Optics for Astrometry: a brief introduction

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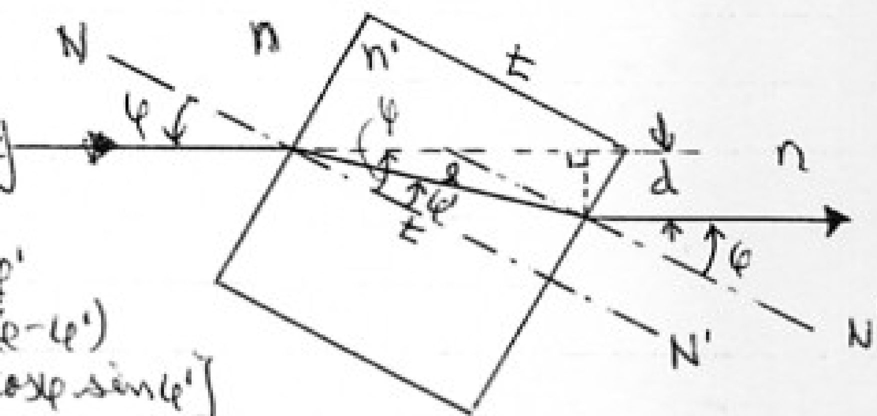
Filters and windows -1

Plane-parallel plates – eg. Filters + CCD windows

Lateral shift

① $\sin\varphi' = \frac{n}{n'}\sin\varphi$

d = displacement of ray



② $d = l\sin(\varphi-\varphi')$

③ $t/l = \cos\varphi' \Rightarrow l = \frac{t}{\cos\varphi'}$

③ into ② + expand $\sin(\varphi-\varphi')$

$$d = \frac{t}{\cos\varphi'}[\sin\varphi\cos\varphi' - \cos\varphi\sin\varphi']$$

or $$d = t\sin\varphi\left[1 - \frac{\sin\varphi'}{\sin\varphi}\cdot\frac{\cos\varphi}{\cos\varphi'}\right]$$

sub. ①

$$\boxed{d = t\sin\varphi\left[1 - \frac{n}{n'}\,\frac{\cos\varphi}{\cos\varphi'}\right]}$$

for small angles $\sin\varphi \to \varphi$; $\cos\varphi \to 1$

$$\boxed{d \simeq t\varphi\left[1 - \frac{n}{n'}\right]}$$

for air, $n \simeq 1$ and glass $n \simeq 1.5$

$d \simeq t\varphi\,[(1.5-1.0)/1.5] = t\varphi\left[\frac{.5}{1.5}\right] \simeq \boxed{t\varphi/3 = d}$ Very useful

Filters and windows - 2

<u>Focus change</u>

Diverging beam originates at Q_1
refracted at surface ①, exits at ②
Entrance $\varphi_{①}$ = exit $\varphi_{②}$

$\sin\varphi_{Q_1} = \frac{h}{s_1}$; $\sin\varphi_{Q_1'} = \frac{h}{s_1'}$

$\frac{\sin\varphi_{Q_1'}}{\sin\varphi_{Q_1}} = \frac{h/s_1'}{h/s_1} = \frac{s_1}{s_1'}$

but $\varphi_{Q_1'}$ is the refracted ray of φ_{Q_1}, so $\frac{\sin\varphi_{Q_1'}}{\sin\varphi_{Q_1}} = \frac{n}{n'} \Rightarrow \frac{s_1}{s_1'} = \frac{n}{n'}$

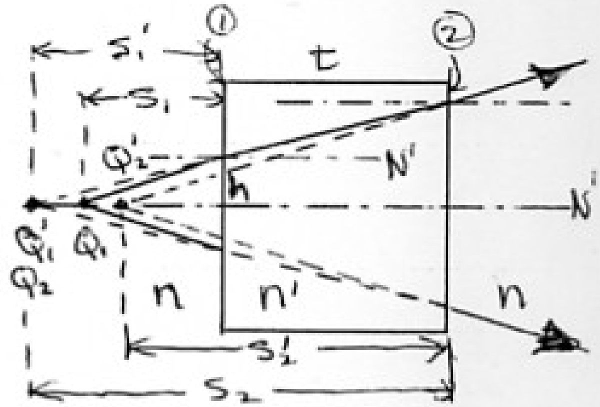
Surface #2: $s_2 = s_1' + t = s_1\frac{n'}{n} + t$

By analogy, $s_2' = s_2\frac{n}{n'} = \left[s_1\frac{n'}{n} + t\right]\frac{n}{n'} = s_1 + t\frac{n}{n'}$

The focus change is: $Q_1 - Q_2' = \Delta f = s_1 - \left[\left(s_1 + t\frac{n}{n'}\right) - t\right]$

or, $\boxed{\Delta f = t\left(1 - \frac{n}{n'}\right)}$ or for glass ($n = 1.5$) + air $\boxed{\Delta f \simeq t/3}$

The focus is moved away from the objective.



Filter flatness

Filter flatness required for Astrometry (van Altena + Monnier) (1968, AJ 73, 649)

Snell's law: $\frac{\sin\theta}{\sin\theta'} = \frac{n'}{n}$

$\theta << 1 \rightarrow \theta' \simeq \frac{n}{n'}\theta$

Final image/ray position

$x_f \simeq t\theta' + d\theta''$

$$\boxed{x_f \simeq t\frac{n}{n'}\theta + d\frac{n'}{n''}\theta'}$$

Positional error at f

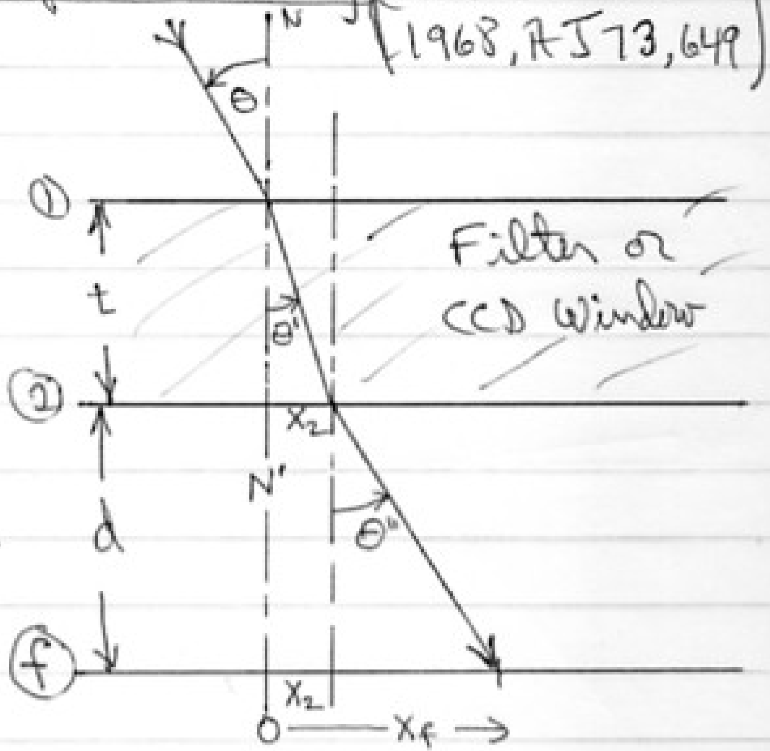
$\sigma_{x_f}^2 \simeq \left(t\frac{n}{n'}\right)^2\sigma_\theta^2 + \left(d\frac{n'}{n''}\right)^2\sigma_{\theta'}^2$

$n = n'' = 1.0$

$\sigma_{x_f} \simeq \sigma_\theta\left[\left(t/n'\right)^2 + \left(dn'\right)^2\right]^{1/2}$

$t << d$

$$\boxed{\sigma_{x_f} \simeq dn'\sigma_\theta}$$



Filter flatness

Example: $n' = 1.5$ ← 1/50th pixel

OPTIC camera $\sigma_{x_f} \simeq \pm 0.25\,\mu m \rightarrow d\sigma_\theta \leq 0.17 \times 10^{-3}$

R-band $\lambda \simeq 0.8\,\mu m$ 1 wave/inch $= 3.15 \times 10^{-5}$ rad.

d (mm)	σ_θ (rad)	σ_θ ''	σ_θ (wave/inch)
10	0.17×10^{-4}	3''.5	0.53
50	0.34×10^{-5}	0''.7	0.11
100	0.17×10^{-5}	0''.35	0.05
150	0.11×10^{-5}	0''.23	0.03

Summary:

① Filter/window flatness tolerances are VERY tight!

