Atmospheric Limits to Accuracy in Ground-Based Astrometry

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7/25/2005a

Michelson Summer School

Pasadena, CA

Overview

- Atmospheric intro
- Wide angle astrometry
  - Two-color technique
  - Differential chromatic refraction
- Narrow angle astrometry
- Very narrow angle (interferometric) astrometry
  - Dual beam
  - Single beam
- Other comments

Atmospheric Turbulence Effects on Astrometry

- Atmospheric turbulence is the fundamental error source for ground-based measurements
- Turbulence (among its other effects) perturbs the phase of the incoming light, reducing coherence
- Types of coherence loss
  - Wavefront distortion: beam diameters larger than $\mathbf{r_0}$ become distorted
    - » $\mathbf{r_0}$ ~ 10 cm at visible wavelengths
  - Fringe motion: coherent integrations longer than τ_0 become blurred
    - » τ_0 ~ 10 ms at visible wavelengths
  - Anisoplanatism: measurements over angles larger than θ_0 become uncorrelated (in an rms sense)
    - » θ_0 ~ 2 arcsec at visible wavelengths
- Ameliorations for some applications
  - Infrared operation ($\mathbf{r_0}, \tau_0, \theta_0 \propto \lambda^{6/5}$)
  - Adaptive optics (increases $\mathbf{r_0}$)
  - Phase referencing (increases τ_0)
  - Multi-conjugate AO (increases $\mathbf{r_0}, \theta_0$)
- For astrometry, we look at these issues slightly differently than for imaging

Atmospheric Turbulence Primer

1) Refractivity: $n-1=N=A\frac{P}{T}+BQ$, where A & B are dry and wet refractivity coefficients, P is total pressure, T is temperature, Q is water vapor density.

2) Temperature and water vapor fluctuations exist on all scales; these fluctuations change the refractivity: $\delta N=-A\frac{P}{T^2}\delta T+B\delta Q$.

3) Refractivity structure function: $D_n(\mathbf{x_1}, \mathbf{x_2}) = \left\langle \left(N(\mathbf{x_1}) - N(\mathbf{x_2}) \right)^2 \right\rangle$.

4) Homogeneity, isotropy: $D_n(\mathbf{x_1}, \mathbf{x_2}) = D_n(\mathbf{r}) = D_n(r)$.

5) Kolmogorov: $D_n(\mathbf{r}) \sim r^{2/3}$ for r > inner scale l_o (cm's) and < outer scale L_0 (10's m).

6) Integrated to one dimension, structure function for phase $D_S(r) \sim r^{5/3}$.

7) Correlation function: $B_n(\mathbf{r}) = \left\langle \tilde{N}(\mathbf{x_1})\, \tilde{N}(\mathbf{x_1}+r) \right\rangle = B_n(0) - \frac{1}{2} D_n(r)$.

8) Power spectrum is Fourier transform of correlation function.

9) Spatial power spectrum $\Phi_n(\boldsymbol{\kappa}, h) \propto C_n^2(h)\kappa^{-11/3}$, where $\boldsymbol{\kappa}$ is spatial frequency and h is height in atmosphere; $1/L_0 < \kappa < 1/l_0$.

10) Most results are derived from integrals of this spectrum.

11) Space-to-time conversion via frozen turbulence assumption: r = Vt; V = wind speed.

12) All answers involve powers of 1/3.

Variances, Power Spectra, and Performance Analyses

- Be careful about distinguishing between variances and power spectra
  - For a long integration time T, where the power spectrum is white at low frequencies, i.e., $W(f) = W(0)$, the error variance $\varepsilon^2(T) = W(0) / 2T$
  - This is usually different from σ^2 / T

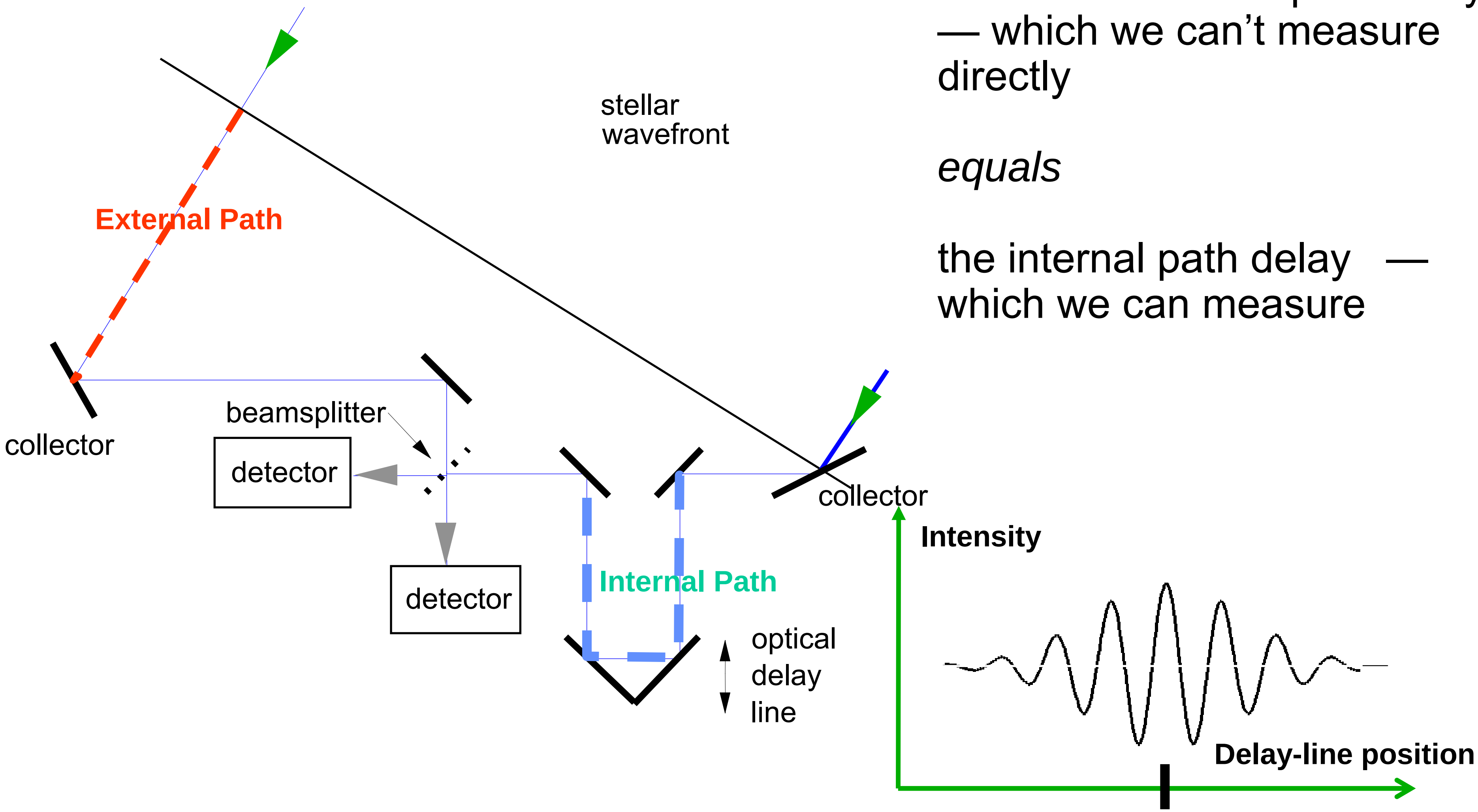
- Performance analyses
  - Easiest to do in context of interferometry, as astrometry with an interferometer is easy to describe geometrically
  - Results similar for a telescope with the same diameter as the interferometer baseline B
    - » NB: collector size d for an interferometer drops out of performance estimates for typical integration times, so point apertures are ordinarily assumed

