Data reduction – a Bayesian Perspective

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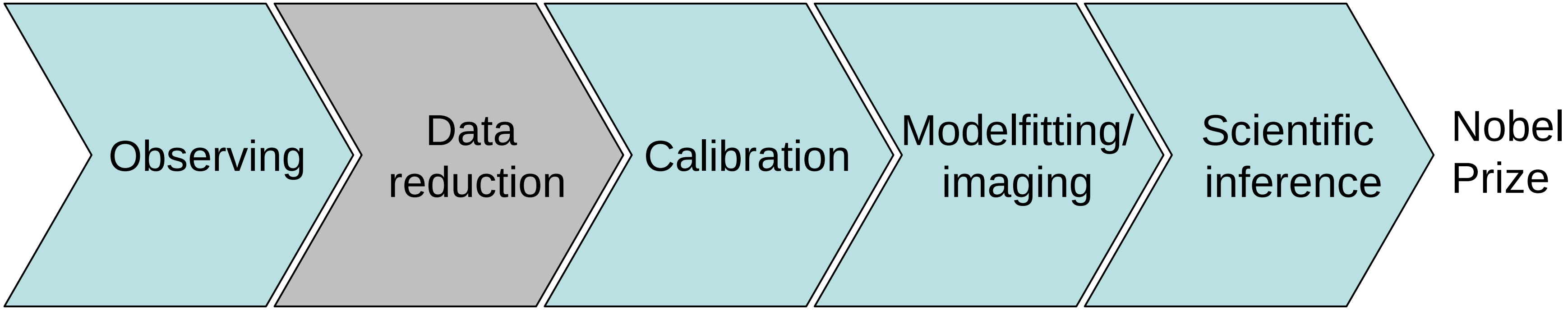
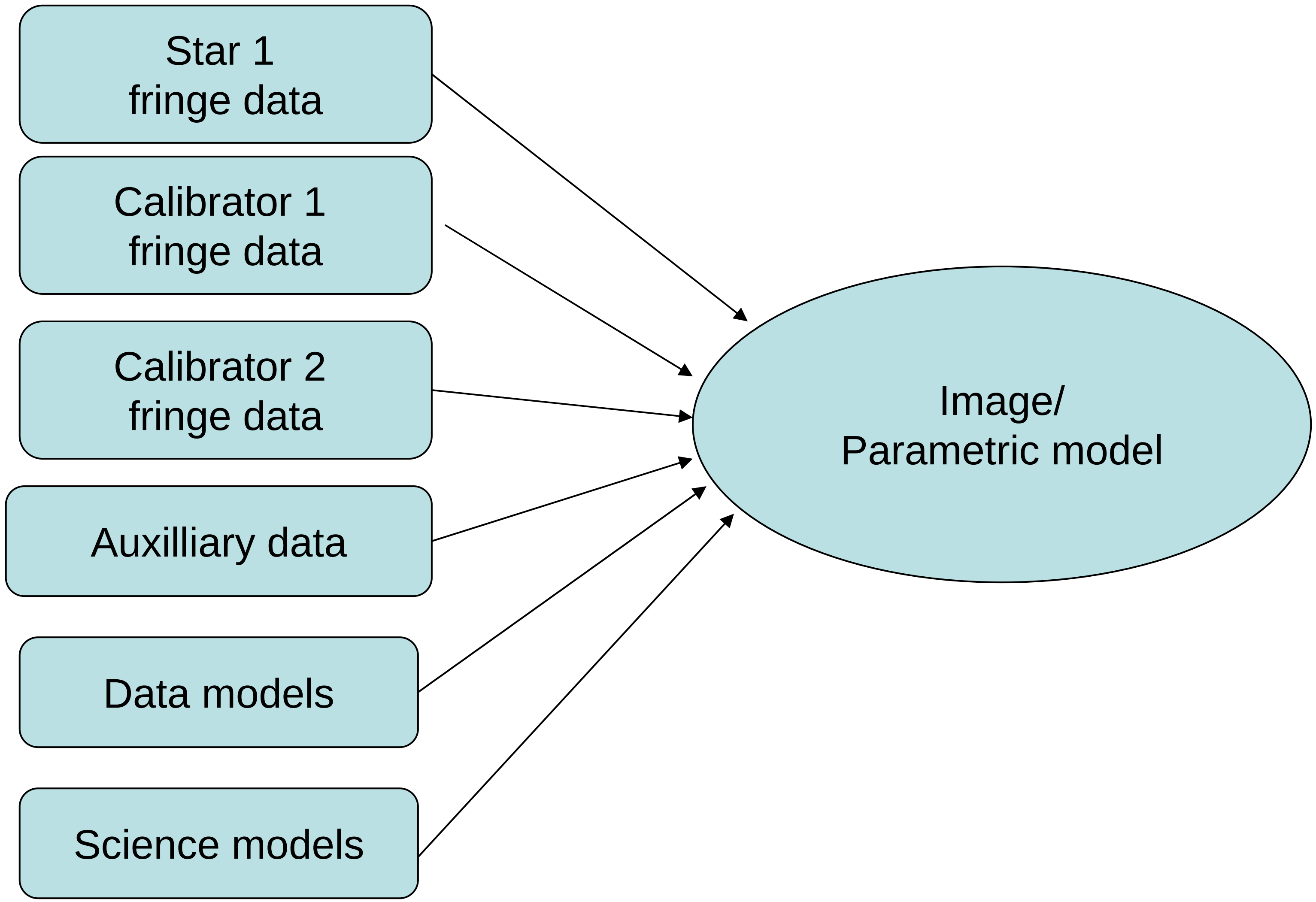
We are at an intermediate stage in the interferometric observation process