② Place the best/flattest surface toward the detector

The Seidel Aberrations

Aberration

If the image distances and ~~directions~~ intersections of the rays ~~all~~ coincide for all zones in the optics, they are free of aberrations.

The deviation of the Gaussian formulae from providing an exact representation of the ray tracing is represented by the "Third-Order Theory" and its difference from the Gaussian "First-Order Theory" with paraxial rays.

The difference, 3rd − 1st order, is characterized by the "Seidel sums" which represent the different aberrations of chromatic light.

Seidel sums:

S_1 = spherical aberration
S_2 = coma
S_3 = astigmatism
S_4 = curvature of the field
S_5 = optical field angle distortion
S_6 = chromatic aberration

First-order theory

Of the 6 basic formulae, 3 contain sines

<u>1st order theory</u> $\sin\theta \rightarrow \theta$

$$\boxed{\frac{n}{s}+\frac{n'}{s'}=\frac{n'-n}{r}}$$

the Gaussian formula from the 1st-order theory

<u>3rd order theory</u>

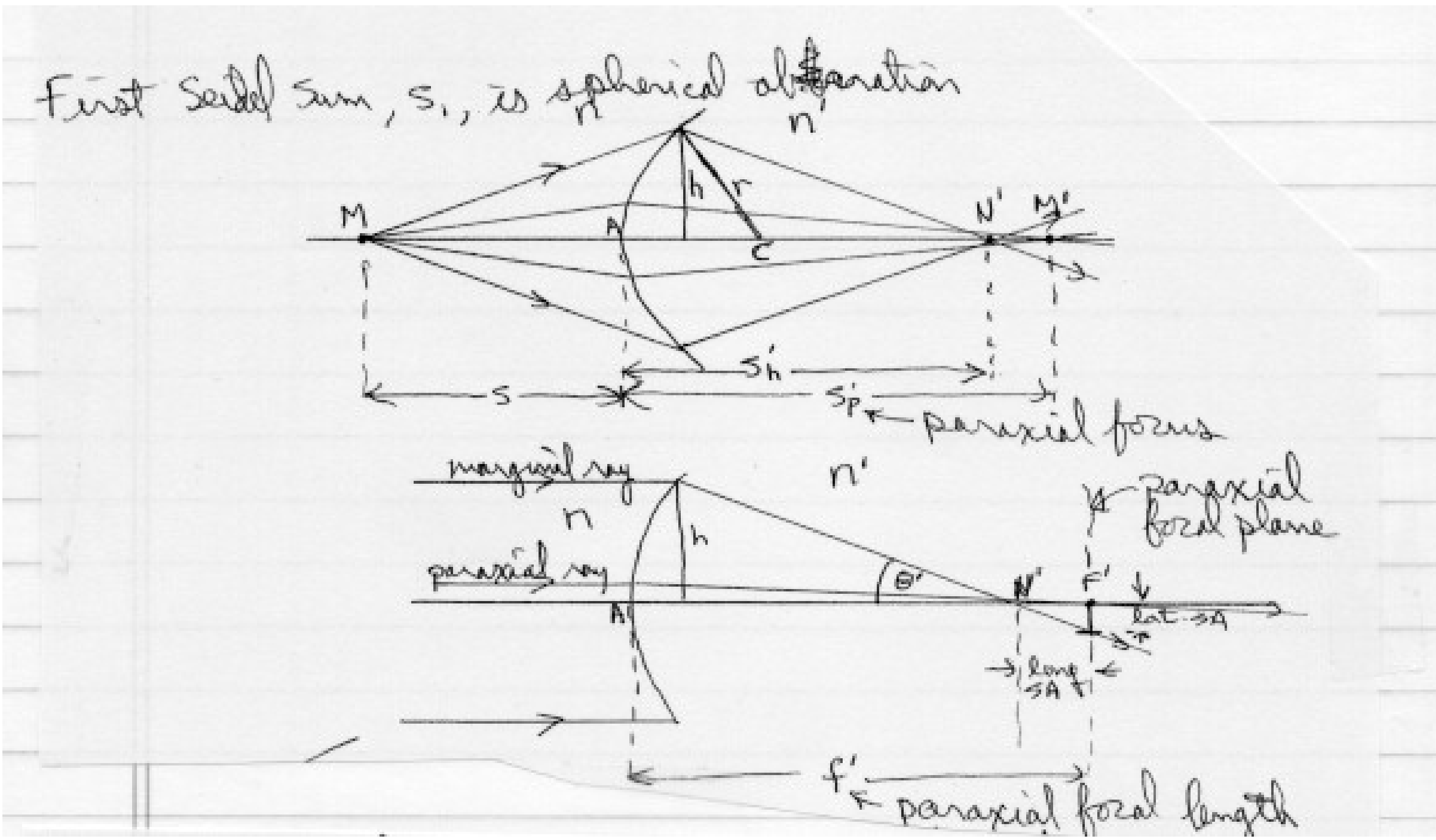
$$\sin\theta \rightarrow \theta - \theta^3/3!$$

$$\frac{n}{s}+\frac{n'}{s'_h}=\frac{n'-n}{r}+\underbrace{\left[\left(\frac{h^2n^2r}{2f'n'}\right)\left(\frac{1}{s}+\frac{1}{r}\right)^2\left(\frac{1}{r}+\frac{n'-n}{ns}\right)\right]}$$

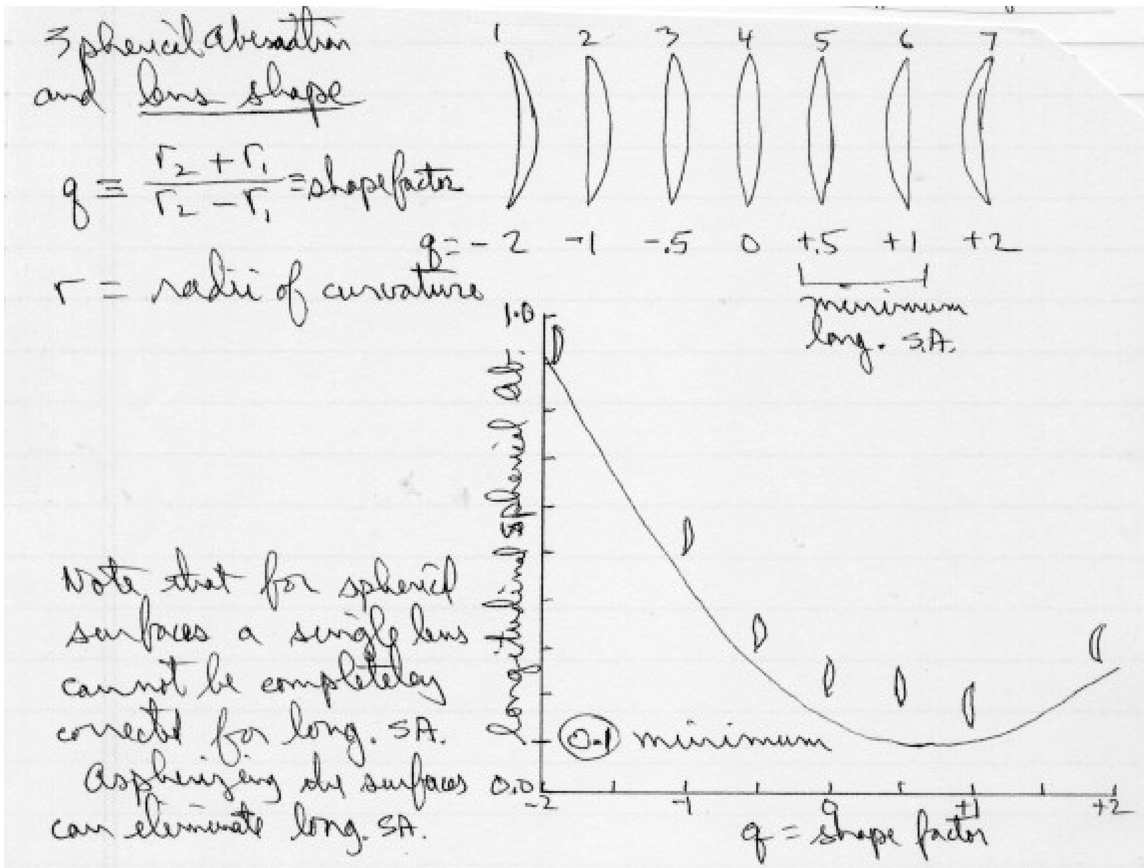
s'_h ≡ image distance for ~~paraxial~~ rays at a height h at lens.