Detecting Fringes with an Interferometer

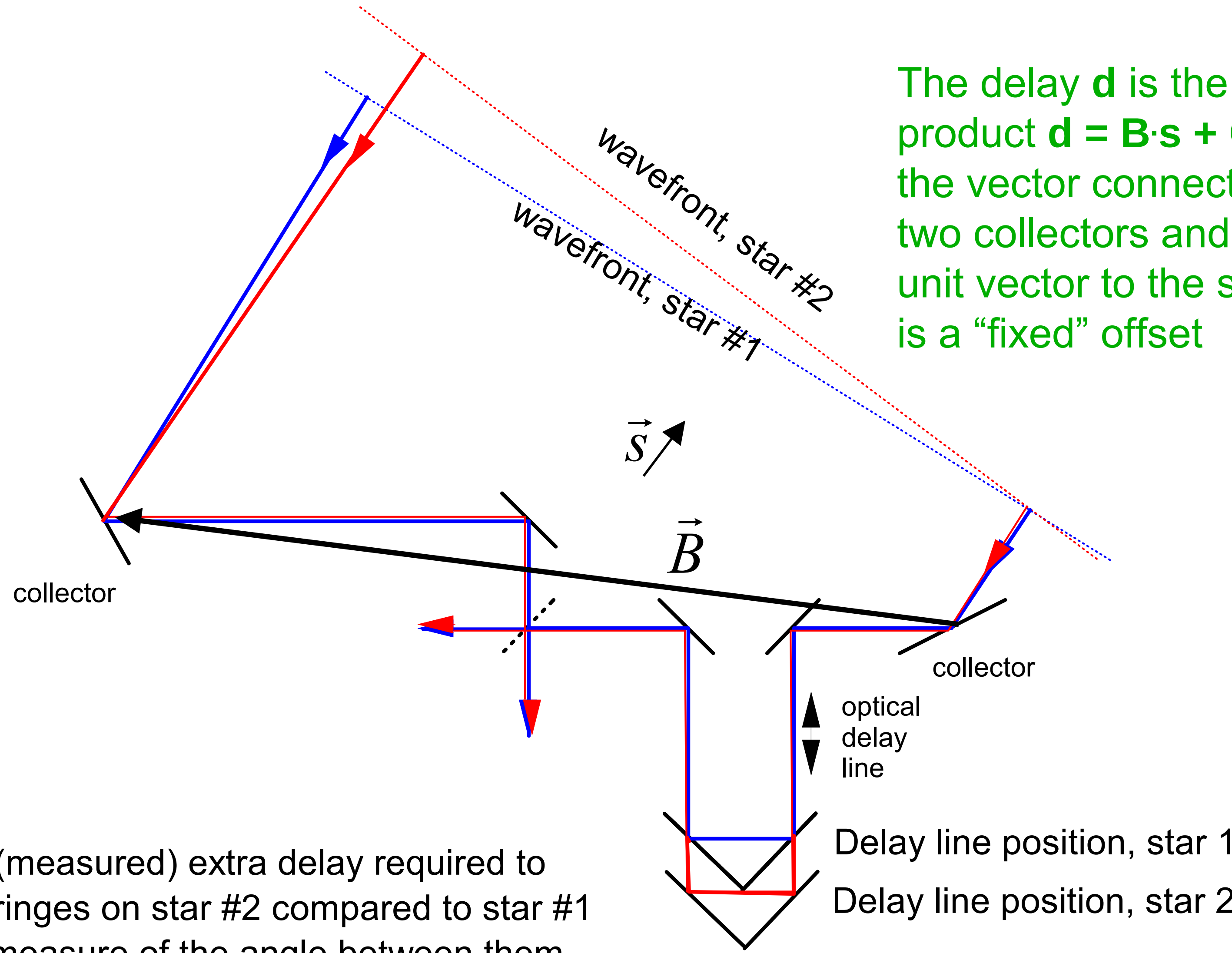
- When the external path delay — which we can't measure directly

  equals

  the internal path delay — which we can measure



Astrometry with an Interferometer



The delay **d** is the dot product **d = B·s + C** of the vector connecting the two collectors and the unit vector to the star; **C** is a "fixed" offset

The (measured) extra delay required to get fringes on star #2 compared to star #1 is a measure of the angle between them