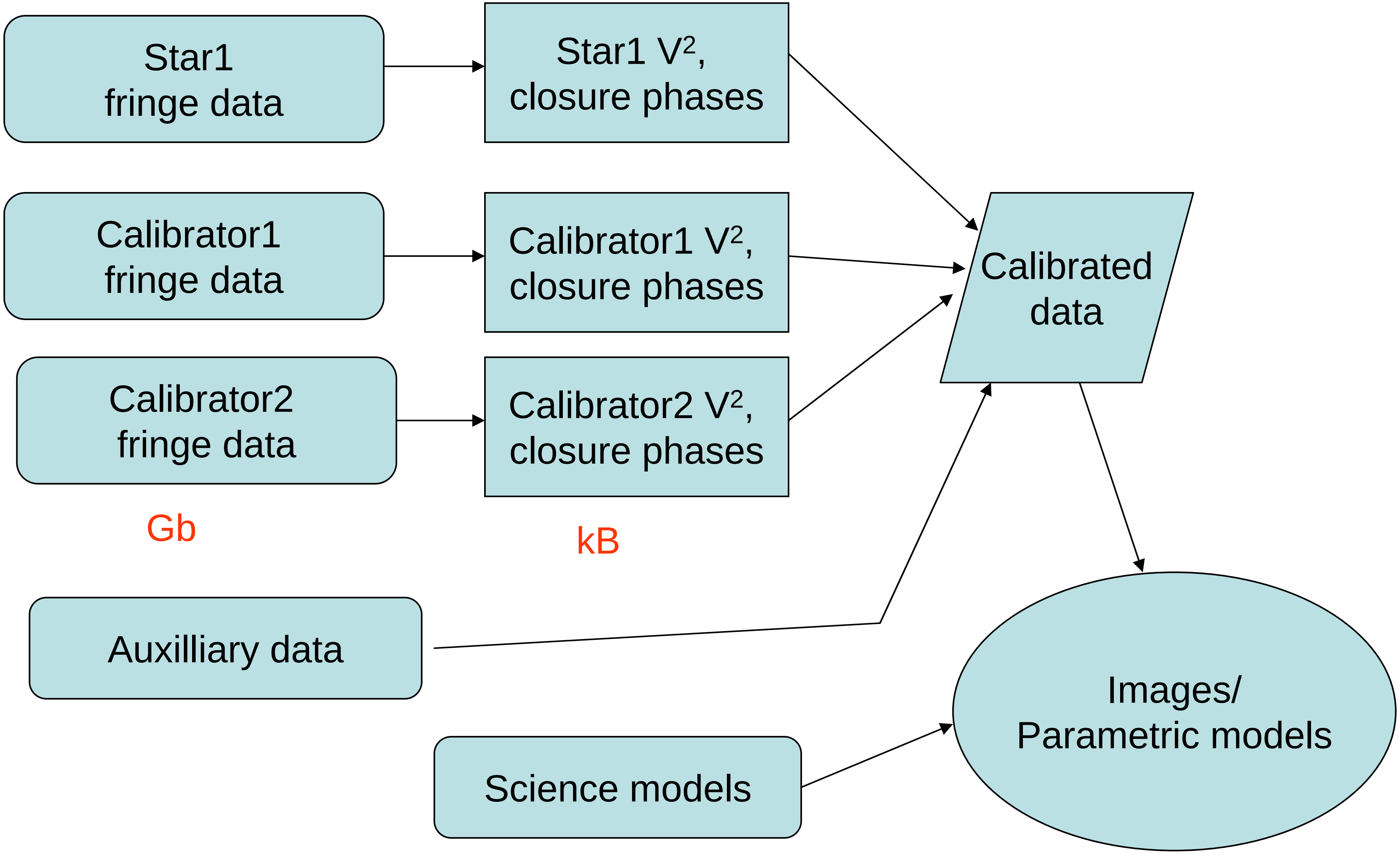


We want to extract the information about the source and throw away the noise

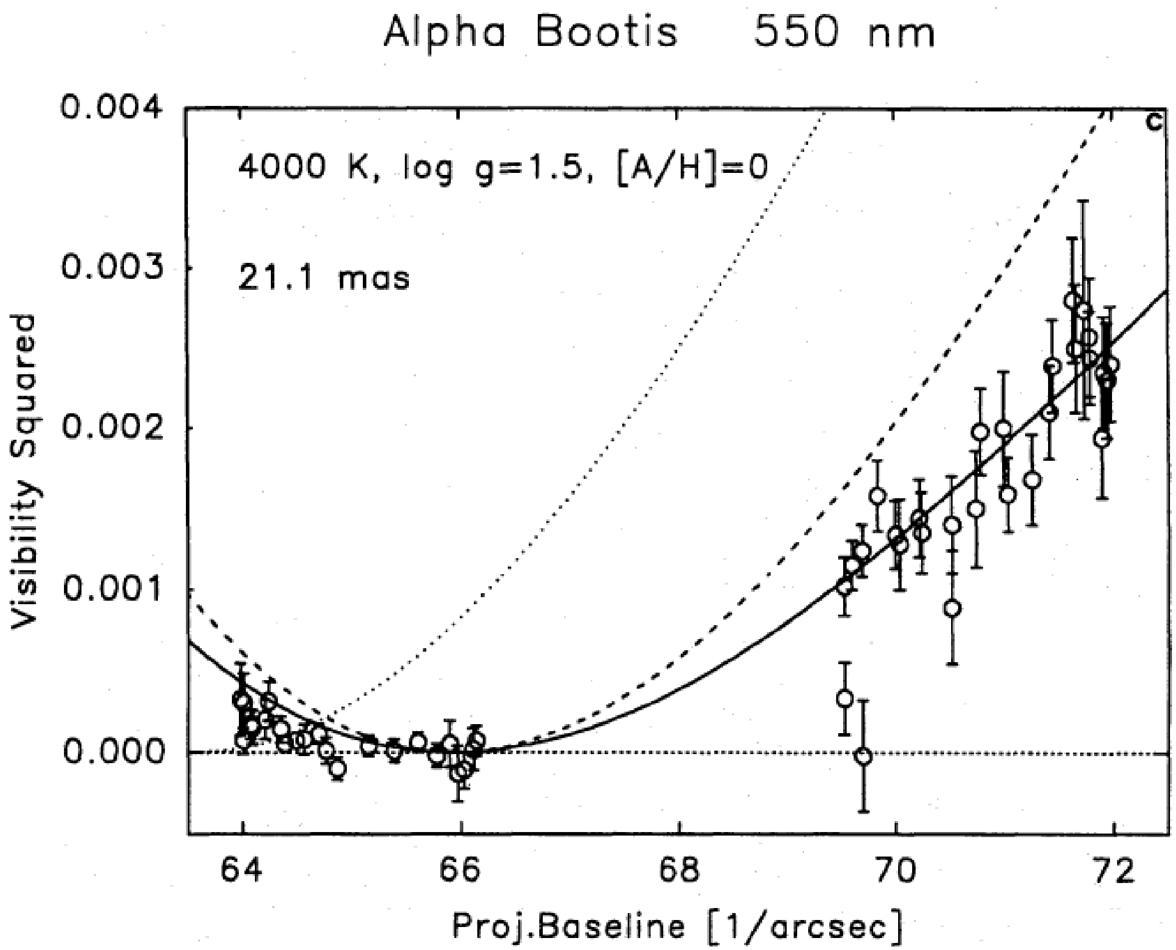
Bayesian inference theory tells us that the best way to do this is in a single global step



In practice, we adopt a multi-step approach

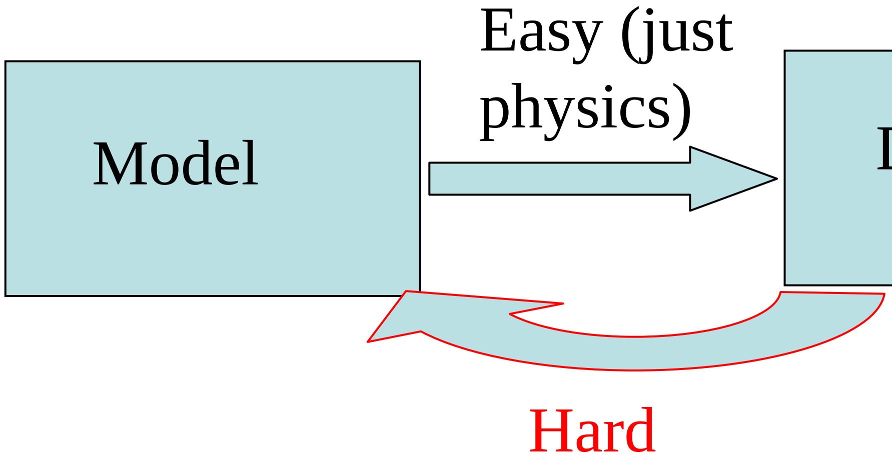


How far are we from making the best use of the interferometric data that we have?

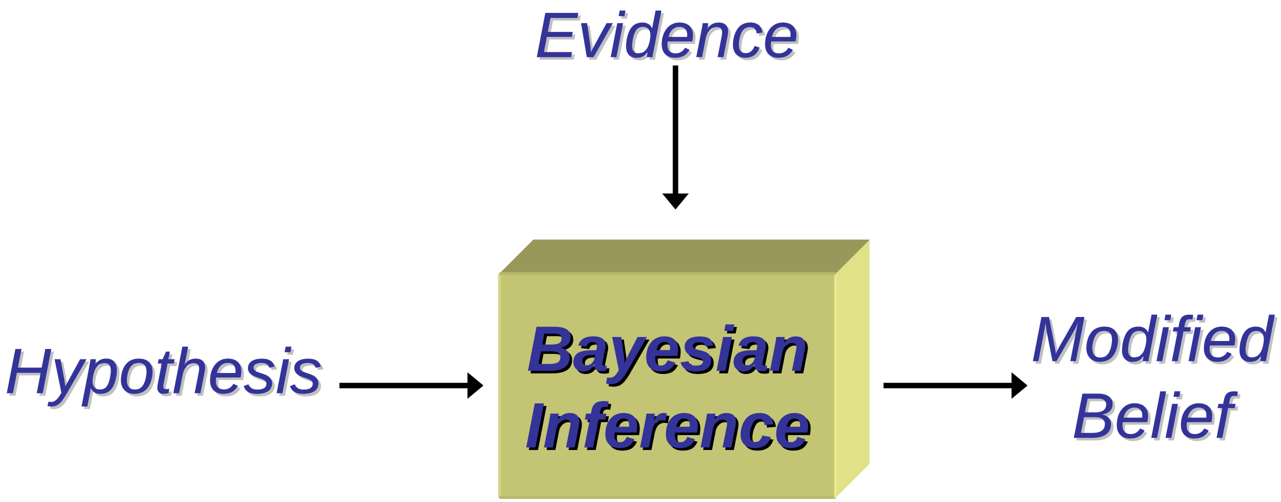


Quirrenbach et al 1996

We need a systematic way of solving “inverse problems”



Bayesian inference formalizes the Scientific Method



Bayes' Theorem tells us how to modify our degree of belief in our models given new data

