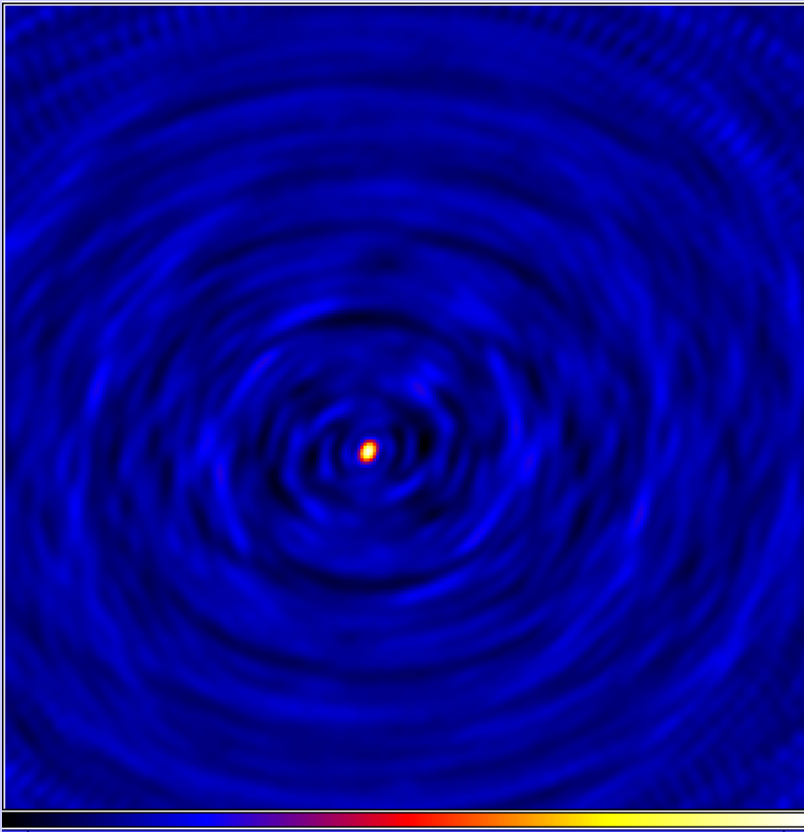
$$\iint V(u, v) \exp[+i2\pi(u\alpha + v\beta)] du dv = I_{norm}(\alpha, \beta).$$

- But since what we measure is a **sampled** version of $V(u, v)$, what we actually recover is the so-called “dirty map”:

$$\begin{aligned} \iint S(u, v) V(u, v) \exp[+i2\pi(u\alpha + v\beta)] du dv &= I_{dirty}(\alpha, \beta) \\ &= I_{norm}(\alpha, \beta) * B_{dirty}(\alpha, \beta), \end{aligned}$$

where $B_{dirty}(l, m)$ is the Fourier transform of the sampling distribution, and is known as the **dirty-beam** (i.e. the PSF of the experiment).