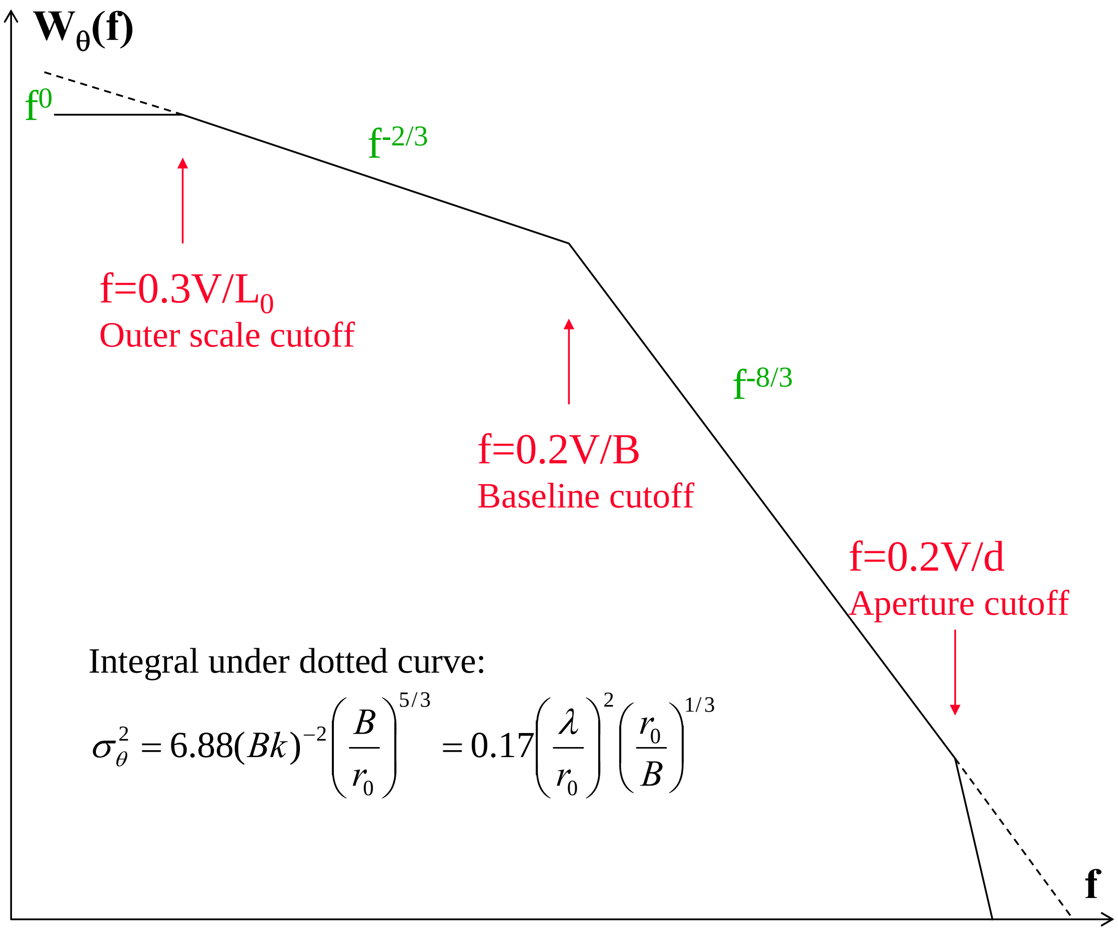
Types of Astrometric Measurements

- Wide angle (~ 1 deg - 10’s deg fields)
  - Typically transit instruments, optical interferometers
- Narrow angle (~ 1 arcmin - 1 deg fields)
  - Typically telescopes with imaging or quasi-imaging back ends (CCD, Ronchi-ruling, photographic plate)
- Very narrow angle (~ < 1 arc min fields)
  - Typically optical interferometers
    - » Dual beam (~ 1 arcsec – 1 arcmin fields)
    - » Single beam (< 1 arcsec fields)

- Atmospheric effects decrease monotonically, but non-linearly, with the size of the field

Wide Angle Astrometry

- Measurements of absolute positions of stars, or of sequential differences between widely-separated stars
- Performance analyses based on power spectrum of angle of arrival



Wide Angle, Cont.

- For infinite outer scale,

$$\varepsilon(T) = 0.5\left(\frac{\lambda}{r_0}\right)\left(\frac{r_0}{VT}\right)^{1/6}$$

$\varepsilon(T) \sim 0.2\ T^{-1/6}$ arcsec [assuming 1” seeing]`

$\sim$ 70 mas for T = 1000 sec of integration time

- Salient points
  - No dependence on baseline or telescope diameter
  - Very slow dependence on integration time – error is not white
  - This result is slightly pessimistic

    For (common) slightly sub-Kolmogorov turbulence,
    T-1/4 dependence. Better, but still slow
  - Better to average over multiple nights to get white-noise behavior

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Wide Angle, Cont.

- With finite outer scale L_0

$$\varepsilon(T) = 0.3\left(\frac{\lambda}{r_0}\right)\left(\frac{r_0^{1/3} L_0^{2/3}}{VT}\right)^{1/2} \quad , \quad L_0 > B, \quad T \gg L_0/V$$

  - Error is now white ($T^{-1/2}$ dependence)
  - Still no baseline dependence (until $L_0 < B$)
  - For L_0 = 50 m, $\varepsilon(T)$ = 8 mas for T = 1000 sec
    - » This result may be slightly optimistic, as power spectrum not perfectly white at low frequencies

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Two-Color Astrometry

- For bright stars at visible wavelengths, one can improve on the accuracy of a wide-angle measurement with a two-color approach
- Consider a distance-measuring example
  - $y(\lambda) = x + \alpha x N(\lambda)\Delta T$,
    - » where x is the true distance, ΔT is a temperature fluctuation, $N(\lambda)$ is the (dispersive) refractivity, and α is a constant
  - Measure the distance at two wavelengths λ_1, λ_2
  - If $N(\lambda)$ is wavelength dependent, we can solve for both x and ΔT

$$\boldsymbol{x} = y_1 - (y_2 - y_1)\ N_1/(N_2 - N_1)$$

$$= y_1 - (y_2 - y_1)\ /\ d_T \Delta\lambda/\lambda$$

Multiplier is ~90 for $\lambda_1, \lambda_2 = 0.5, 0.7$ um

- If there were only temperature turbulence, two-color would provide perfect correction for turbulence errors, at the expense of some noise multiplication