data

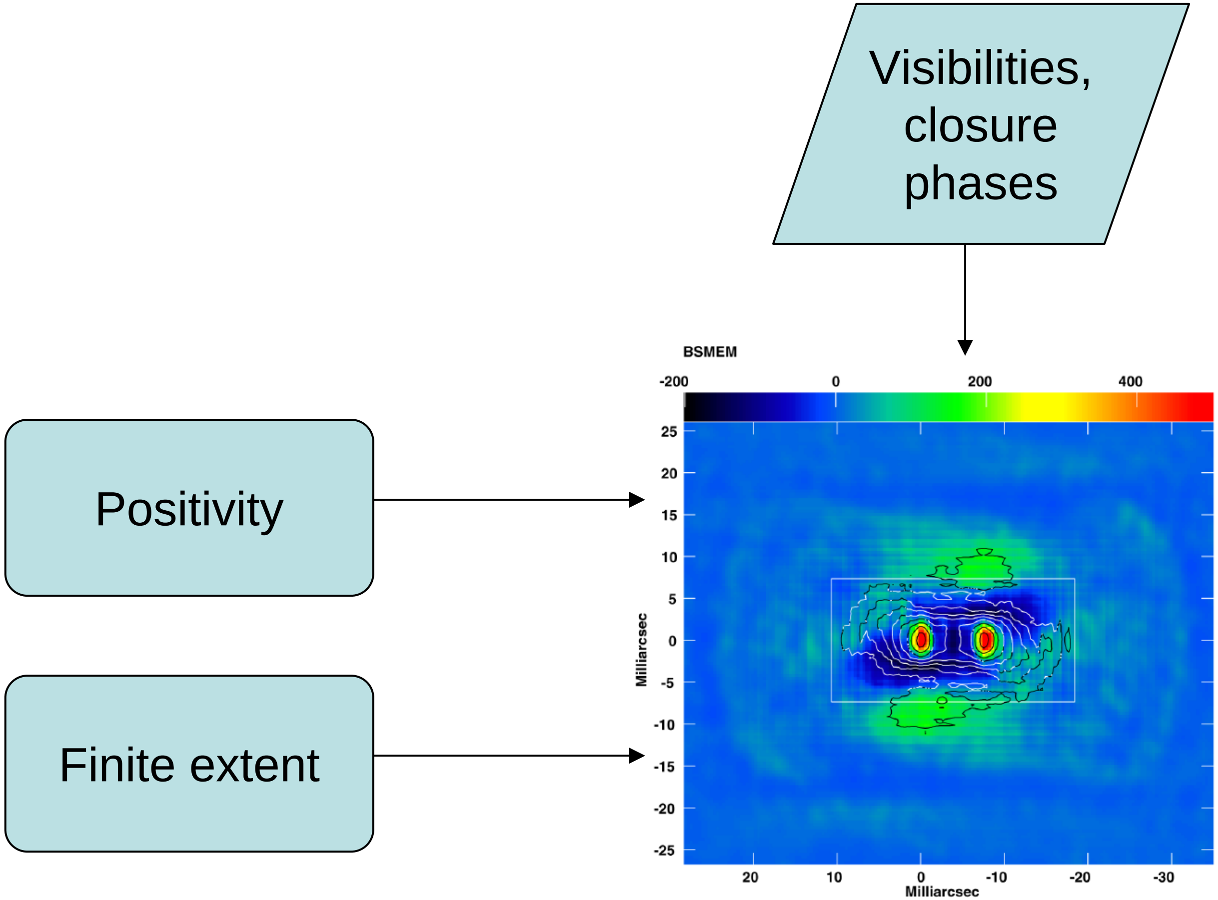
$$P(B_k \mid A) = \frac{P(A \mid B_k)P(B_k)}{\sum_{j=1}^{n} P(A \mid B_j)P(B_j)}$$

model

Solving inverse problems is straightforward, but can be computationally intensive

- Recipe:
  - Generate all possible models (tedious but possible).
  - Find the likelihood that each model would have generated the data (easy).
  - The one which best predicted the data wins (modulo prior information).

Bayesian methods allow us to get more out of the data when we have *a priori* constraints



"Frequentist" estimators are an ad-hoc, non-Bayesian, way of solving inverse problems

estimator — data

$$\tilde{B}(a_1, a_2, \ldots)$$

model

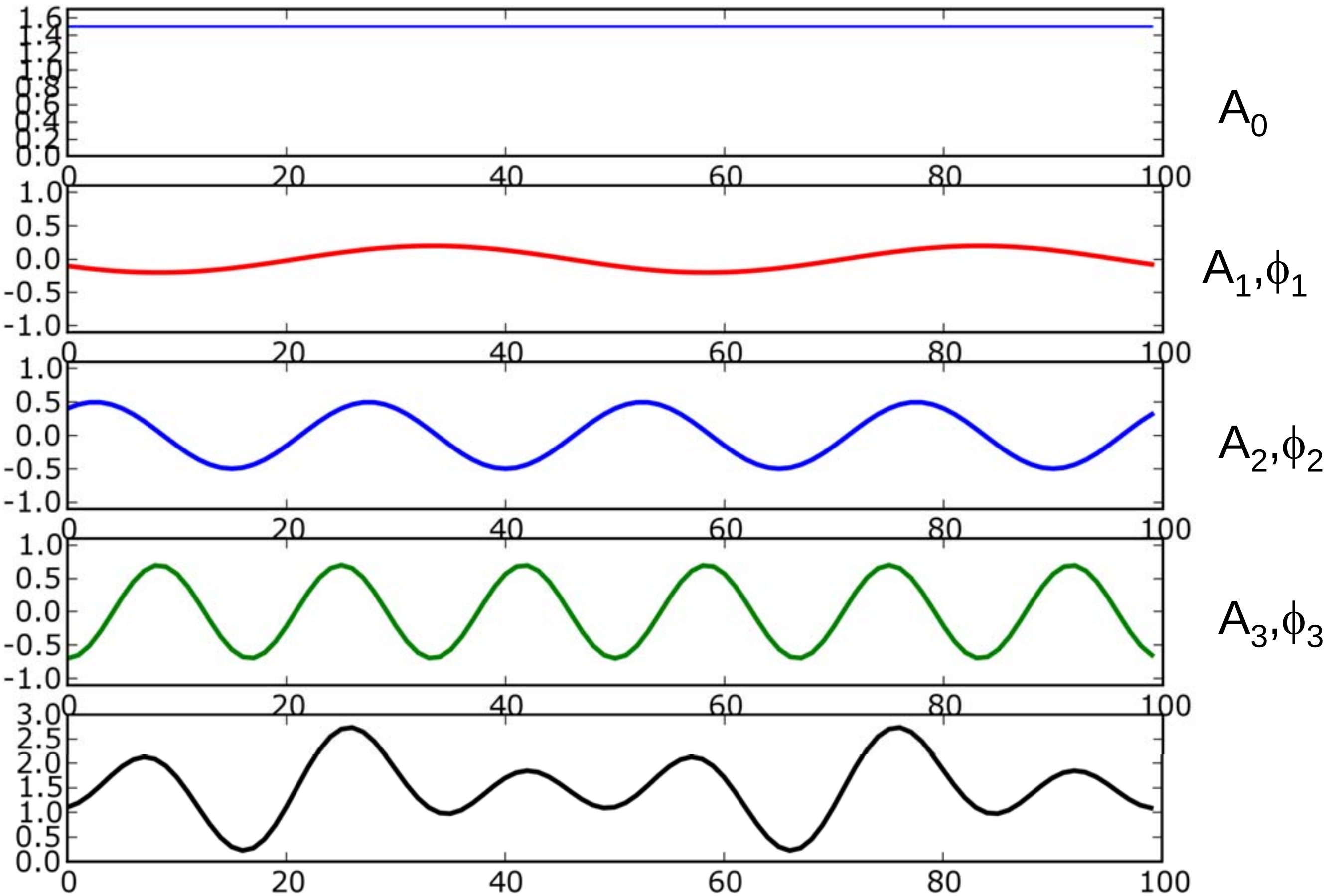
$$\langle \tilde{B} \rangle \to B$$

unbiased

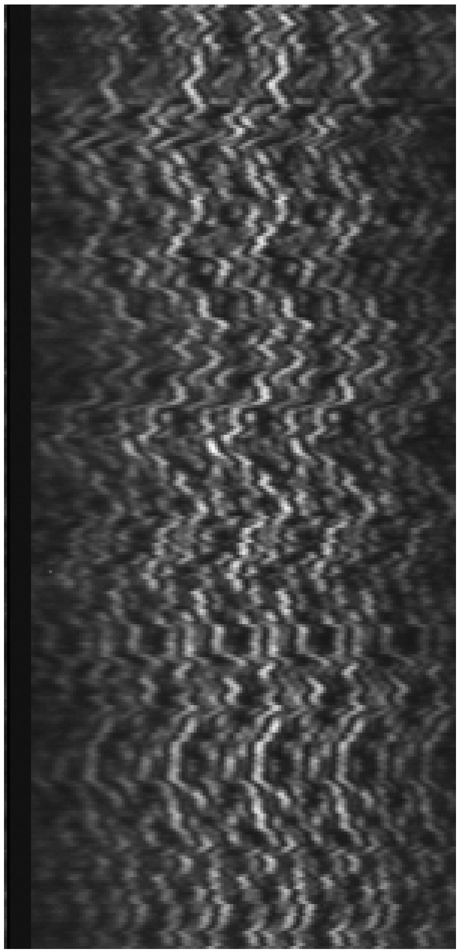
$$\mathrm{var}\left(\tilde{B}\right) < \mathrm{var}\left(\tilde{B}_2\right)$$

efficient

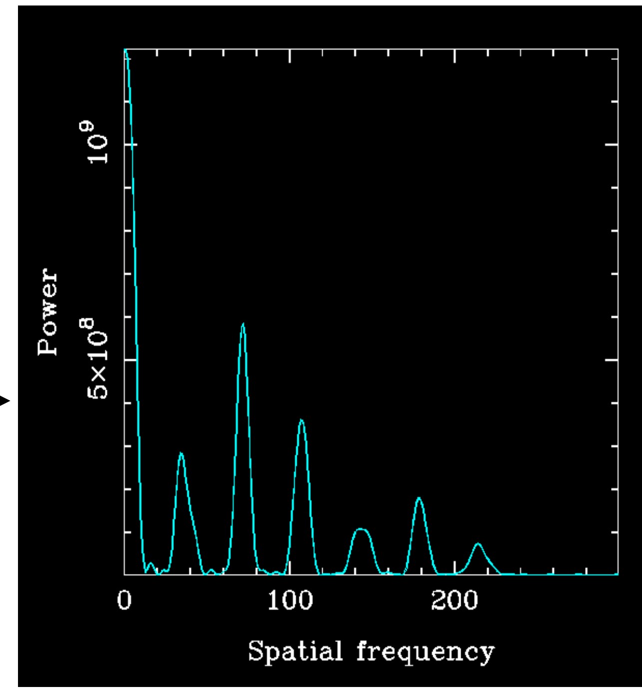
Consider a fringe pattern from an idealized phase-stable interferometer