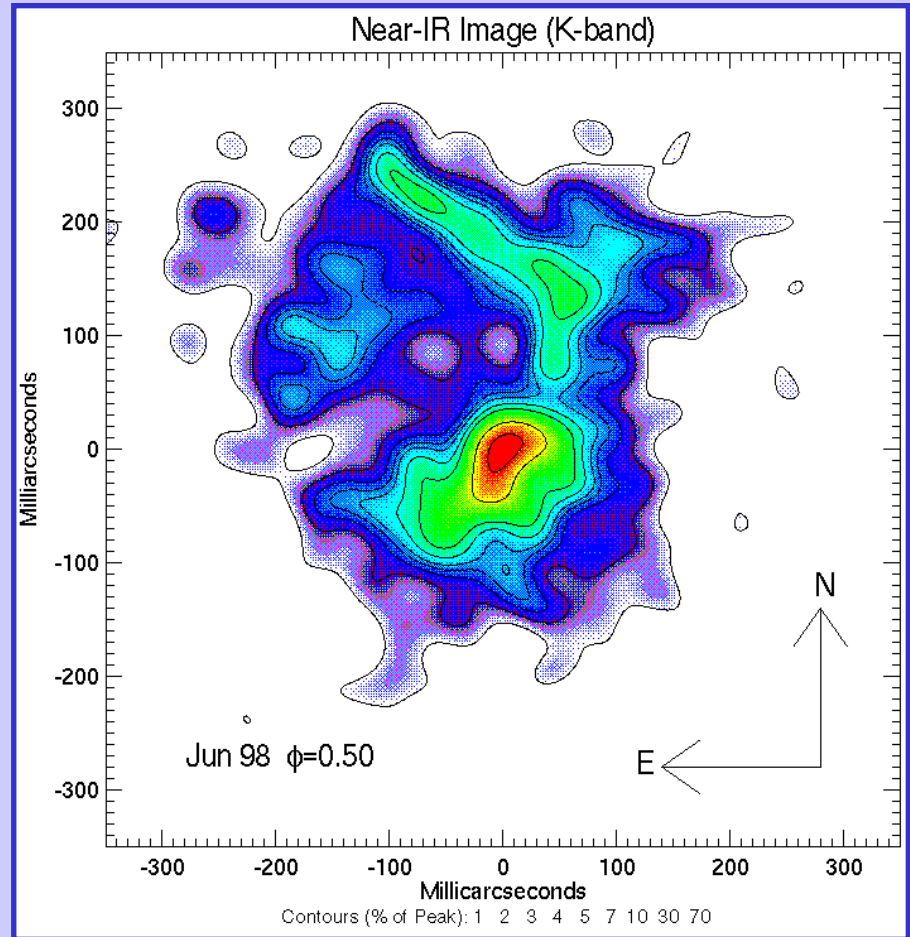
The effect of the sampling distribution



- Interferometric PSF's can often be horrible. Correcting an interferometric map for the Fourier plane sampling function is known as **deconvolution** and is broadly speaking straightforward.

A real astronomical example

- K-band image of IRC+10216. Image courtesy of Peter Tuthill and John Monnier.
- The angular resolution is of order 50 mas.



Key ideas 9

- Imaging with an interferometer => measuring the visibility function for a wide range of baselines.
- It also => measuring its amplitude and phase.
- The map you get will only contain information corresponding to the baselines you measured.
 - This applies to conventional imaging as well
- There is no such thing as the “correct” image.

Sensitivity

- What does this actually mean for an optical/infrared interferometer?
- The “source” has to be bright enough to:
 - Allow **stabilisation** of the interferometer against any atmospheric fluctuations.
 - Allow a reasonable signal-to-noise for the fringe parameters to be build up over some total convenient integration time. This will be measured in **minutes**.
- Once this achieved, the faintest features one will be able to interpret reliably will be governed by S/N ratio and number of visibility data measured.
- In most cases (i.e. photon noise dominated regime), the sensitivity of an interferometer will scale like some power of the **measured fringe contrast** \times **square root of number of photons detected while the fringe is being measured**.
 - This highlights fringe contrast and not throughput as being critical.
 - This also highlights the difficulty of measuring resolved targets where V is low.

Key ideas 10

- The sensitivity of optical/infrared interferometry is strongly constrained by the seeing.
- It mirrors the sensitivity of an adaptive optics system, where a bright enough reference source is needed to monitor the atmospheric fluctuations.
- For resolved targets, where the fringe contrast is low, high photon rates are needed.

Quiz 6

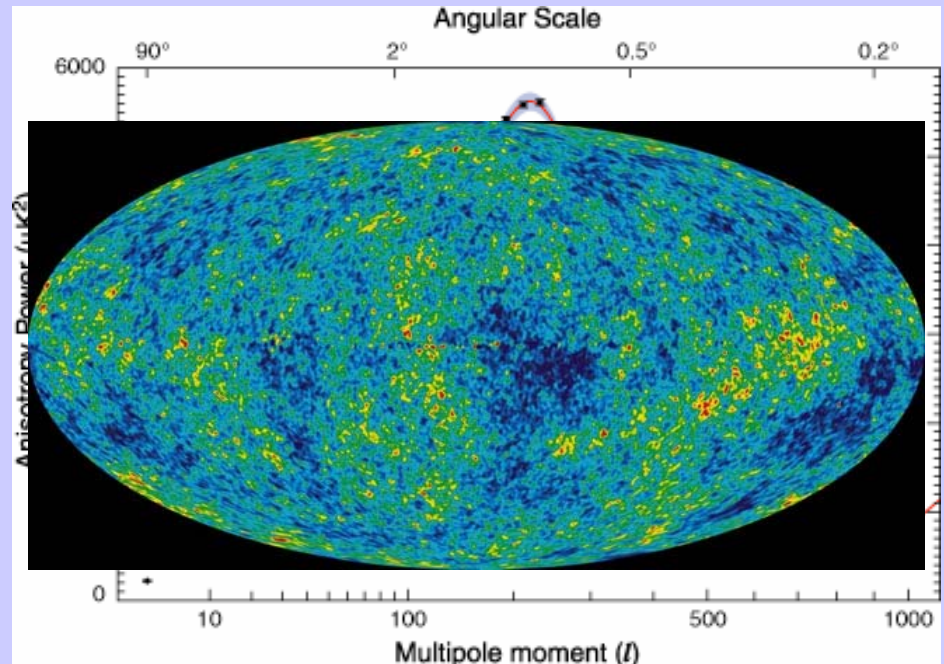
1. Why might the total integration time for an measurement of the visibility function with an interferometer be limited to a few minutes? (*You may wish to consider the rotation of the Earth and the how much u and v have to change so that you are measuring an independent value of the Fourier transform of the target.*)
2. What are the typical spatial and temporal scales associated with the atmosphere for observations made at 1 micron? How do these compare to the equivalent scales at centimetric radio wavelengths (e.g. at the VLA site in New Mexico) and at sub-mm wavelengths (e.g. at the ALMA site in Chile). (*You will need to bug the lecturers for this information or do some literature sleuthing to find the answers.*)
3. If the S/N for some interferometric observation of an unresolved target ($V=1$) is X , how much brighter (in magnitudes) must a target be to be measured with the same S/N if it is sufficiently resolved that its visibility function, V , is equal to 0.1? (*Assume that you are working in the regime where the S/N scales like $V\sqrt{\text{flux}}$.*)