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Two-Color Limitations

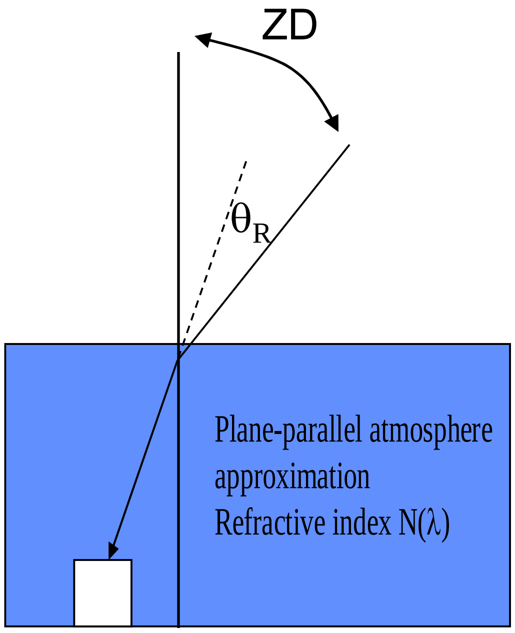
- While temperature turbulence dominates the refractivity fluctuations in the visible, there is a smaller refractivity term attributable to water vapor fluctuations
  - Size of water-vapor term is ~1/20 to 1/10 of temperature term (site dependent)
- The water vapor term is amplified by the two-color method
  - Dispersion d_Q for water-vapor (under assumption of constant total pressure) is negative, and larger than that for temperature
  - Two color amplifies the water-vapor fluctuations by $1-d_Q/d_T$, ~2.75 in the visible
- Thus, two-color can provide a maximum factor of ~4 - 8 improvement in astrometric accuracy, depending on the strength of the water vapor seeing

Wide-Angle Summary

- The fundamental atmospheric limit for wide-angle astrometry at a good site with two color is ~~ 5-10 mas in one night – dependent on detailed seeing
- However, most measurements are systematic limited
  - 13-23 mas accuracy from 4 yrs of M3OI data – likely systematic dominated (Hummel et al. 1994; Lindegren 1995 [Hipparcos comparison])
  - 6-10 mas formal errors obtained from 5 nights of M30I data (Shao et al. 1990) [-> 13-22 mas nightly errors]; also likely systematic limited

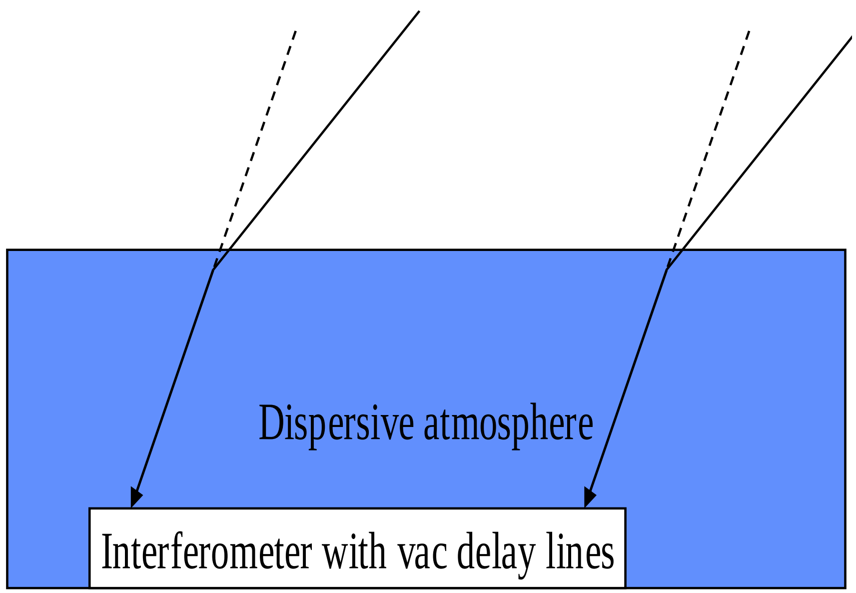
Differential Chromatic Refraction

- Refraction of light through the atmosphere introduces an angular displacement of ~1 arcsec/deg near the zenith
  - It's (of course) dispersive
  - In the visible, $\theta_R \sim (59" + 0.36"/\lambda^2) \tan(ZD)$
  - Causes image elongation for a finite bandpass
- At 30 deg ZD
  - V (0.55 um), $\Delta\lambda$ = 0.01 um: $\Delta\theta_R$= 25 mas
  - R (0.70 um), $\Delta\lambda$ = 0.01 um: $\Delta\theta_R$= 12 mas
- Notes
  - Effect is smaller at longer wavelengths, infrared
  - A (the) major issue for narrow-angle astrometry, too
    - » Can easily dominate the error budget