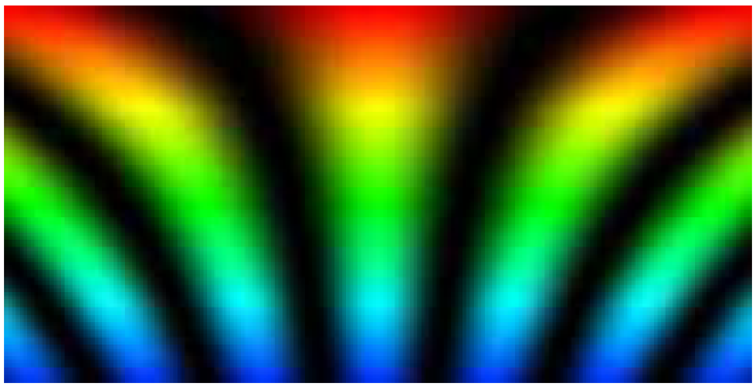
A Fourier-based estimator is intuitively appealing

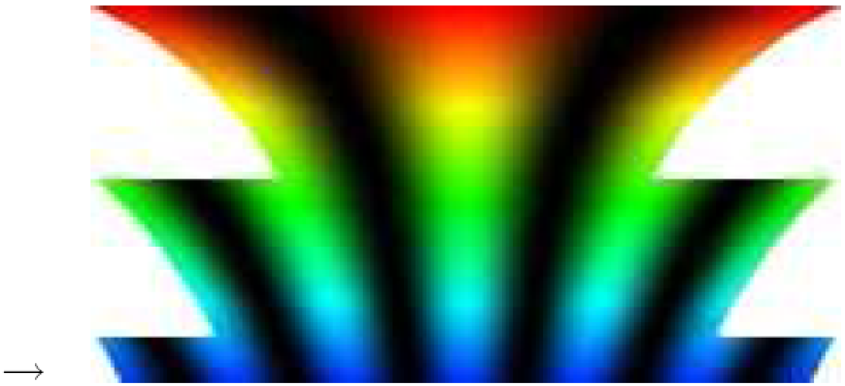


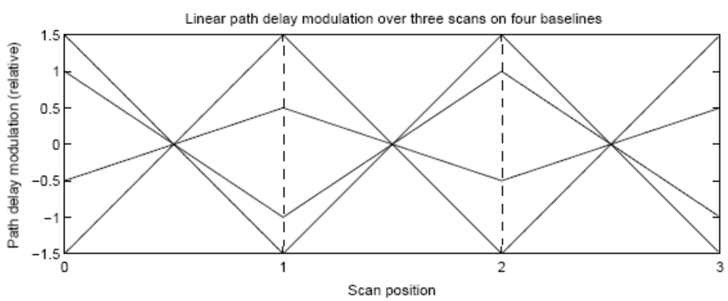
FT

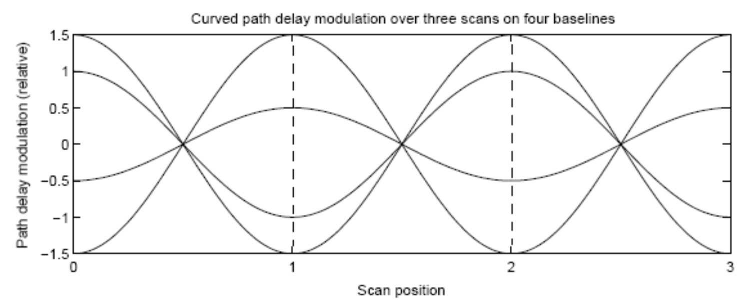


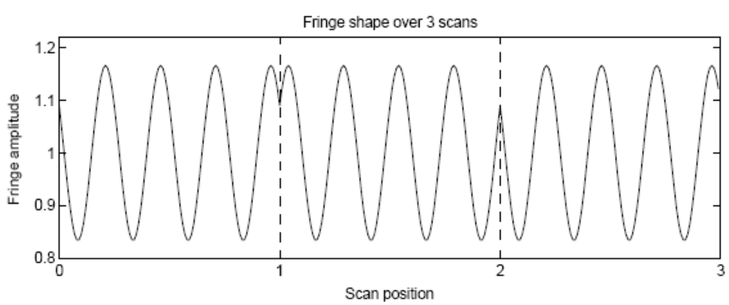
We run into problems as we consider more realistic scenarios

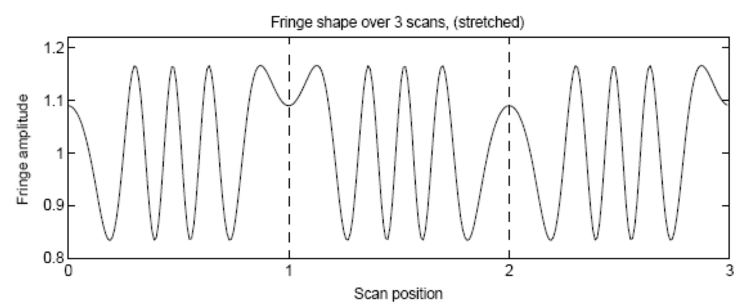












Instead, we explicitly define a forward model and from this compute a Bayesian inverse

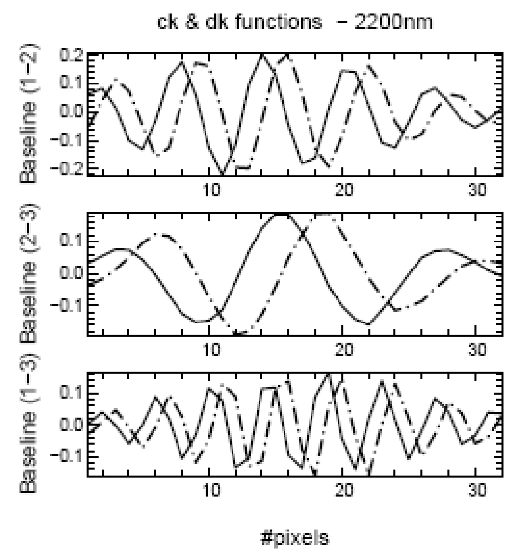
pixel data

cos,sin amplitudes

$$p_m = \sum_n W_{nm} b_n + \text{noise}$$

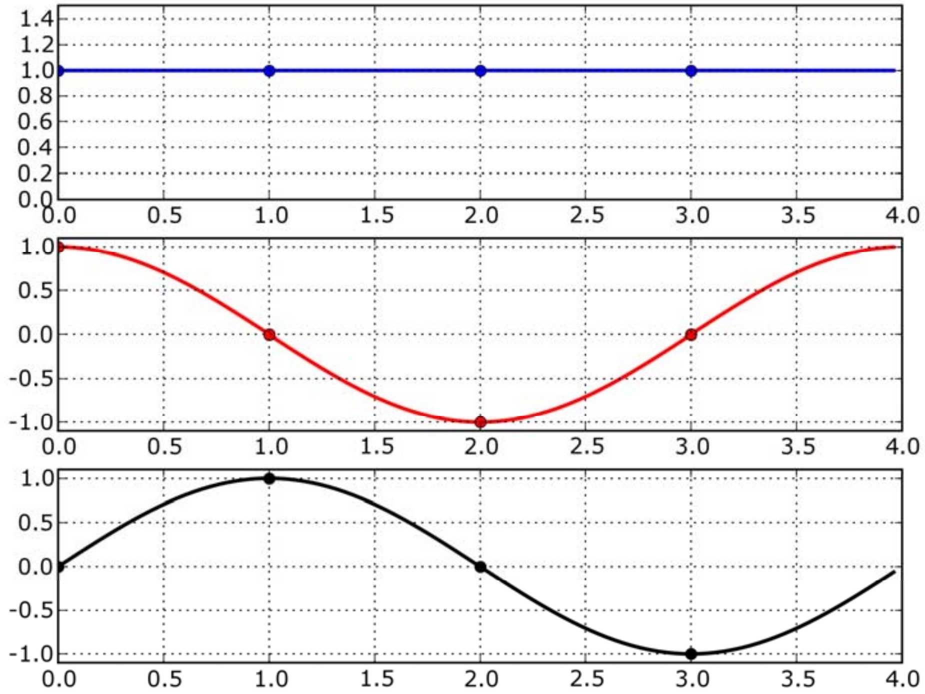
SVD inverse

$$\tilde{b}_n = \sum_m H_{mn} p_m$$



A simple example is the ABCD scenario

$$W_{nm} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$



$$H_{mn} = \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \\ 1/2 & 0 & -1/2 & 0 \\ 0 & 1/2 & 0 & -1/2 \end{bmatrix}$$