[ ] is the deviation of the 1st-order theory from the 3rd-~~theory~~ order-theory $\underline{\propto h^2}$ height of ray

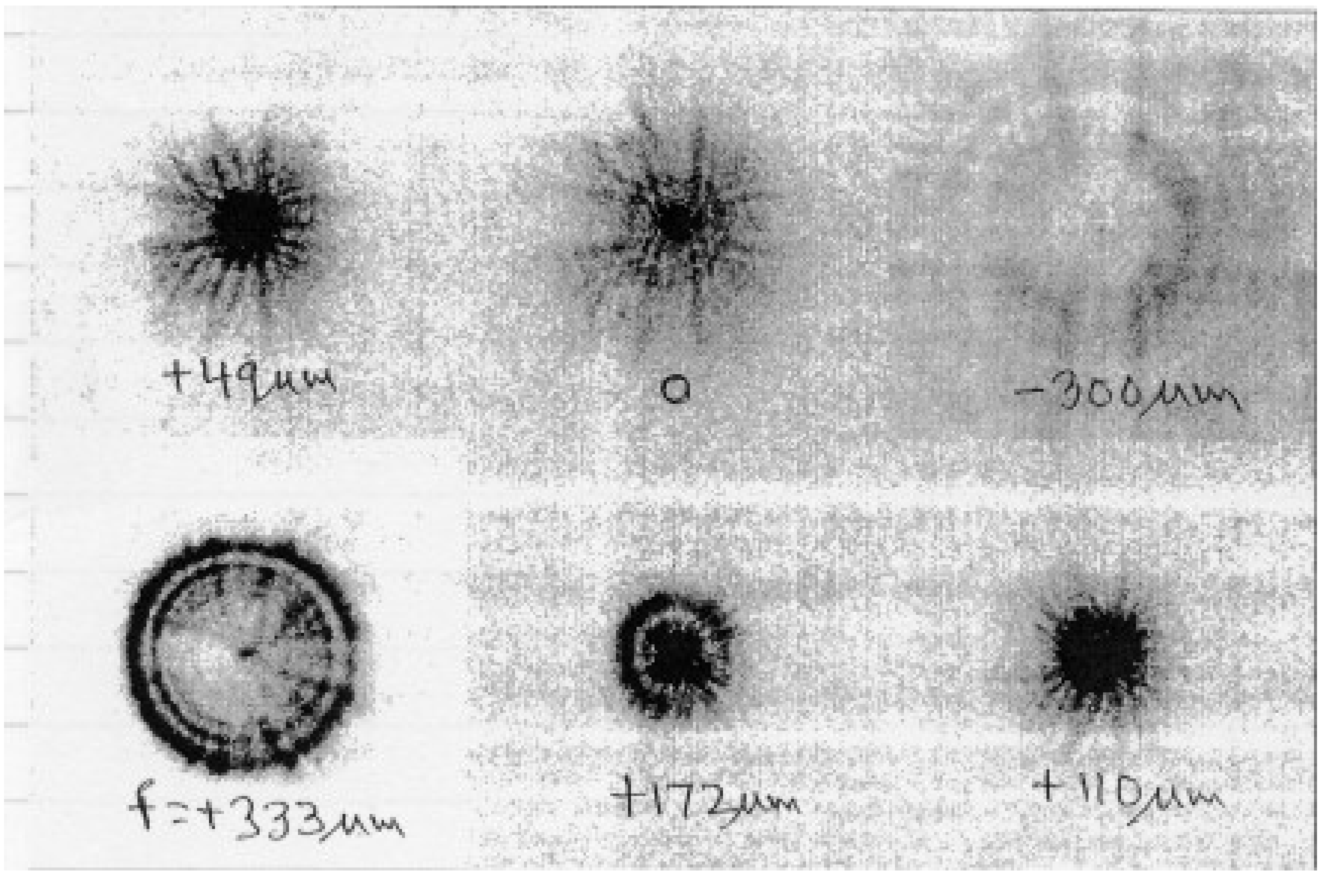
S1 - Spherical aberration



S1 and lens shape



S1 and the HST



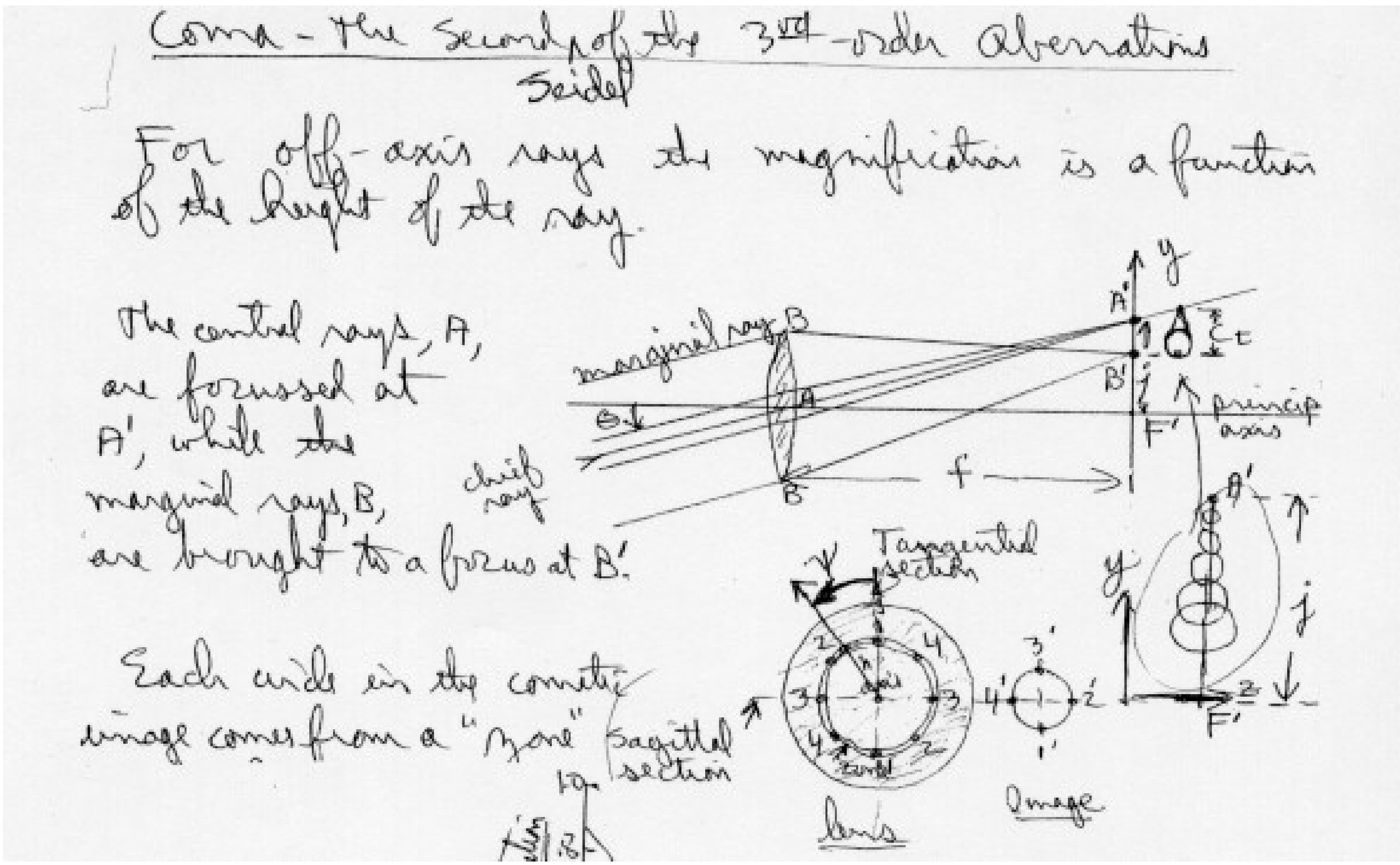
S1 and astrometry

Astrometric consequences of Spherical aberration

PSF varies over field-of-view
photometry very difficult
gradient in PSF decreased ∴ image
centering precision is lower

Focus ill defined
Plate scale not constant

S2 - Coma

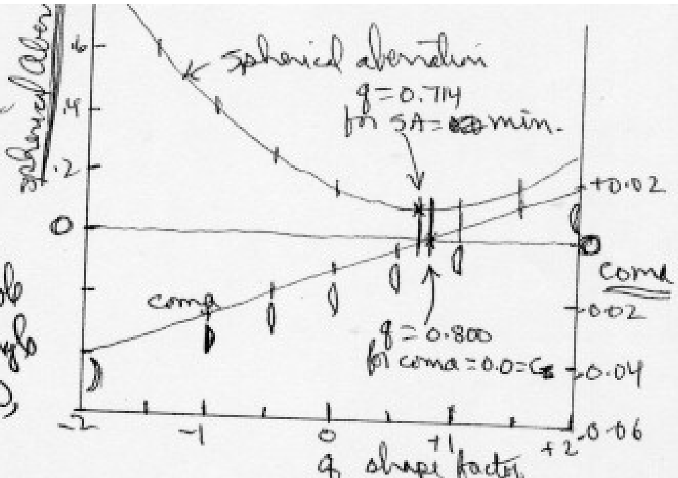

Coma - the second of the 3rd-order aberrations (Seidel)

For off-axis rays the magnification is a function of the height of the ray.

The central rays, A, are focussed at A', while the marginal rays, B, are brought to a focus at B'.

Each circle in the comatic image comes from a "zone".

S2 - minimizing coma



Since Coma crosses the axis, it is possible to design a simple single lens with coma = 0.0, but SA $\neq$ 0

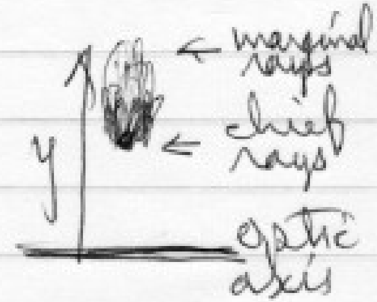
Note that for coma = C_s = 0.0 the shape factor = 0.80 which is essentially identical to that for a minimum of SA where q = 0.714, so a plano-convex lens for parallel (incoming) light is an excellent choice.

<u>Aplanatic</u> — a lens that is free of both spherical aberration <u>and</u> coma

S2 and astrometry

<u>Astrometric consequences of Coma</u>

Image becomes more deviant from chief ray position as you approach the marginal rays.



↳ position of image $\propto$ distance from optical axis

a scale change

Brighter stars "fill out" distorted image