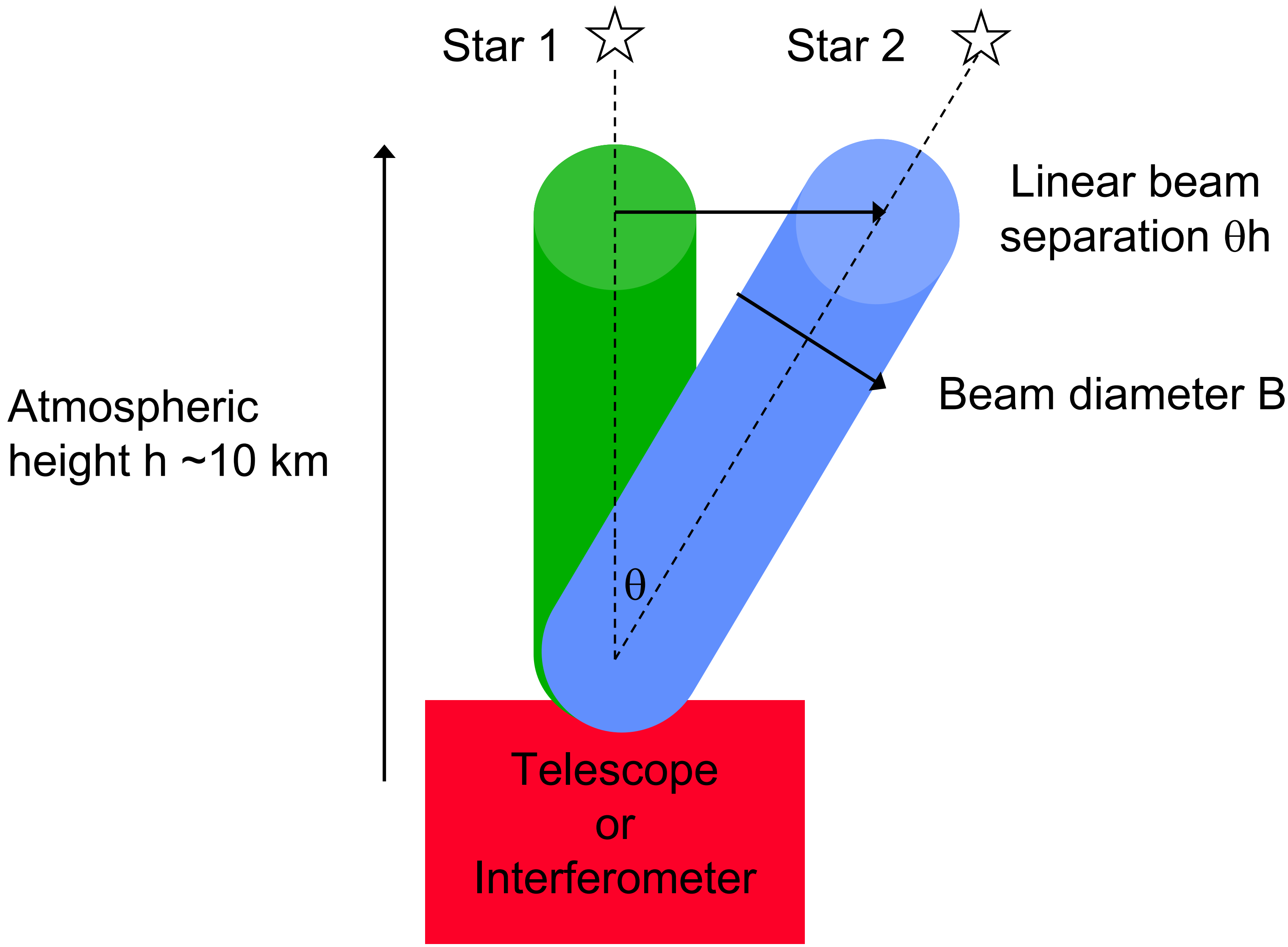


Differential Chromatic Refraction, Cont.

- Not an issue for an interferometer with vacuum delay lines (in limit of plane parallel atmosphere)
  - Extra OPD from being off zenith introduced (and compensated) in vacuum
  - Pathlengths of two arms of interferometer through atmosphere remain matched

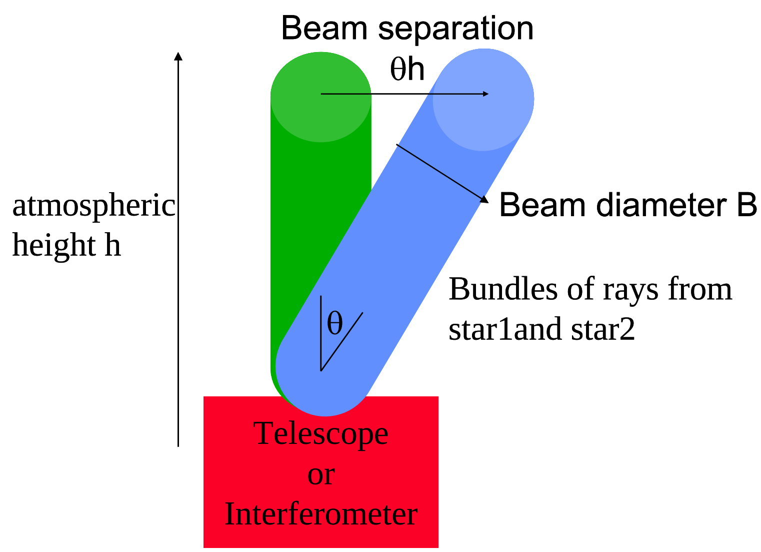


Geometry for a Narrow-Angle Measurement



Narrow Angle Astrometry

- Rays from different stars traverse different paths through the atmosphere
- Intuitively, expect error (difference in measured angle between two stars) to depend on separation in atmosphere, as well as on amount of overlap of the beams
- Traditional anisoplanatism analyses usually address rms phase difference between the two stars: $\sigma^2 \approx 0.2(\theta/\theta_0)^{5/3}$
- However, for astrometry, we care about the detailed shape of the error power spectrum, and need to be a bit more careful in analysis



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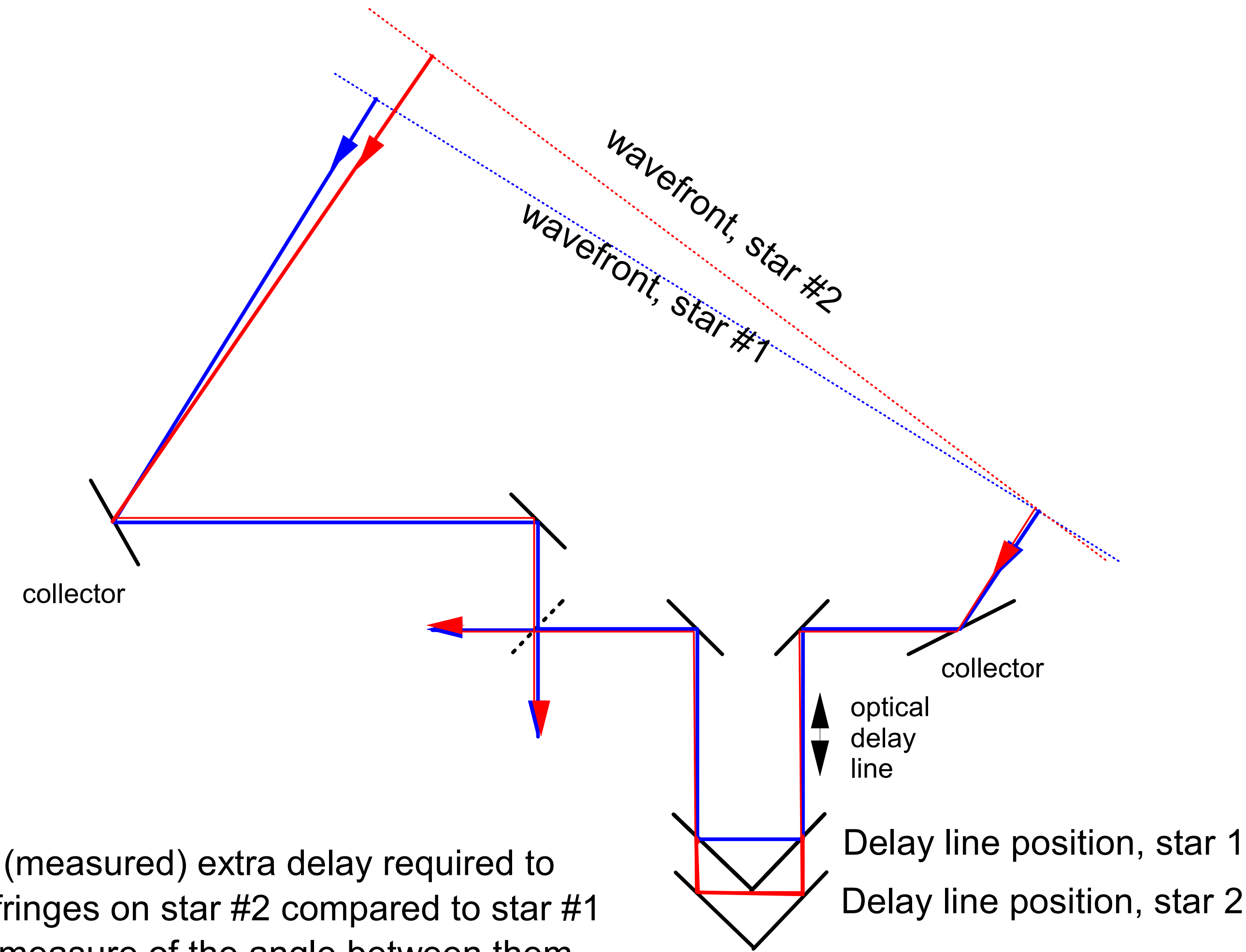
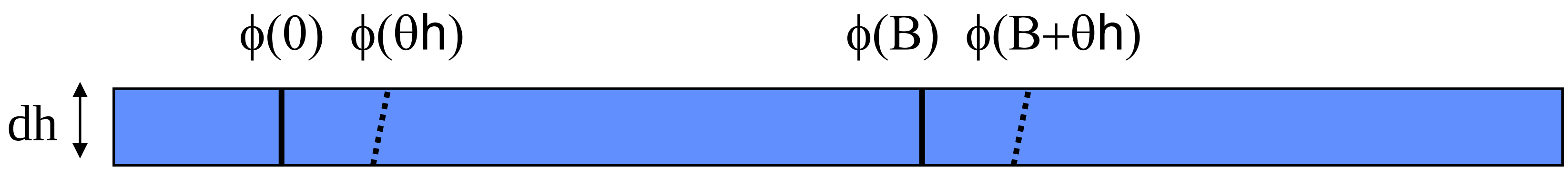
Differential Astrometry with an Interferometer