Crawl, walk, run

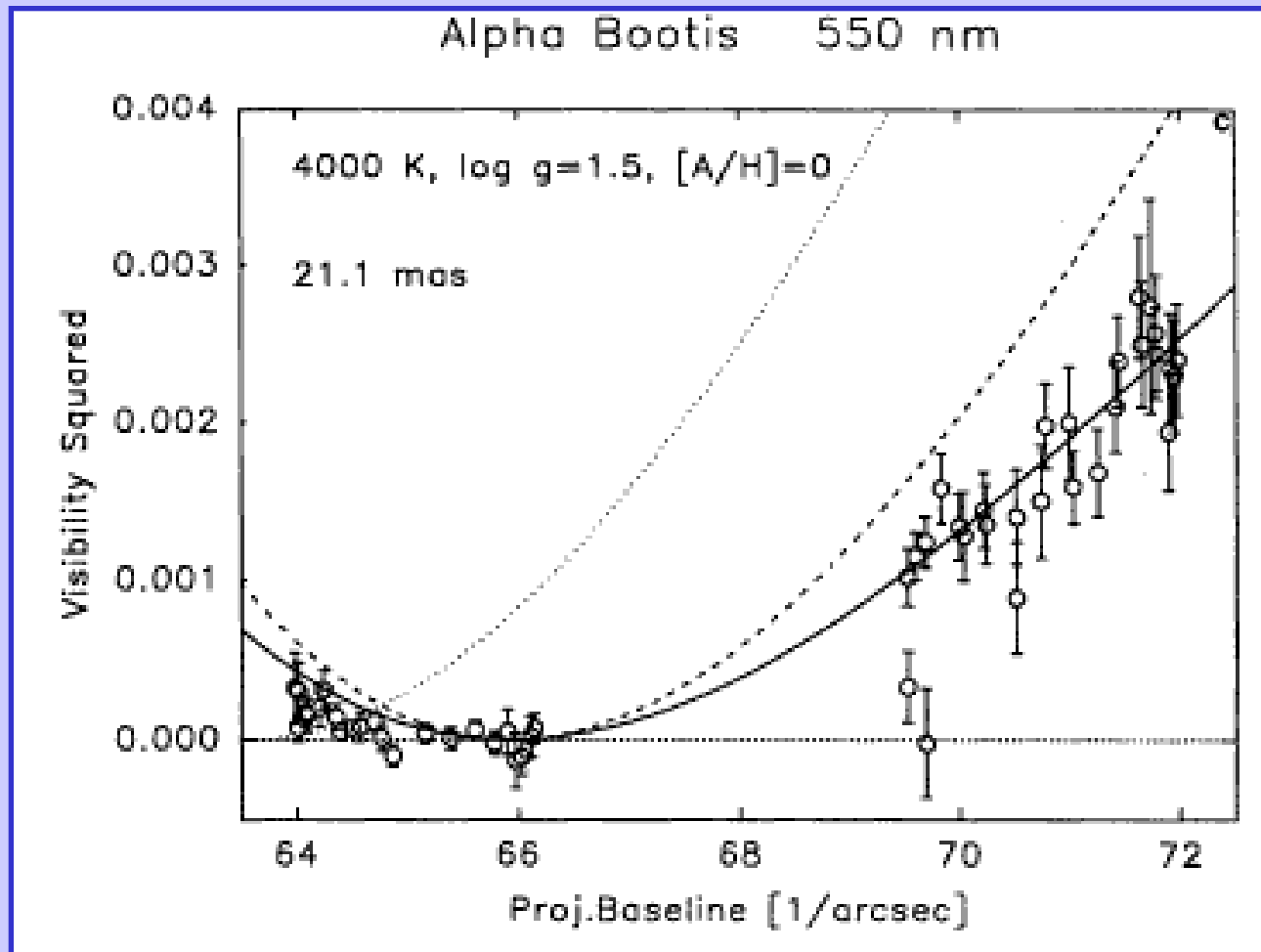


Recap

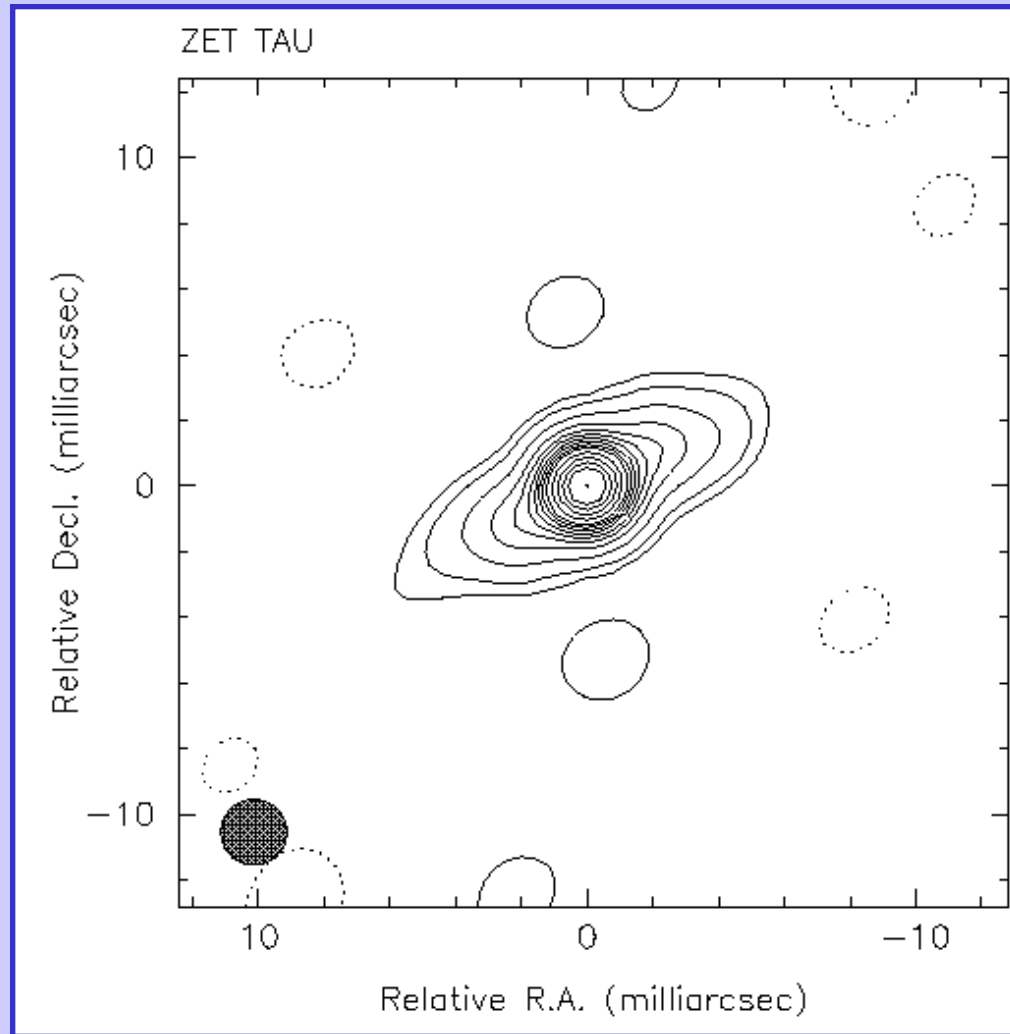
- Once we have measured the visibility function of the source, we have to interpret these data. This can take many forms:
 - Small numbers of telescopes:
 - Model-fitting.
 - Moderate numbers of telescopes:
 - Model-fitting.
 - Rudimentary imaging.
 - Large numbers of telescopes:
 - Model-fitting.
 - Model-independent imaging.
- Don't forget that imaging is not a requirement for good science:



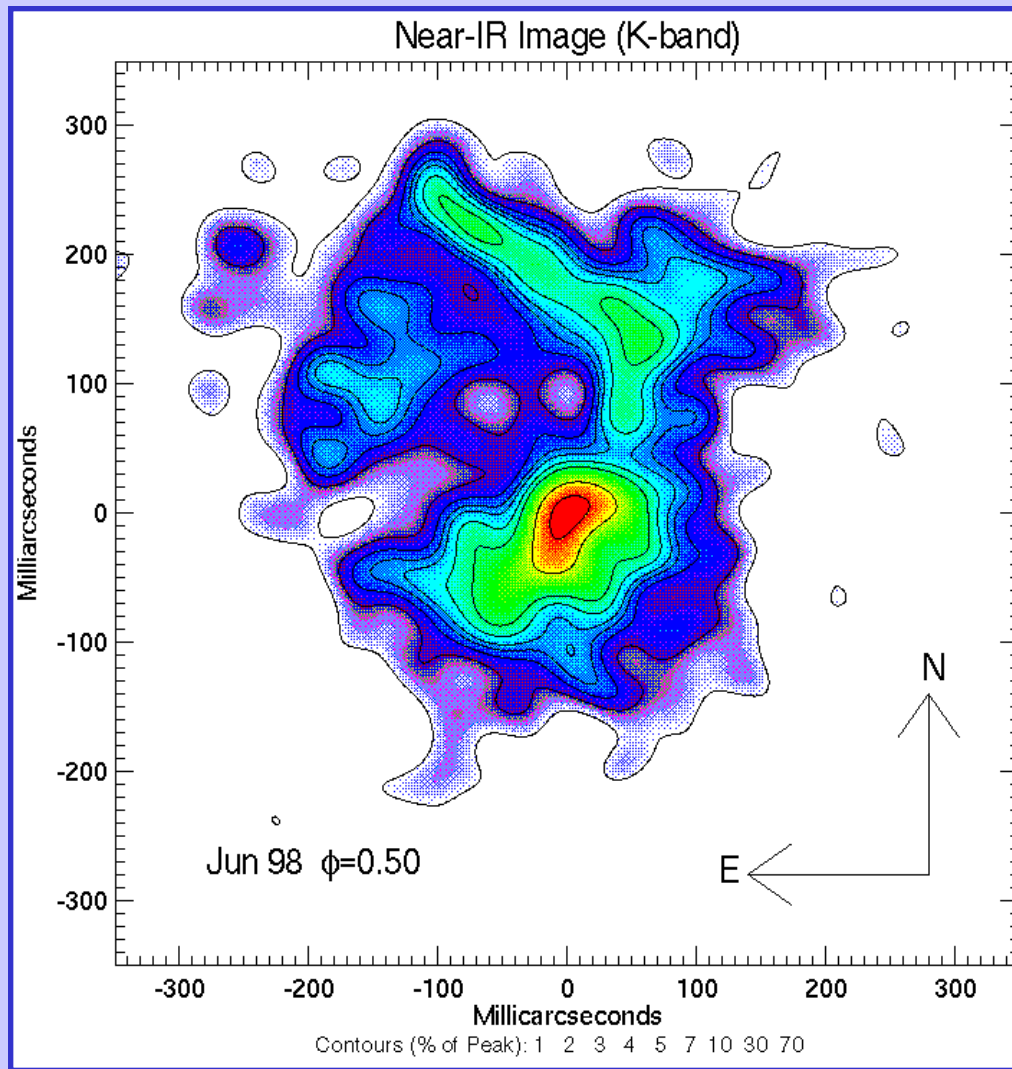
Interferometric science – 2 telescopes



Interferometric science – 5 telescopes



Interferometric science – 21 telescopes



Summary

- Interferometers are machines to make fringes.
- The fringe modulation and phase tell you what you are looking at.
- More precisely, these measure the amplitude and phase of the FT of the source brightness distribution.
- A measurement with a given interferometer measures one value of the FT of the source brightness distribution.
- Multiple baselines are obligatory to study a source reliably.
- Once the measurements have been made, reliable interpretation is the next critical step.

Throughout this lecture I have, largely, ignored the impact of the atmosphere.
How this impacts things is the subject of your next journey!