↳ position of image $\propto$ brightness of image

a scale change

coma $\propto$ brightness $\times$ coordinate $m \cdot x$ and $m \cdot y$

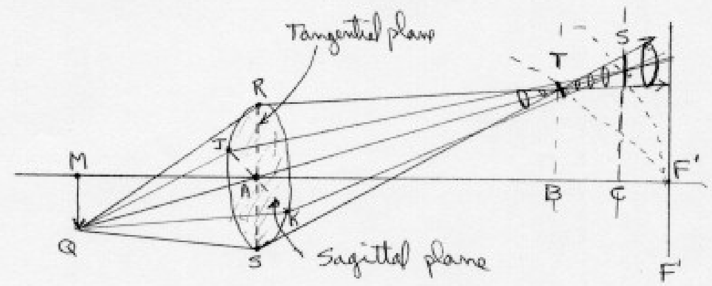
caution: highly correlated with scale + mag eqn.

S3 - Astigmatism

<u>Astigmatism</u> – the third Seidel-Sum, S_3 (39)

Take a lens with $SA \doteq coma \doteq 0$ for paraxial rays.

Failure of the rays from an object lying some distance from the optical axis to yield a point image.



Take the image plane at **B**, where the rays from R + S intersect at a point. Since the rays from J + K and the points inbetween do not intersect until the image plane at C, the image will be blurred into a line instead of a point. Likewise with the image plane at C, where the rays from J + K intersect, those from R + S + inbetween spread out into a line S; this is the sagittal focus.

S3 and image shape

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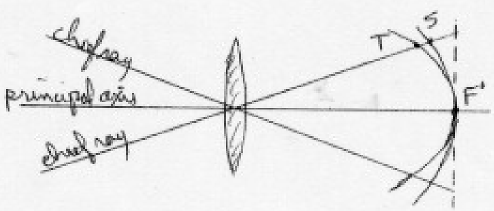
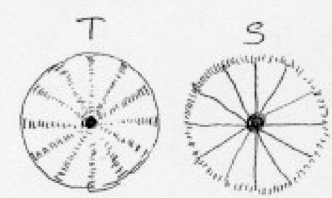




Astigmatic images of a spoked wheel

N.B. Lens shape is relatively unimportant to the amount of astigmatism, but the longer the focal length, f, the smaller the astigmatism.

S3 and astrometry

Astrometric consequences of astigmatism

Images are elliptical, but with no positional change.

Tang

Sag

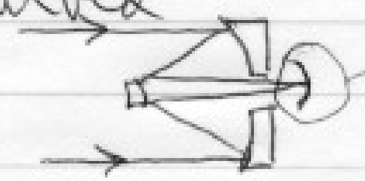
Field is curved

↳ Field Flattener required

↳ Introduces Optical Field Angle Distortion

Astigmatism is the dominant aberration in our R-C

∴ Field is curved



S4 - Field Curvature

J.P.