The (measured) extra delay required to get fringes on star #2 compared to star #1 is a measure of the angle between them

Deriving the Error Variance

- Compute error by layer, $\delta = (\phi(0) - \phi(B)) - (\phi(\theta h) - \phi(B+\theta h))$



- Can derive the error variance as

$$\varepsilon^2(T) \propto \frac{1}{B^2}\int \mathrm{d}h \int \mathrm{d}\kappa \Phi(\kappa, h)\left(1-\cos(B\kappa)\right)\left(1-\cos(\theta h\kappa)\right)$$

$$\propto \frac{1}{B^2}\int \mathrm{d}h C_n^2(h) \int \mathrm{d}\kappa \kappa^{-11/3}\left(1-\cos(B\kappa)\right)\left(1-\cos(\theta h\kappa)\right)$$

- The last two terms are filter functions, proportional to κ^2 near origin
- Astrometric error behavior depends on relative sizes of B and θh

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Narrow-Angle Case
θh > B

$$\varepsilon^2(T) \propto \theta^{2/3}\left(\int \mathrm{d}h\, C_n^2(h)\, h^{2/3}\right) T^{-1}$$

- Notes
  - Error is white
  - High altitude turbulence weighted more than low-altitude turbulence (wide-angle case has uniform height weighting)
  - Weak angle dependence
  - No baseline dependence

$$\varepsilon(T) \approx 1.1\theta^{1/3} T^{-1/2} \mathrm{arcsec}$$

- θ = 1.5' separation: ~5 mas in 5 min
- But can do better

Narrow Angle, Cont.

- However, the strict differential result is a bit pessimistic
  - For small fields, big telescopes, start to transition to next regime
    - » 10' : $\theta h = 30$ m
    - » 1' : $\theta h = 3$ m
  - Field-averaged results are much better than a simple differential measurement
    - » Narrow angle measurements typically made with an imaging device which captures many stars
      - Allows solution for higher-order terms
      - Intuitively, one measures the position of the target star with respect to the centroid of the reference star cluster
        - This centroid can be very close the target star
    - » Provides a factor of 2-3 improvement over simple differential measurement

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Narrow Angle Summary

- Narrow angle atmospheric limit ~~0.5-2 mas for single field-averaged measurement
- Field averaging gains 2-3x over simple differential measurements
- DCR effects are major systematic
- With big telescopes can move to very narrow angle regime

Field *(arc min)*	Tel. *(m)*	Measured performance *normalized to 5 min integration* *(mas)*	Narrow angle prediction (No field avging)	Very narrow angle predict (No field avging)	
50	0.2	8 differential	12	-	1)
3	1.5	1-2 field-avg'd	4.8	-	2)
1.5	5	0.5-1 field-avg'd	3.8	1.3	3) 4)

1) Stone et al. 2003
2) Harris et al. 2005; also Monet et al. 1992. Harris results quoted for a exposure times from a few to 40 minutes; adopting 5 min for comparison with model
3) Shaklan this workshop; Pravdo and Shaklan 1996. Normalizing data to 5 minutes for comparison with model
4) The field size / telescope diameter for these measurements is such that the very narrow angle results apply