It is easy to modify the Bayesian method for different forward models

- Uneven sampling intervals
- Hot/dead pixels
- Overlapping fringe peaks

The problem becomes more complex when we include atmospheric phase perturbations

"Nuisance parameters"

Science parameters

$$\{b_1, b_2, \ldots, b_n\}$$

$$\left\{ \begin{array}{c} \{\phi_{1,1}, \phi_{2,1}, \ldots\} \\ \{\phi_{1,2}, \phi_{2,2}, \ldots\} \\ \vdots \end{array} \right\}$$

thousands ↓

We pose the problem in terms of estimating parameters immune to atmospheric piston

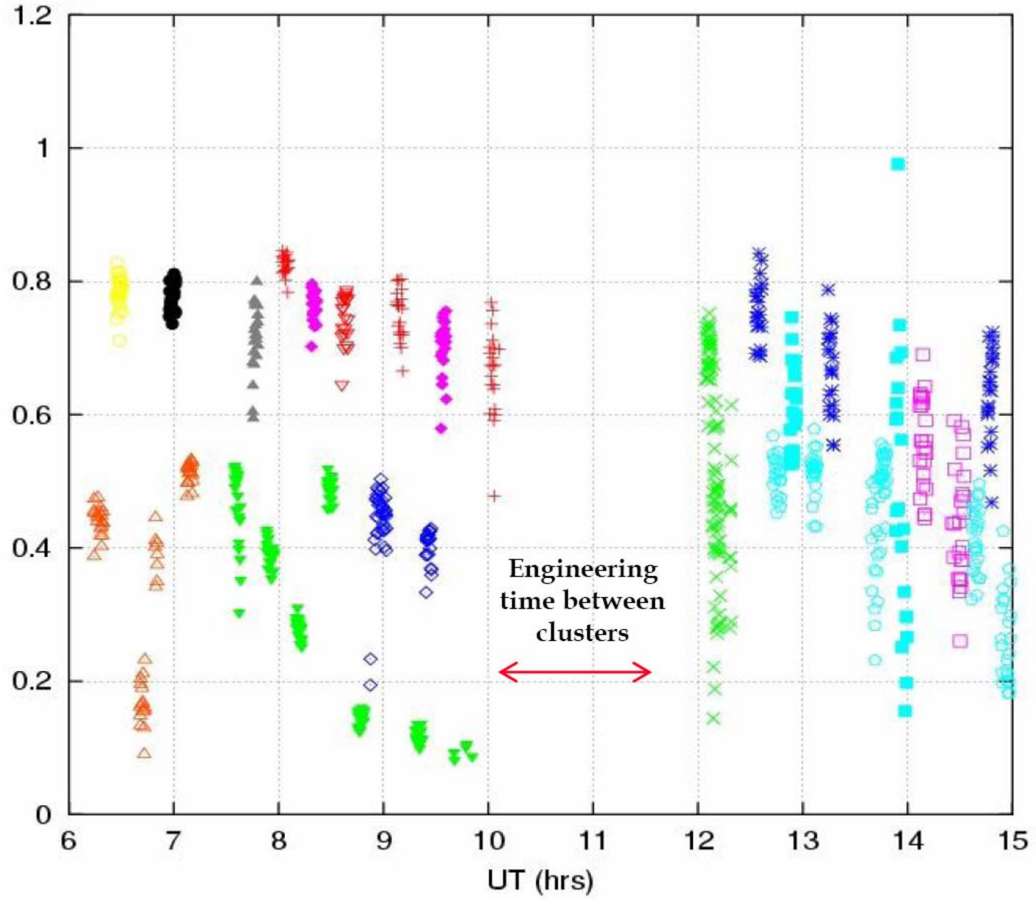
Power spectrum $\left|V^2\right|$

Bispectrum/triple product $V_{12}V_{23}V_{31}$

Differential phase/phase-referenced $V_1V_2^*$

Spectral, multi-spectral, fringe tracker, dual-star

This leads to the concept of "incoherent averaging"



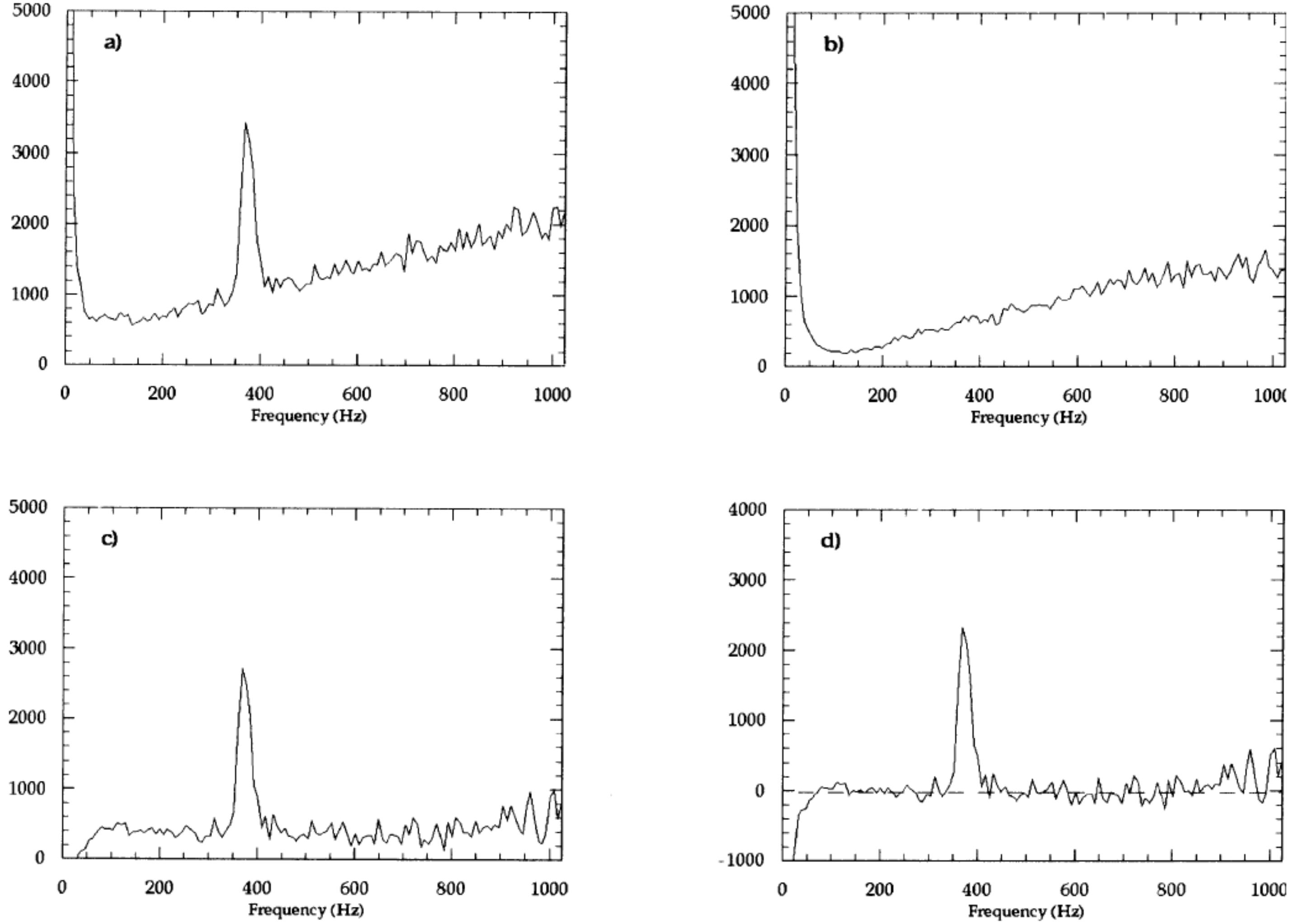
Ad-hoc combination of estimators leads to bias terms

$$\left\langle \left| \tilde{V} \right|^2 \right\rangle = \left| V \right|^2 + N$$