4th Seidel aberration is Curvature of the Field

and use of a field flattener introduces the 5th Seidel aberration: OFAD

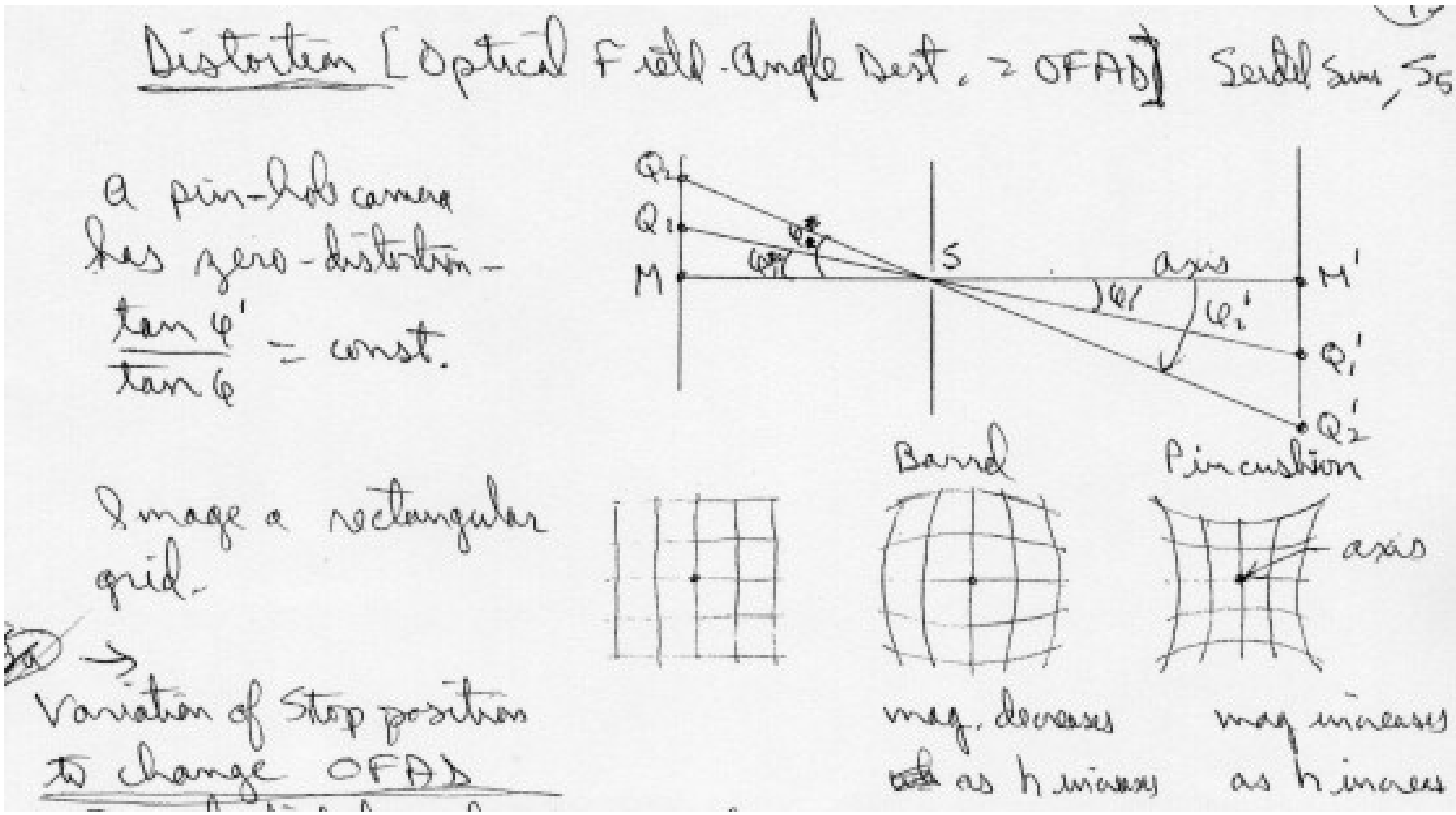
$$\text{OFAD} \propto d_1 r^3 + d_2 r^5$$

$$\text{or} \propto d_1\left[\sqrt{(x-x_c)^2+(y-y_c)^2}\right]^3 + d_2 \cdots$$

an error in the OFAD center (x_c, y_c) introduces quadratic terms

$$\Delta(\text{OFAD}) \propto 3d_1 r^2 + \text{hot}$$

S5 - OFAD


Distortion [Optical Field-Angle Dist. = OFAD] Seidel Sum, S_5
a pin-hole camera has zero-distortion –
$\frac{\tan \varphi'}{\tan \varphi}$ = const.
Q_2
Q_1
M
S
axis
M'
φ_1'
Q_1'
Q_2'
Image a rectangular grid.
Barrel
Pincushion
axis
→ Variation of Stop position to change OFAD
mag. decreases as h increases
mag increases as h increases

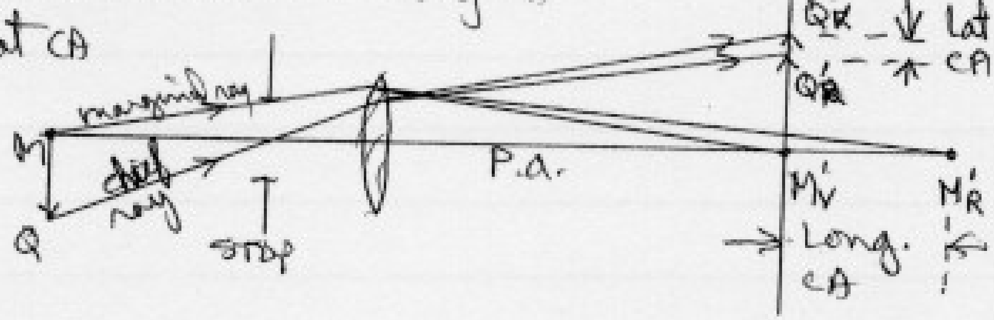
S6 - Chromatic aberration

Chromatic Aberration

Longitudinal chromatic aberration = Long CA

Latitudinal CA = Lat CA

scale = $f(\lambda)$



Long CA = $M'_V M'_R$

Lat CA = $Q'_V Q_R$

Lateral CA $\propto$ X and $\propto$ color index

$\propto X \cdot CI$

Longitudinal CA not too important for astrometry

Reflecting telescopes

Schroeder
Astron. Optics Ch 6

(after p.55)

(R1)

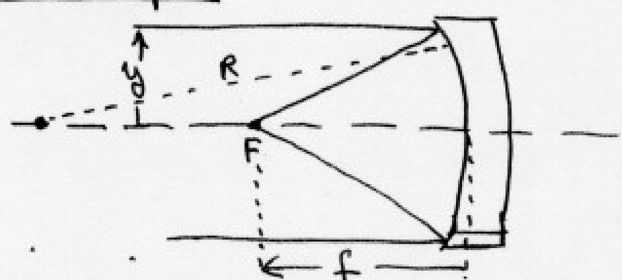
Reflecting Telescopes –

focal ratio

$F = f/2y$

$f = R/2$

$\therefore F = R/4y$



Conic Sections (Sch. Sect. 3.5)

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} = \frac{2}{R}$ paraxial rays

or $(s+s')/ss' = 2/R$

b) or $ss'/(s+s') = R/2$

Take an ellipse $e \equiv \frac{c}{a}$

$c^2 = a^2 - b^2$

$1 - e^2 = 1 - \frac{c^2}{a^2}$

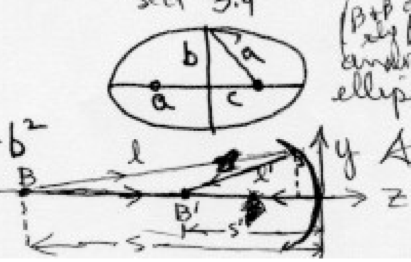
From ellipse analogy $\ell + \ell' = s + s' = 2a$