Very Narrow-Angle Case
θh < B

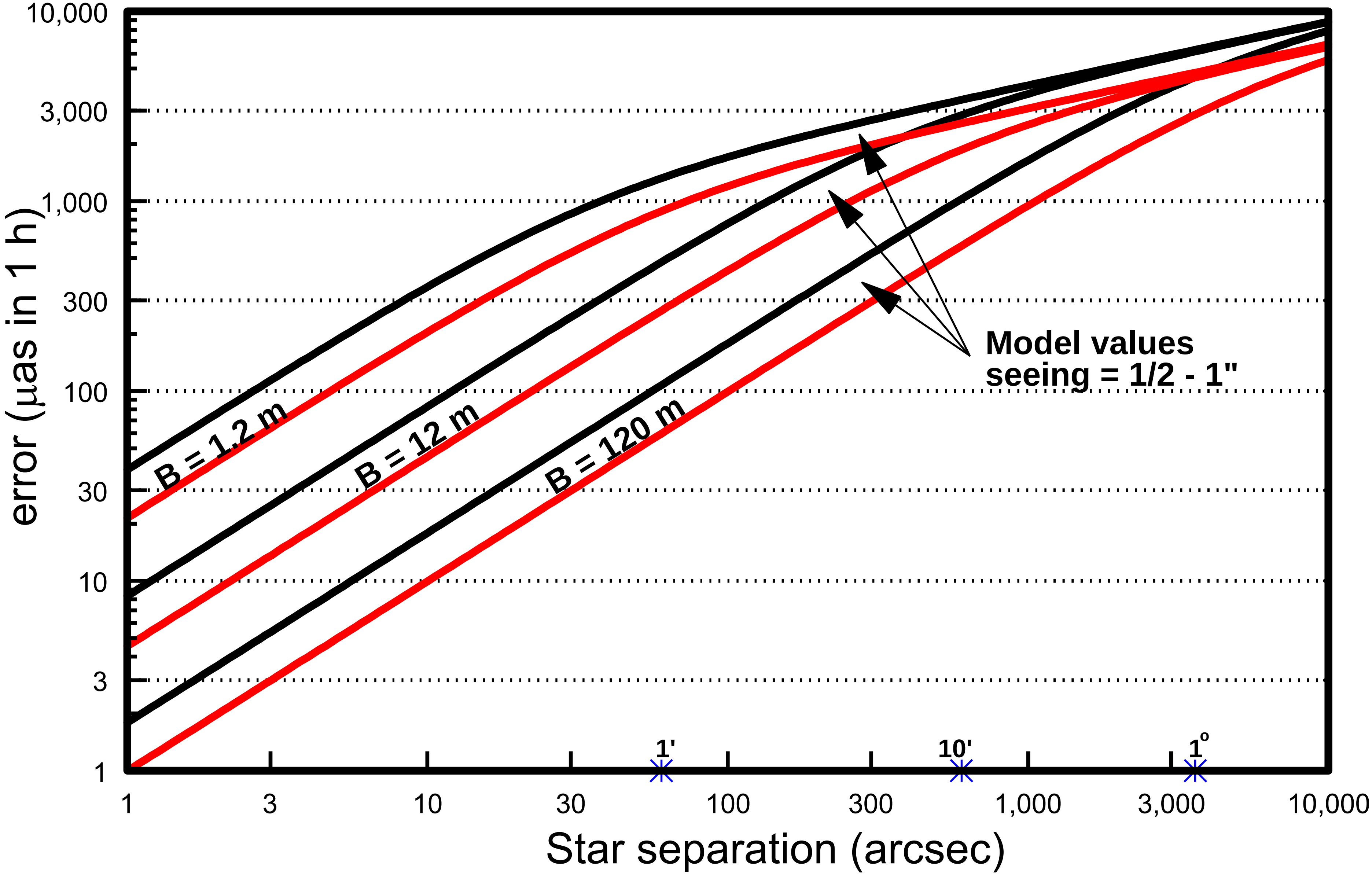
$$\varepsilon^2(T) \propto B^{-4/3}\theta^2\left(\int \mathrm{d}h C_n^2(h) h^2\right) T^{-1}$$

- Notes
  - Error is white
  - High altitude turbulence weighted very strongly: now as h^2
  - Error standard deviation linear with star separation
  - Nearly linear baseline dependence
  - Well suited for interferometers
    - » Interferometers can achieve long baselines
      - Long baselines also be used to improve SNR on faint stars

$$\varepsilon(T) \approx 300 B^{-2/3} \theta T^{-1/2} \text{arcsec}$$

- θ = 20” separation, 100 m baseline: 80 uas in 5 min
- θ = 1” 4 uas in 5 min

Atmospheric Limits to a Narrow-Angle Measurement



Very Narrow Angle Measurements

- Short-term differential data generally matches model predictions
  - 3” separation, 12 m baseline (M3OI, Colavita 1994)
  - 20” separation, 110 m baseline (PTI, 1998 (unpublished))
- External errors not yet at fundamental limits
  - PTI dual-beam: α Cyg, 30” separation, 100 uas night-to-night rms over 7 nights (cf. Lane et al. 2000)
  - PTI single-beam: PHASES binary, 0.25” separation, 16 uas night-to-night rms over 4 nights (Lane & Muterspaugh, et al. 2004)