PHOTON NOISE
AND DFT ONLY

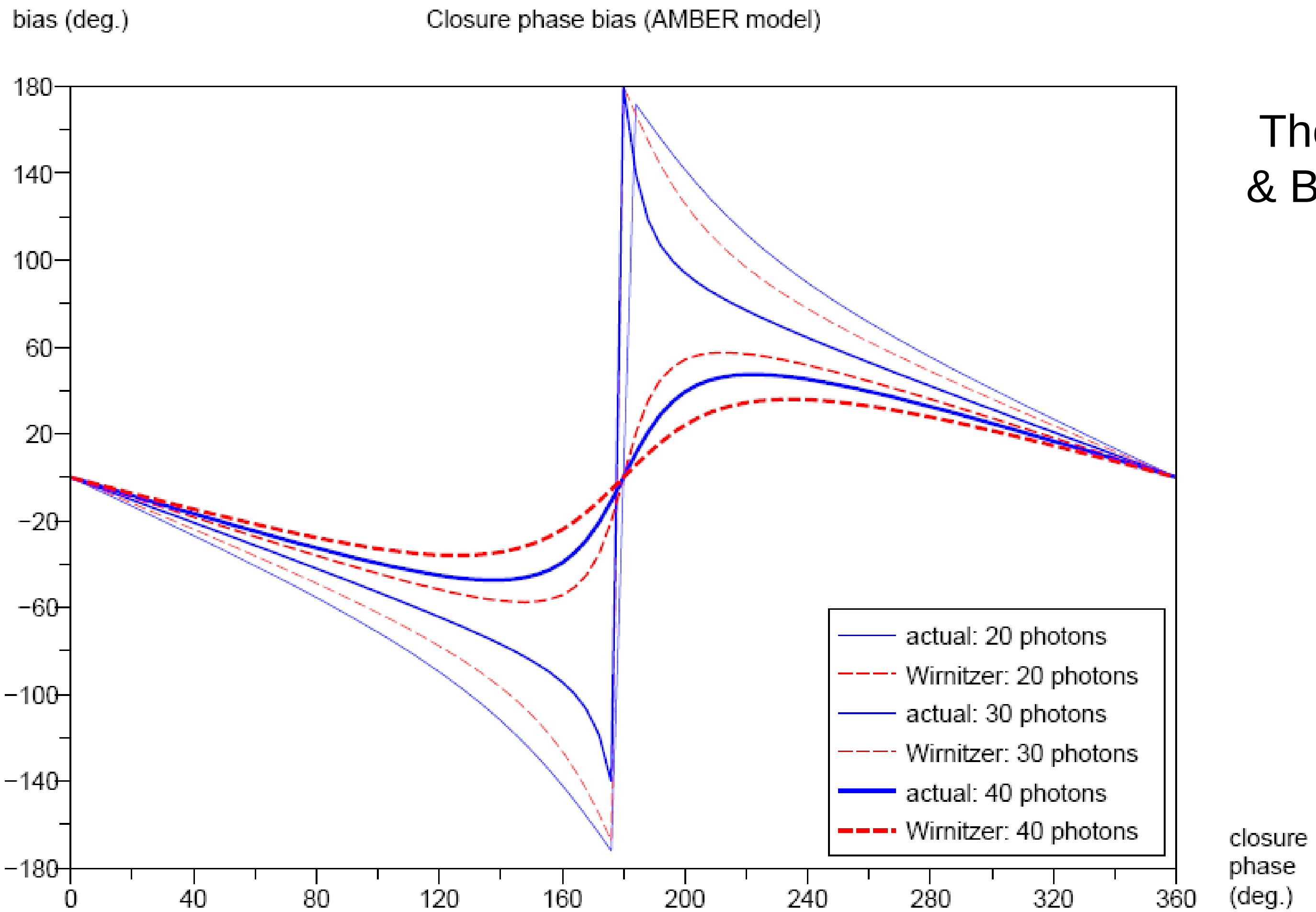
$$\left\langle \tilde{V}_{12} \tilde{V}_{23} \tilde{V}_{31} \right\rangle = V_{12} V_{23} V_{31} + \left| V_{12} \right|^2 + \left| V_{23} \right|^2 + \left| V_{31} \right|^2 - 2N$$

We compensate for biases with theoretical and empirical bias-subtraction methods



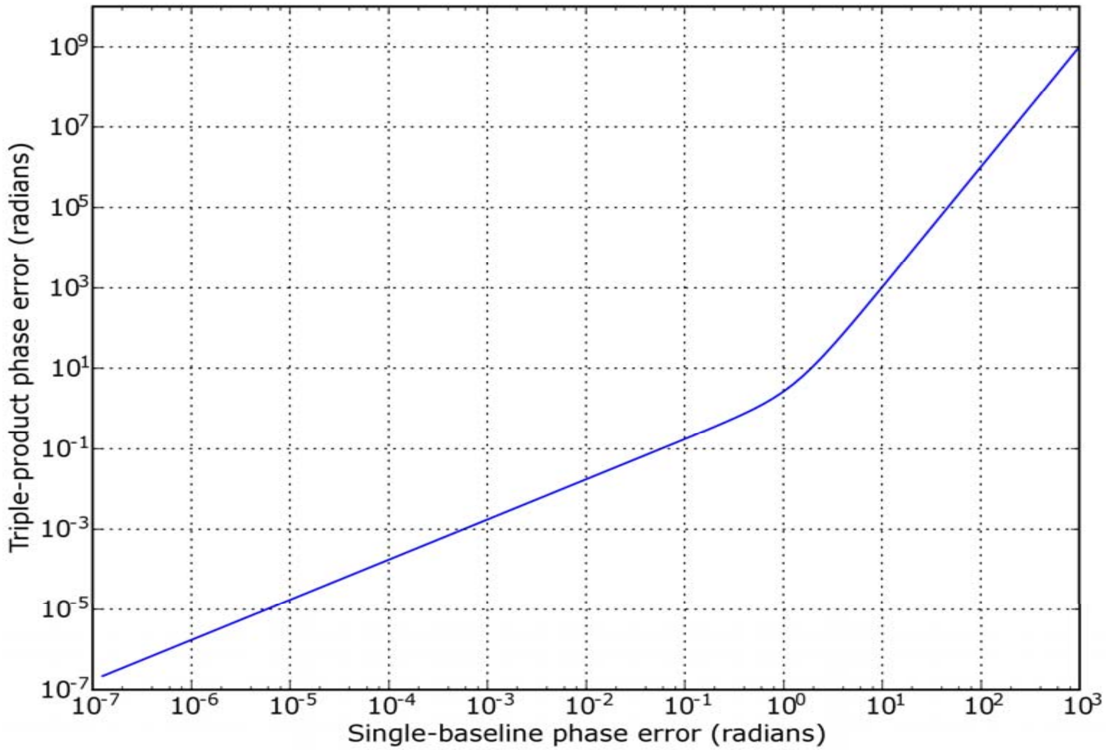
Perrin 2003

Proper treatment of bias terms is essential, especially for the bispectrum



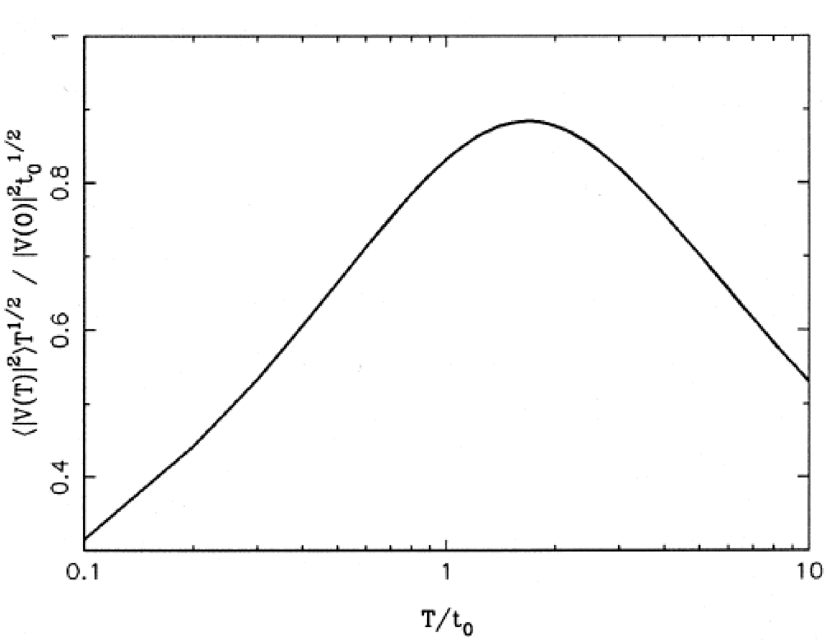
Thorsteinsson & Buscher 2004

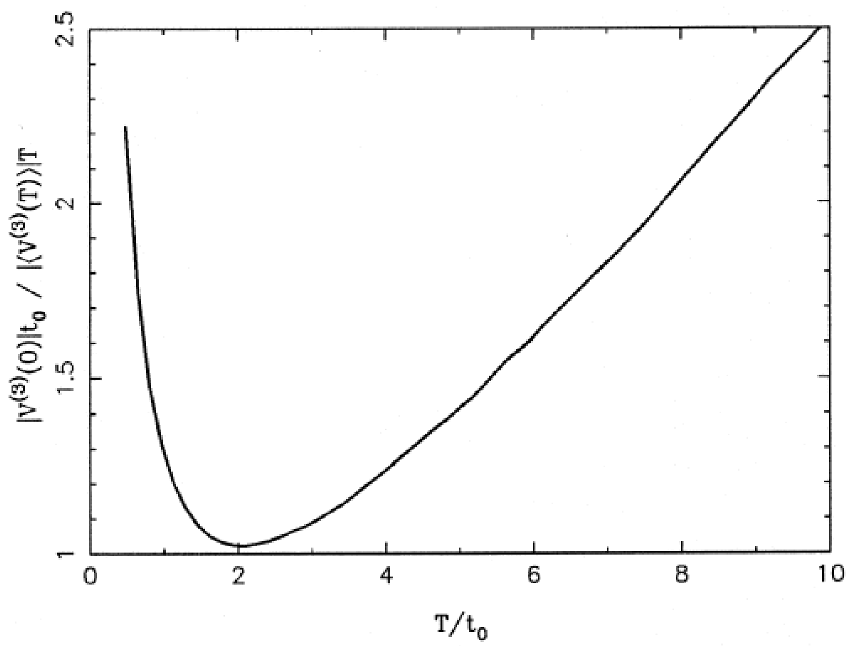
The noise on these estimators is a non-linear function of the detector noise