Ellipse centered at a

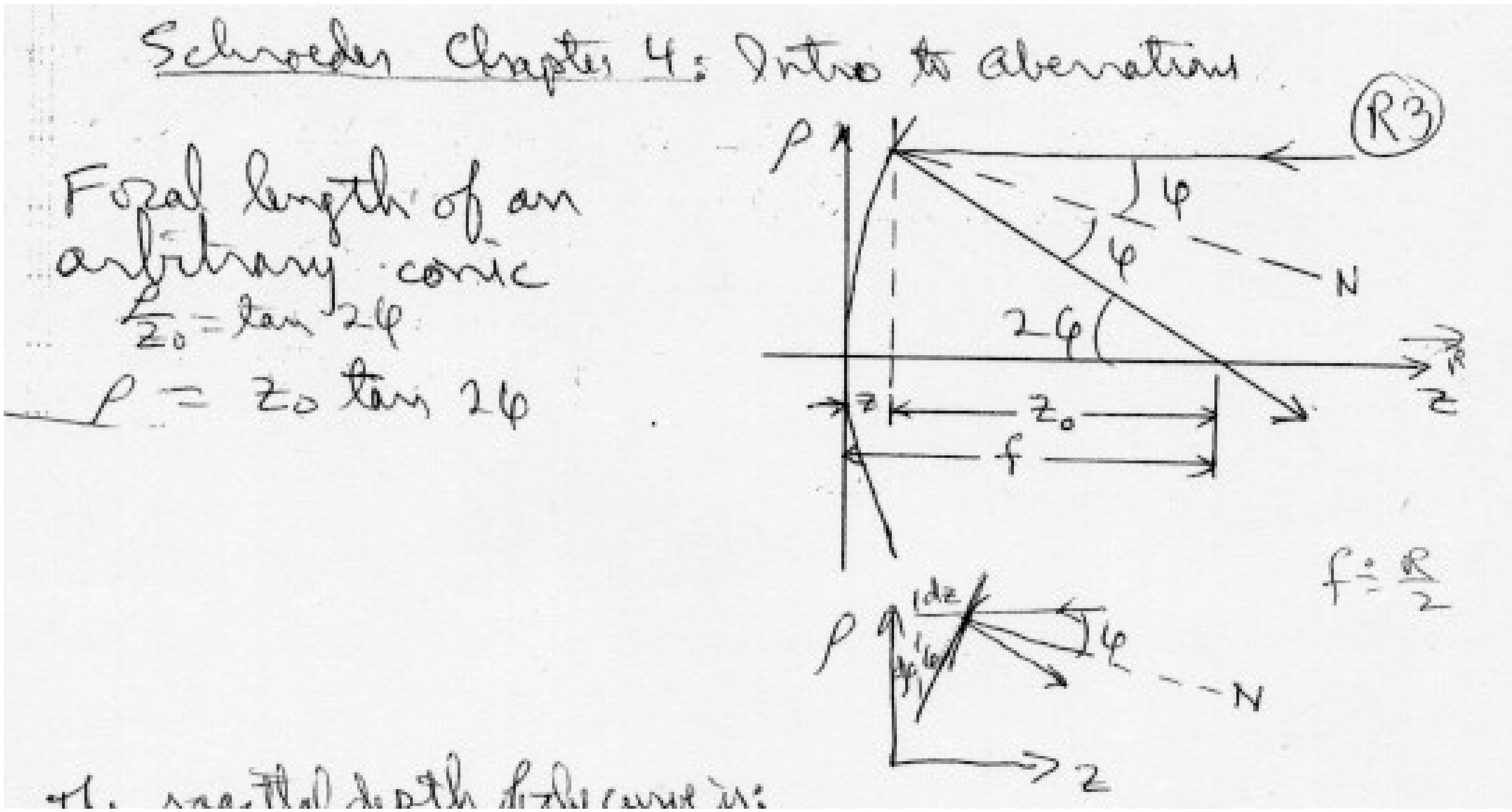
$\frac{(z-a)^2}{a^2} + \frac{y^2}{b^2} = 1$

Sect 3.4

(B + B' are [illegible] foci) analogy to ellipse



Aberrations in reflectors



The sagittal depth

The sagittal depth of the curve is:

$$\boxed{z \simeq \frac{\rho^2}{2R} + \frac{\rho^4}{8R^3}(1+K) + \frac{\rho^6}{16R^5}(1+K)^2}$$

$$\boxed{f \doteq \frac{R}{2} - \frac{(1+K)\rho^2}{4R} - \frac{(1+K)(3+K)\rho^4}{16R^3} - \dots}$$

$\rho = 0$ (paraxial) $\rightarrow$ $f = \frac{R}{2}$ as before!

$$\Delta f = f(\rho) - f(\text{paraxial})$$

$$= -\frac{(1+K)\rho^2}{4R} - \frac{(1+K)(3+K)\rho^4}{16R^3} - \dots$$

$\Delta f = f(\rho^2)$ so blur is symmetric about optical axis

Paraboloid $(K = -1) \rightarrow \Delta f = 0$ as before + all others have blur!

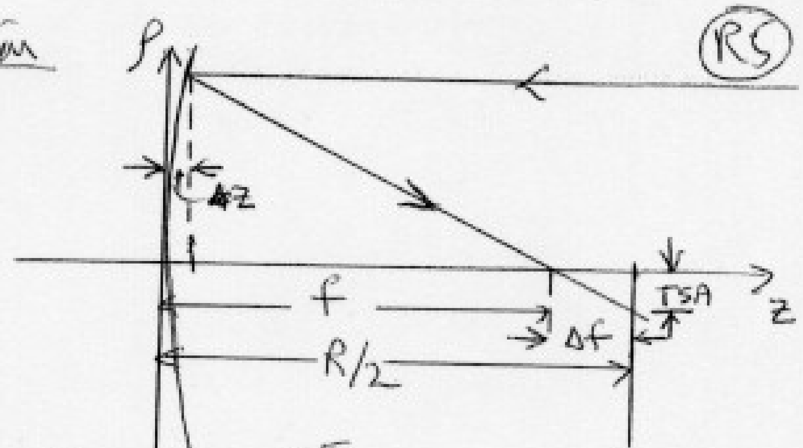
Spherical aberration

Spherical Aberration

RS

similar Δs

$$\frac{TSA}{\Delta f} = \frac{\rho}{f - z}$$



sub. Δf, f, z
+ expand giving

$$TSA \doteq \underbrace{-(1+K)\frac{\rho^3}{2R^2}}_{3^{rd}\text{-order } TSA} - \underbrace{3(1+K)(3+K)\frac{\rho^5}{8R^4}}_{5^{th}\text{-order } TSA} + \cdots$$

Paraboloid ($K = -1$) $\rightarrow$ $TSA = 0$

Sphere ($K = 0$) $\rightarrow$ $TSA = -\frac{\rho^3}{2R^2} - \frac{9\rho^5}{8R^4} + \cdots$

Hyperboloid ($K < -1$) $\rightarrow$ TSA = positive, so $f > R/2$

Higher-order aberrations

angular aberration (R6)

$$AA = \underbrace{3a_1 \frac{y^2\theta}{R^2}}_{\text{coma}} + \underbrace{2a_2 \frac{y\theta^2}{R}}_{\text{astigmatism}} + \underbrace{a_3\theta^3}_{\text{distortion (OFAD)}}$$