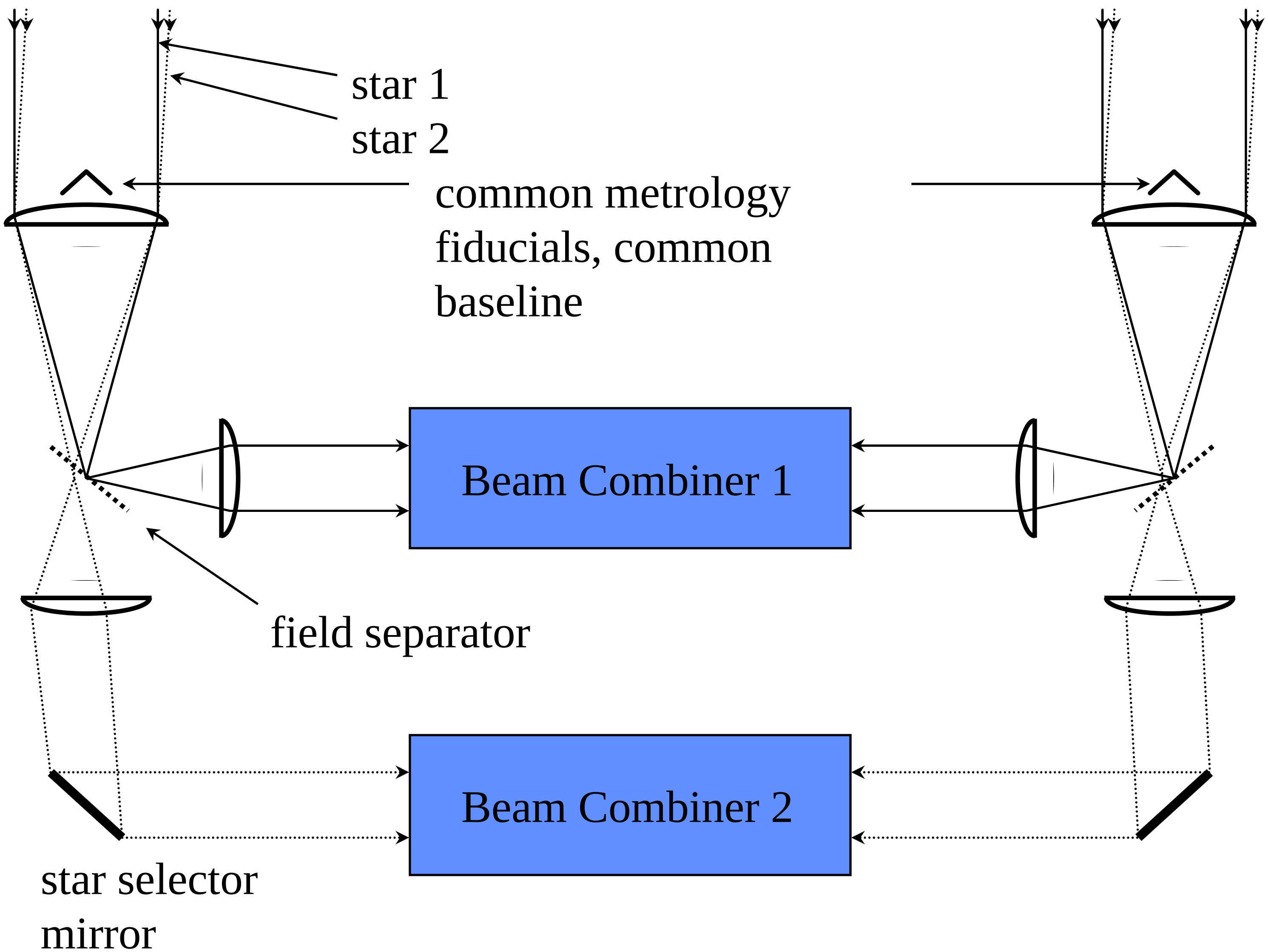
More Discussion

- Outer scale
  - With an interferometer, baseline can be longer than the outer scale
    - » Improves performance: for outer scale L_0 = ½ of baseline length B, performance improved by 2X
      - Outer scale estimate on Mauna Kea from Keck AO measurements ~40 m
- Field averaging
  - More challenging with an interferometer, but mathematically, the error for a target with respect to two references depends on angular difference of target star to their centroid
- Simultaneity
  - The stars must be observed essentially simultaneously to achieve this limit, e.g., $\Delta t\, V < \theta h$. 100 ms delay same error as 20" separation
  - Requires phase referencing (where all measurements are made with respect to phase of one star), and dual-star feed for wide separations

Implementing a Narrow-Angle Measurement for Separations > λ/d – Dual Beam

- Measurement of the two stars must be essentially simultaneous to exploit the common-mode nature of the atmosphere over small fields
  - But…instantaneous interferometer field of view is typically only a few arcsec
- Measurements should use as many common optics as possible to minimize systematic errors
  - I.e., want to use the same basic interferometer to measure both stars
- One implementation: dual-star astrometry

Dual-Star Concept



Dual-Star, Cont.

- With the dual-star concept, the same baseline is used for the two stars
  - Delay equation: $x = \mathbf{B} \cdot \mathbf{s} + C$; 4 instrument parameters
  - Baseline knowledge requirements (**B**; 3 parameters) are greatly reduced from those needed for an absolute measurement
- Laser metrology to common corner cubes monitors offset C

Phase Referencing

- In a small field, although the primary star will be bright (chosen to be nearby to maximize the astrometric signature), the secondary star will generally be faint and not trackable with short integration times
  - Use phase referencing to stabilize the optical path to allow long coherent integrations to increase sensitivity
- Phase referencing is the temporal analog of adaptive optics (AO)
  - AO
    - » AO uses a reference star (or laser guide star) to measure atmospheric wavefront distortions
    - » Uses a deformable mirror to correct distortion on reference star and in vicinity of reference star
  - Phase referencing
    - » Uses a reference star (the primary star in this case) to measure atmospheric fringe motion
    - » Uses an optical delay line to correct motion on reference star and in vicinity of reference star

Implementing a Narrow-Angle Measurement for Separations ~ λ/d: Single Beam

- Both stars are propagated in a single field of view, and stars are separated in delay space
- A) Pupil is divided in power with a beamsplitter into phase reference beam and measurement beam
  - Phase reference beam used for fringe tracking
  - Phase-referenced measurement beam is swept between two fringe packets
  - See PHASES talk later in week by Matthew Muterspaugh

  -or-
- B) Pupil is divided spatially
  - One half used for fringe tracking
  - Other half to measure separation

- All measurement approaches – single and dual beam - rely on phase referencing on target, and switching between target and reference on other beam to measure difference with same configuration

Atmospheric noise is not the only term in the error budget

- In particular, for very narrow angle measurements, the atmospheric term can be among the smaller terms

A sample systematic error budget; doesn't show all terms

Dual-Star Systematics Error Budget

			nm per arm	nm total	uas total	
unmodeled pivot noise	25.0	um	1.9	2.7	5.5	
pivot beacon to pivot transfer	25.0	um	1.9	2.7	5.5	
DSM CC to beacon transfer	25.0	um	1.9	2.7	5.5	
baseline solution	35.0	um		2.6	5.4	
DCR					5.0	
beamwalk of secondary over field			2.5	3.5	7.3	
alignment of metrology to starlight	0.5	arc sec	1.8	2.5	5.2	
alignment drift	0.5	arc sec	1.8	2.5	5.2	
metrology stability	1.00E-08	fractional	0.1	0.1	0.2	
metrology polarizer mount gradient	0.04	K	2.0	2.8	5.8	
fringe-measurement accuracy	0.005	rads	1.8	2.5	5.1	
beamwalk stability in propagation	1.5	mm	2.3	3.2	6.6	
				TOTAL:	**18.8**	**uas**

Summary

- Atmospheric turbulence is the fundamental limit for astrometry from the ground
- Errors decrease non-linearly with the size of the field, from milli-arcseconds for wide angle to micro-arcseconds for very narrow angle
  - For wide angles, accuracy is not dependent on instrument size
  - For very narrow angles, accuracy is strongly dependent on instrument size
  - Systematics can easily dominate over atmospheric errors in all regimes
    - » Wide angle – e.g. baseline stability
    - » Narrow angle – e.g. chromatic effects
    - » Very narrow angle – e.g. fringe measurement accuracy, beam walk

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