- For SNR>> 1 $\sigma_\theta^2(T_{123}) \cong \sigma_\theta^2(V_{12}) + \sigma_\theta^2(V_{23}) + \sigma_\theta^2(V_{31})$
- For SNR<< 1 $\sigma_\theta^2(T_{123}) \cong \sigma_\theta^2(V_{12})\sigma_\theta^2(V_{23})\sigma_\theta^2(V_{31})$

This leads to a critical choice between coherent and incoherent integration





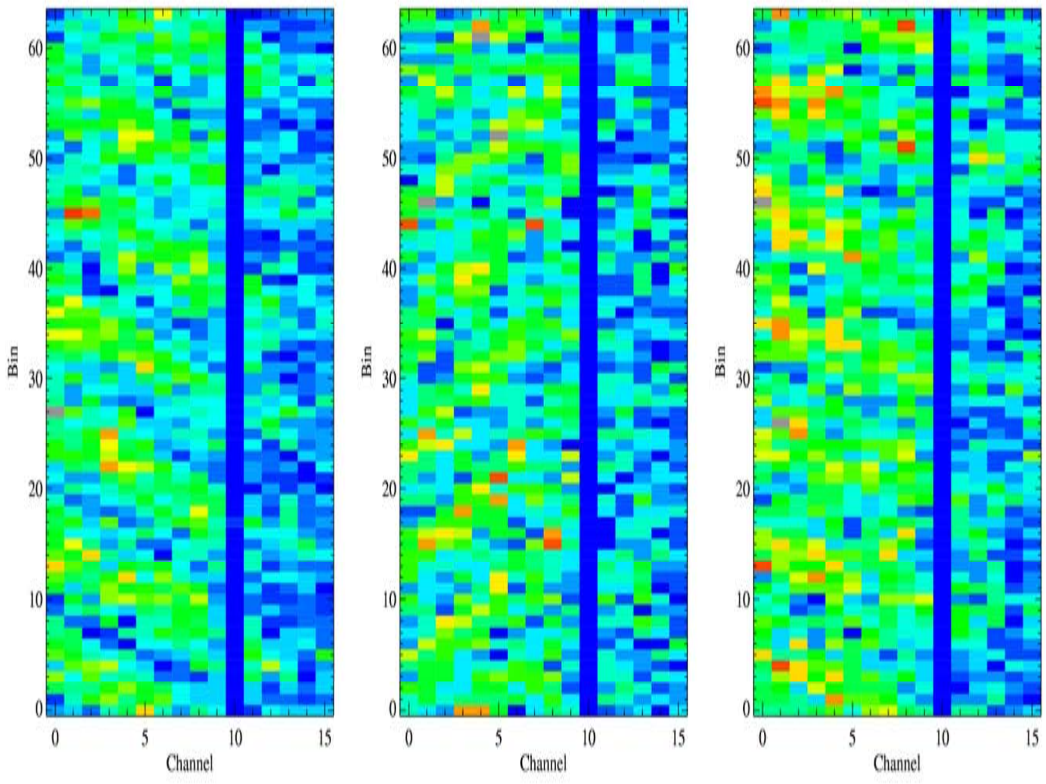
Analysis of the covariances suggests that a more global method may do better

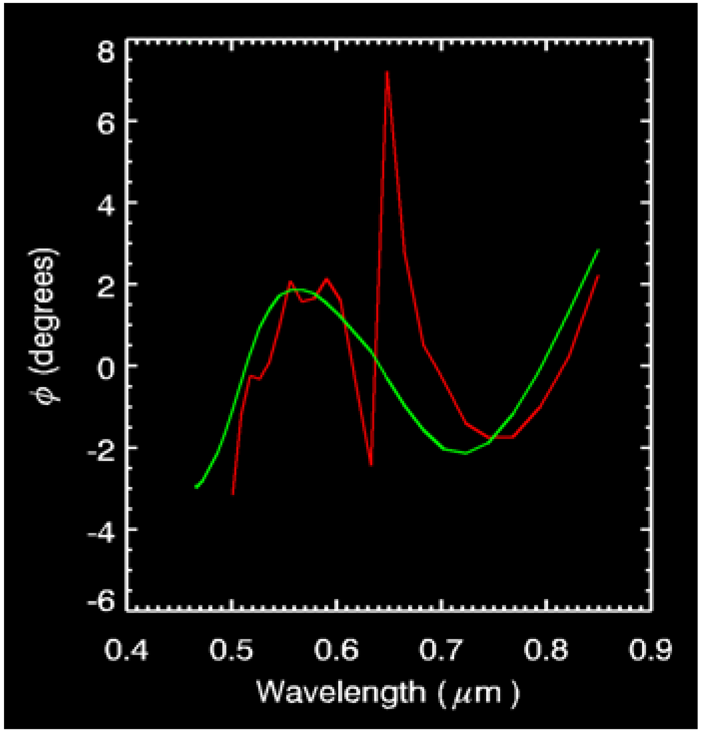
- In the high-SNR regime, the noise on triple products sharing a common baseline is correlated.
- In the low-SNR regime, the noise on all triple products is uncorrelated.

Telescopes	Atmosphere-independent phase quantities	Number of measurable triple products
4	3	4
6	10	20
10	36	120

The engineer, the mathematician and the bucket of water

An example of a more global approach is multi-wavelength fringe-fitting (Jorgensen)

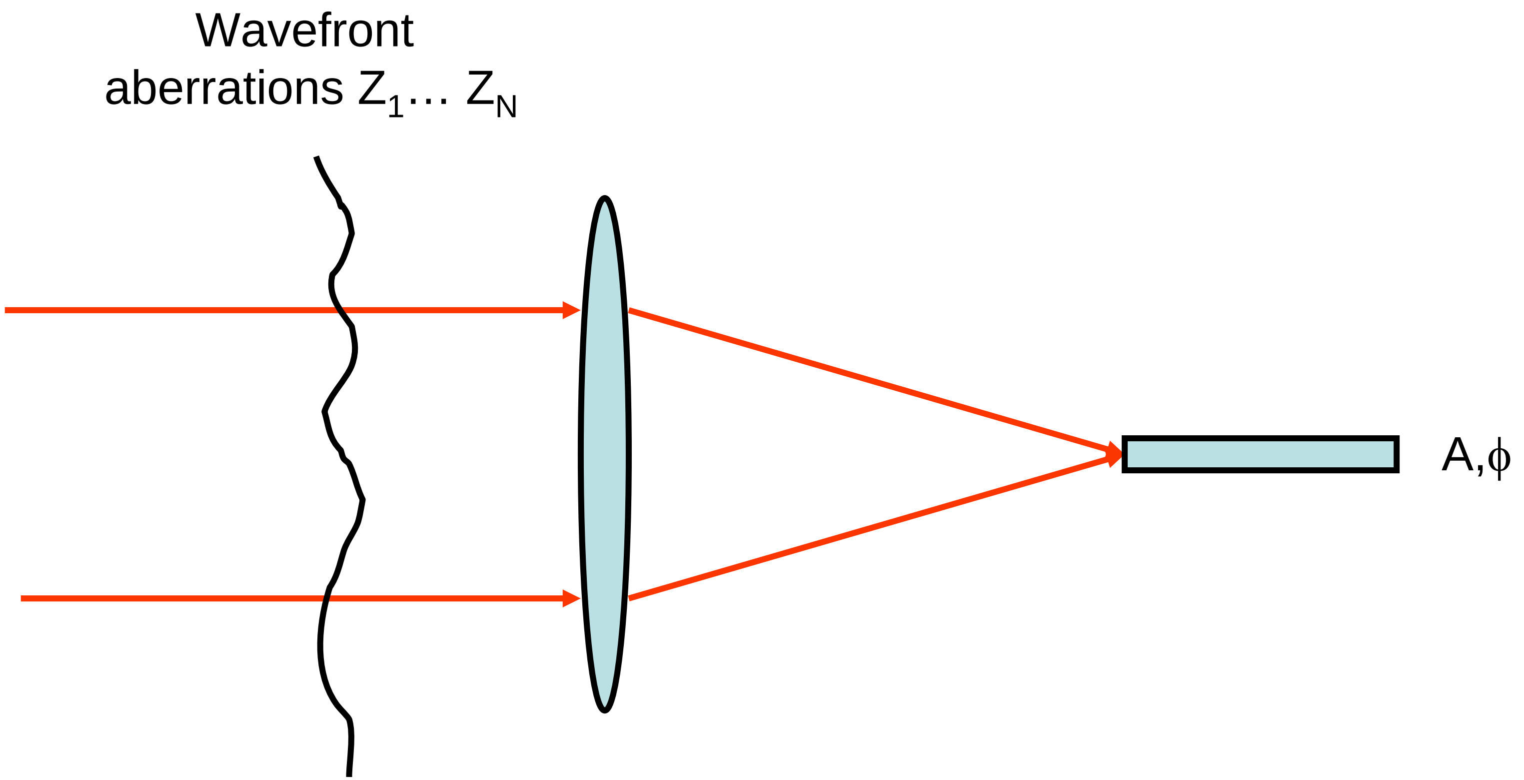




The remaining work is dealing with second-order “nuisance parameters”