SA missing since we used a paraboloid

$$SA = -(1+K)\frac{y^3}{2R^2}$$

So, the 3rd order aberrations are of the form

$$\boxed{\text{aberration} \propto y^n \theta^m}, \text{ where } n+m=3$$

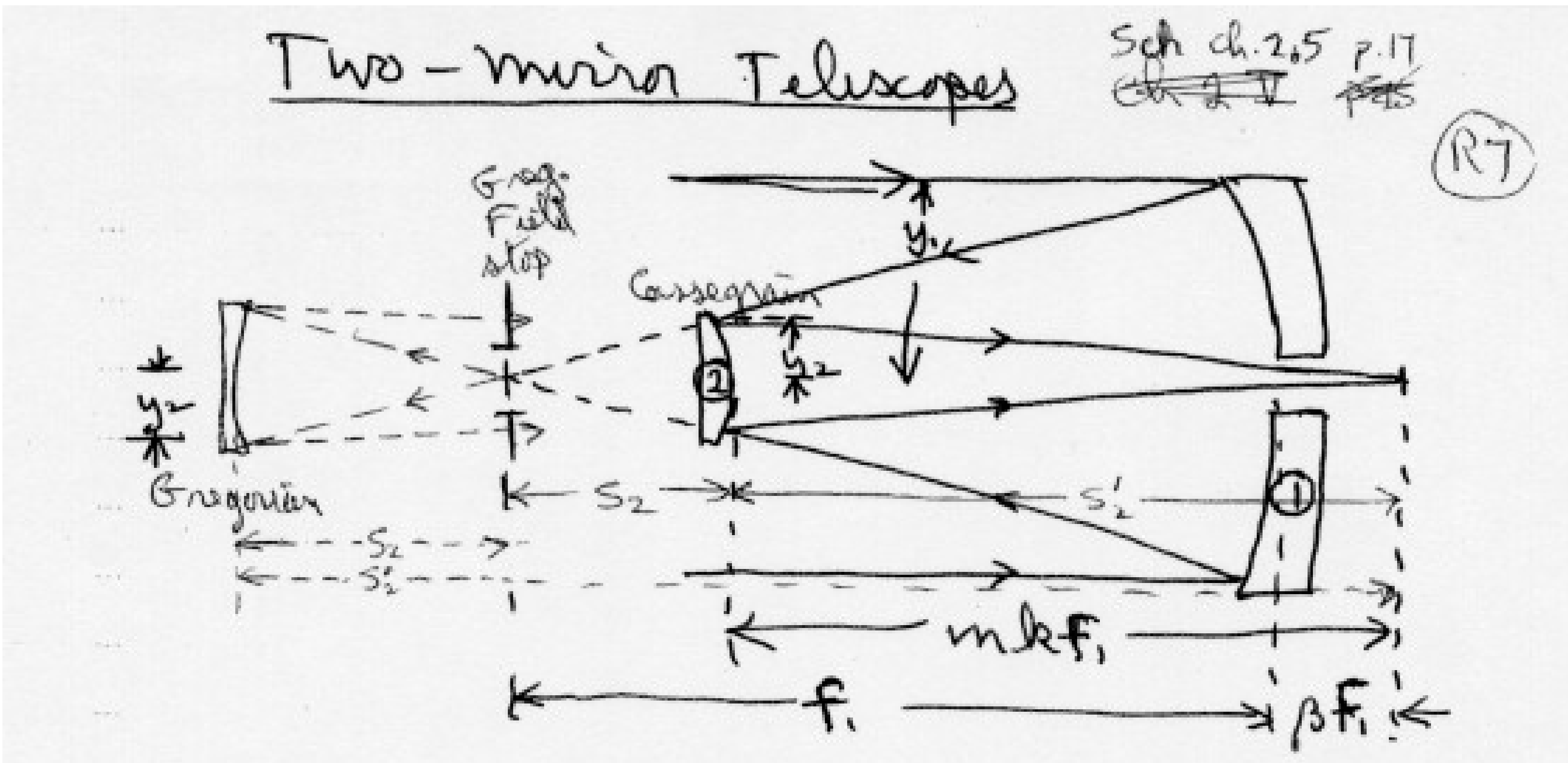
Note: $SA \neq f(\theta)$

$OFAD \neq f(y)$ ∴ image quality is unaffected

Curvature of the field

Note that we rotated the paraboloid around C, or conversely about O, so that C (or F) describes a circle, or curved focal plane.

Two-mirror telescopes



Normalized parameters

Table 6.3 **Normalized Parameters**

Ray heights on M1 + M2 (a) $k = y_2/y_1$ marginal rays

Ratio of vertex radii of curvature (b) $\rho = R_2/R_1$

Ratio of focal lengths. or f/ratios (c) $m = f/f_1 = F/F_1 = -s_2'/s_2$ (the secondary mag.)

mkf_1 = distance from secondary to focal surface

Critical parameter for instrument design is the BFD

βf_1 = back focal distance (BFD)

Calculation sequence

a + b → (1) ; a + c → (2) + (3)

Relationships between parameters,

(1) $m = \frac{\rho}{\rho - k}$; (2) $\rho = \frac{mk}{m-1}$ p.17

$k = \frac{\rho(m-1)}{m}$; $k = \frac{1+\beta}{m+1}$; (3) $\beta = k(m+1) - 1$

Reflector types

Table 6.4 p. 116

m	k	ρ	Type	Secondary	Example
>1	>0	>0	Cassegrain	Convex	MW 1.5, 2.5, ...
$=1$	>0	∞	"	Flat	USNO 1.5-m
$0 \to 1$	>0	<0	"	Concave	
<0	<0	<0	Gregorian	Concave	Arecibo
<0	>0	>0	Inverse Cass.	Concave	

Angular aberrations

<u>The angular aberrations for Two-mirror Telescopes</u> [Table 6.7 (p. ~~106~~ 118)]

$$ASC = \frac{\theta y^2}{4f^2}\left[1 + \frac{m^2(m-\beta)}{2(1+\beta)}(K_1+1)\right]$$

$$AAS = \frac{\theta^2 y}{f}\left[\frac{m^2+\beta}{m(1+\beta)} - \frac{m(m-\beta)^2}{4(1+\beta)^2}(K_1+1)\right]$$

$$OFAD = \frac{\theta^3(m-\beta)(m^2-1)}{4m^2(1+\beta)^2}\left[m+3\beta + \frac{m^2(m-\beta)^2}{2(1+\beta)(m^2-1)}(K_1+1)\right]$$

and the curvature of the field

κ = kappa

$\kappa_p \equiv$ Petzval field curvature

$\kappa_s \equiv$ Sagittal " "

$\kappa_t \equiv$ Tangential " "

$\kappa_m \equiv$ Mean " " $= \frac{1}{2}(\kappa_s + \kappa_t)$

$$\kappa_p = \frac{2}{R_1}\left[\frac{m(m-\beta)-(m+1)}{m(1+\beta)}\right]$$

$$\kappa_m = \frac{2}{mR_1}\left[\frac{(m^2-2)(m-\beta)+m(m+1)}{m(1+\beta)} - \frac{m(m-\beta)^2}{2(1+\beta)^2}(K_1+1)\right]$$

Classical Cass & the R-C

Two special cases

1) Classical Cassegrain — ~~parabolic~~ $SA = 0$
2) Ritchie Cretien — Aplanatic $SA = \text{Coma} = 0$

For the Ritchey-Chretien system both primary and secondaries are Hyperboloids.

Aberrations in Aplanatic telescopes

$ASA = 0$; $ASC = 0$ | $\beta = 0$

$AAS = \frac{\theta^2 y}{f}\left[\frac{m(2m+1)+\beta}{2m(1+\beta)}\right]$ | $\frac{\theta^2 y}{f}\left(m+\frac{1}{2}\right)$

$OFAD = \theta^3 \frac{(m-\beta)}{4m^2(1+\beta)^2}\left[m(m^2-2)+\beta(3m^2-2)\right]$ | $\frac{\theta^3}{4}(m^2-2)$

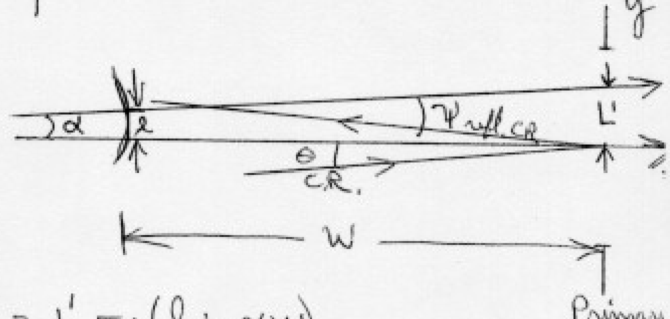
$K_m = \frac{2}{R_1}\left[\frac{(m+1)}{m^2(1+\beta)}\left[m^2-\beta(m-1)\right]\right]$ | $\frac{2}{R_1}(m+1)$

Optical alignment errors

Schroeder p. 132 (R12a)

Alignment Errors in Two-Mirror Telescopes

Suppose we decenter the secondary by l, then that introduces an angular offset of the secondary wrt the primary by α



the distance of the axis of the secondary from the center of the stop (primary) $= L' = +(l + \alpha W)$

$$\Psi = (\theta + \alpha)$$

Signs differ from Schroeder since I have taken the axis of the secondary from the stop instead of vice versa

Ψ is the angle between the reflected chief Ray and the secondary axis.

Tilt and Decenter

Substitute L' and $\bar{\psi}$ into the coma coeffs. in Tables 5.6 + 5.9 to yield

$$ATC = \frac{3y_1^2 k^3}{R_2^2}\left\{\underbrace{\frac{\ell}{R_2}\left[K_2-\left(\frac{m+1}{m-1}\right)\right]}_{\text{decenter}} - \underset{\text{tilt}}{\alpha\left(\frac{m+1}{m-1}\right)}\right\}$$

set $= 0$ ↑

a combination of tilt + decenter yields $ATC = 0$

used in IR telescopes with "field-switching" secondaries to achieve better background subtraction in IR photometry.

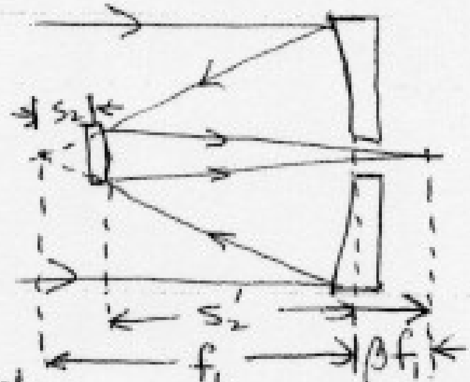
$$\alpha\,(\text{Cass}) = -\frac{\ell}{S_2}$$

$$\alpha\,(\text{RC}) = -\frac{\ell}{S_2}\left[1 + \frac{1}{(m+\beta)(m-1)}\right]$$

Despace

R14

Despace - an error offset of the secondary from the proper secondary-final focal position (s_2'), i.e. focussing!!



$$m = f/f_1 = F/F_1 = -s_2'/s_2$$

$$\text{despace} \equiv ds_2$$

$$ASA(RC) = \frac{m(m^2-1)}{16F^3}\left[1 + \frac{2}{(m-1)(m-\beta)}\right]\frac{ds_2}{f_1}$$

$$ATC = \frac{3\theta}{16F^2}\left[\frac{(2m^2-1)(m-\beta)+2m(m+1)}{1+\beta}\right]\frac{ds_2}{f_1}$$

Alignment error example

	RC	AG
ASA	0.″912	0.″846
ATC	0.″252	0.174

$\frac{ds_2}{f_1} = 0.001$

field angle $= 18'$

$F_1 = 2.5$, $|F| = 10$

$\beta = 0.25$, $|m| = 4$

This is a <u>very</u> important factor and until recently it was a limiting factor in the performance of the WIYN telescope!

For the YALO telescope $f_1 = \cancel{10}\ 2.5$ m so $ds_2 \simeq \cancel{10}\ 2.5$ mm but typically instrument builders have been happy to have their design BFD w/i an inch of the design BFD for the telescope, i.e. ~~twice~~ ten x the tolerance.

Additionally, it is common for the "as built" optics to have a focal length tolerance of 1% from the design while the mechanical tolerances are different. Result can be badly mismatched BFD's

Relative aberrations

R12

	Classical Cass.	Aplanatic Cass	
ASC	$\frac{\Theta y^2}{4f^2}$	0	set a field size; dominant in CC
AAS	$\frac{\Theta^2 y}{f} m$	$\frac{\Theta^2 y}{f}\left(m+\frac{1}{2}\right)$	dominant in AC $m \sim 2\text{-}4$
OFAD	$\frac{\Theta^3}{4}(m^2-1)$	$\frac{\Theta^3}{4}(m^2-2)$	CC $\lesssim$ AC
Km	$\frac{2}{R_1}\left(m+1-\frac{1}{m}\right)$	$\frac{2}{R_1}(m+1)$	CC $\lesssim$ AC

“Sky errors”

The Sky

Refraction: $\Delta x = f(z, HA) + f(x, y, \text{exp.t})$

Differential Color Refr: $\Delta x = f(z, HA, CI)$

Guiding: mag. eqn

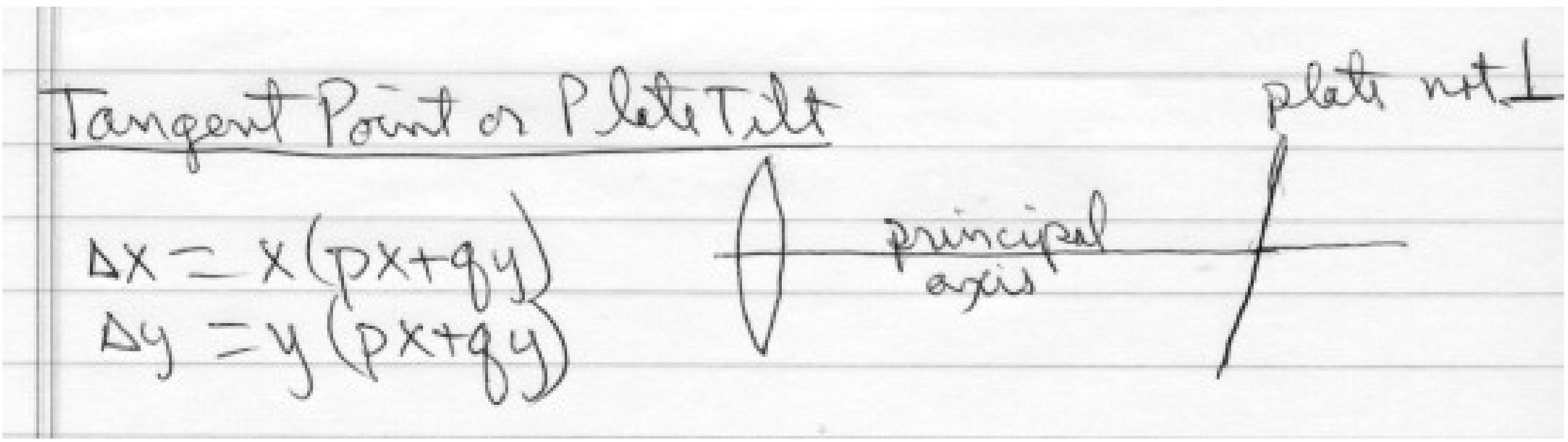
1st order: $\Delta x = x \cdot [\underbrace{a}_{\text{scale}} + \underbrace{\text{mag}}_{\text{coma}} + \underbrace{CI}_{\text{chrom. mag.}}] + y \cdot \underbrace{[b]}_{\text{orientation}} + \underbrace{[c]}_{\text{zero pt,}}$

2nd order: $\Delta x = x \cdot [\underbrace{(px + qy)}_{\text{Tangent pt or plate tilt}} + \underbrace{(d_1 x^2 + d_2 y^2 + d_3 xy)}_{\text{OFAD}}]$

Misc: $\Delta x = \underbrace{\text{mag.}}_{\text{magnitude equation}} + \underbrace{f(x, y)}_{\text{filter/window + CCD flatness}} + \underbrace{\text{chip-to-chip alignments}}_{\text{Mosaic cameras}}$

$+ \underbrace{f(z, HA)}_{\text{Differential Refraction}} + \underbrace{f(x, y, \text{exp.time})}_{\text{non-averaged atmospheric motions - short exps.}} + \underbrace{f(z, HA, CI)}_{\text{Differential Color Refraction}}$

Tangent point location



Astrometric correction formulae

Astrometric Correction "Formulae"

Optical Aberrations

Spherical Ab.: —

Coma: $\Delta X = mag \cdot X$

Astigmatism: —

Curvature of field:

OFAD: $\Delta X = X \cdot (d_1 X^2 + d_2 Y^2) + \cdots$

Chromatic Ab.: Lat: $\Delta X = \overset{CI}{\cancel{mag}} \cdot X$

Telescope Alignment of Optics

Decenter, Tilt > Coma

Despace — Coma + SA

Geometrical Alignment in Focal Plane

Zero Pt.: constant

Scale: $\Delta X = S \cdot X$

Orientation: $\Delta X = \theta y$

Tangent Pt.: $\Delta X = X(pX + qY)$

Detector Problems

Filter flatness: $\Delta X = f(x,y)$

Window flatness: $\Delta X = f(x,y)$

CCD flatness: $\Delta X = f(x,y)$, psf

CTE: mag. eqn + "coma"

Mosaic cameras: chip align/stab.

Schmidt cameras: curvature of field