- Higher-order wavefront errors
- Piston phase changes with time (fringe smearing)
- Non-ideal detector characteristics
- Spectral effects
- Polarisation effects

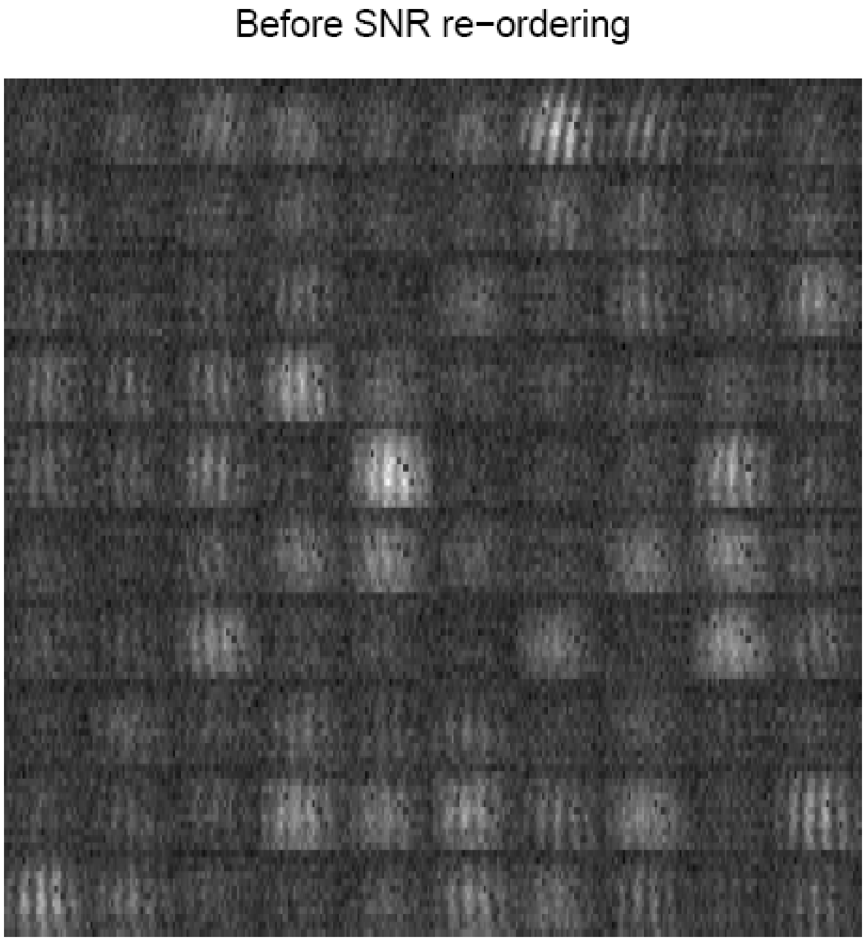
Spatial filtering can reduce the number of nuisance parameters

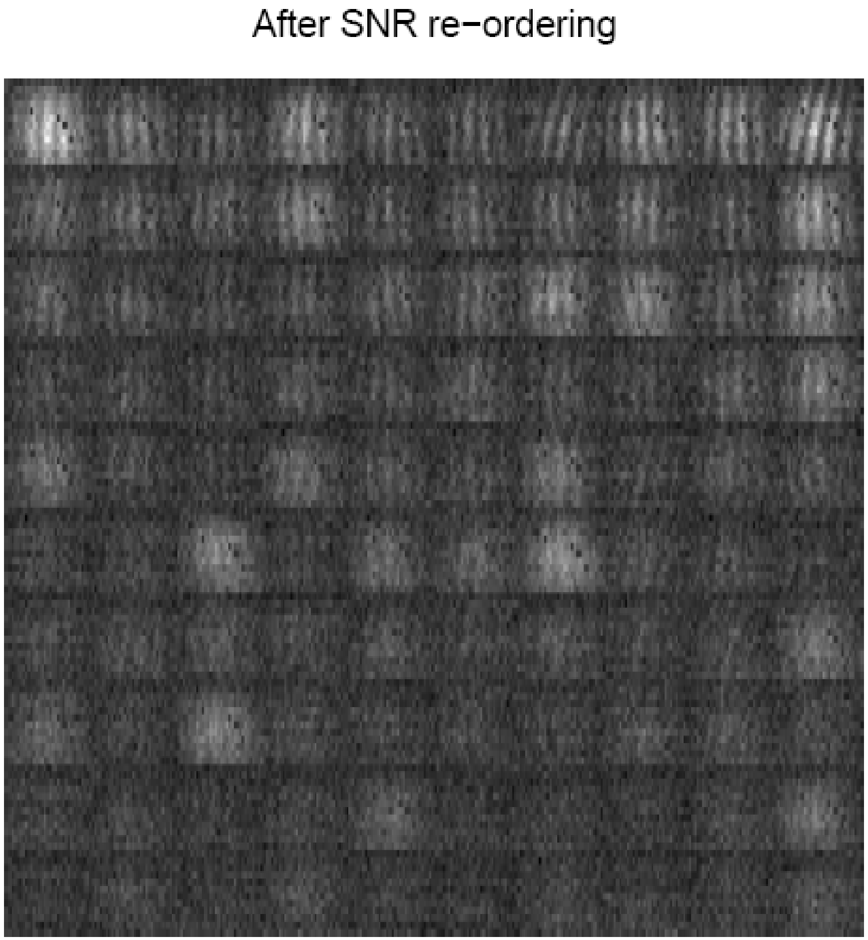


We need to make use of any auxiliary data we have to constrain the nuisance parameters

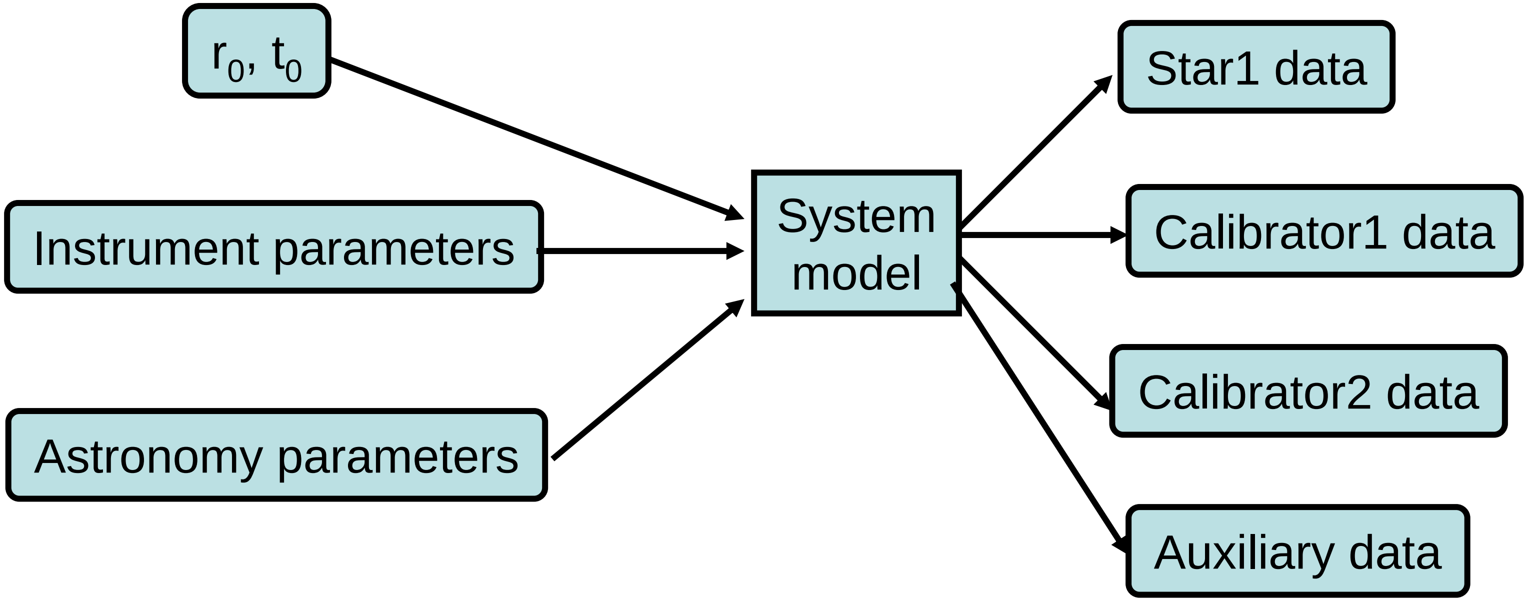
- Fringe tracker/science combiner phase jitter & scintillation
- Tip/tilt jitter
- Spatial filter photometric channels

Data selection, binning, or weighting based on auxilliary data is a powerful tool





Stating the problem in terms of a global model leads to the concept of calibration



Think globally, act locally?