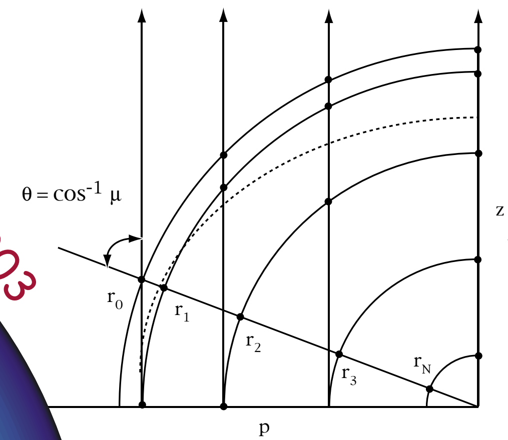
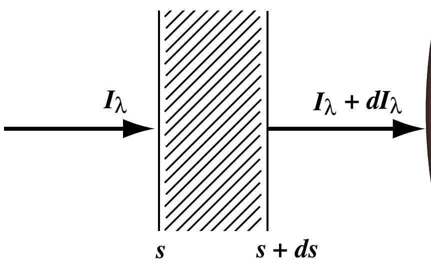
Michelson Interferometry Summer School 2003





Stellar Atmospheric Structure

J. P. Aufdenberg (Harvard-Smithsonian CfA)

On Stellar Atmospheres

"Stars are massive and they have no walls."

Steve Shore: The Tapestry of Modern Astrophysics (2002)

"...the transition from confinement in the stellar interior to open-ended interstellar emptiness...

...will keep you and me busy for years to come."

Rob Rutten: Radiative Transfer and Stellar Atmospheres (2000)

What's to Come...

* Stellar Interiors vs. Stellar Atmospheres

* Parameters and Equations from a Stellar Atmosphere Model

* Spectroscopic Information on Stellar Atmospheric Structure

* From Basic Radiative Transport to Limb-Darkening

    * with diversions for spherical atmospheres and the Sun's temperature structure

* Concept of Radiative Equilibrium

* Real Stars

    * Altair: rapid rotation
    * Deneb: Stellar Winds
    * β Peg: Extended M-giant atmospheres

* Summary & References

Stellar Interiors versus Stellar Atmospheres

Interiors	Atmospheres

Radiative flow of energy

Interiors	Atmospheres
Diffusion Equation	Radiative Transfer Equation

Thermodynamic State: Radiation Field

Interiors	Atmospheres
* Thermodynamic Equilibrium (TE)	*Non-Local TE
*Radiation enclosed by matter at approx. the same temperature	*Matter "sees" radiation of different temperatures
*Radiation field is Planckian	*Radiation field is non-local
*Radiation field is isotropic	*Radiation field is anisotropic

Thermodynamic State: Collisional Processes

Interiors	Atmospheres
	*Radiative processes dominate --> detailed balance
*Saha & Boltzmann Eqns. describe ionization and and excitation	*Saha & Boltzmann don't describe ionization and excitation
* Maxwellian velocity distribution of ions and electrons	*Maxwellian velocities (except chromospheres)

Example Parameters and Equations for Stellar Atmosphere Models*

* In this case, hot stars with winds

Input Parameters

$T_{\rm eff},\ R_\star,\ g(R_\star),\ Z$

$R_{\rm max},\ \dot{M},\ v_\infty,\ \beta_{\rm wind}$

- Effective Temperature
- Reference Radius
- Gravity @ Reference Radius
- Chemical Composition

- Outer Radius
- Mass-Loss Rate
- Max. Stellar Wind Velocity
- Velocity field of the Wind

Atmospheric Structure

Hydrodynamic

$$v = v_\infty \left(1 - \frac{R_\star}{r}\right)^{\beta_{\rm wind}}$$

$$\rho = \frac{\dot{M}}{4\pi r^2 v}$$

Hydrostatic

$$\frac{dP}{d\tau} = \frac{g}{\kappa}$$

Outer Structure

- Velocity Field
- Density structure via continuity equation

Inner Structure

- Pressure gradient goes as gravity/opacity

Equation of State

$$P = P(\rho, T)$$

- LTE ionization and level populations, chemistry

Special Relativistic Radiative Transfer Equation

$$\gamma(\mu+\beta)\frac{\partial I}{\partial r} + \frac{\partial}{\partial \mu}\left\{\gamma(1-\mu^2)\left[\frac{(1+\beta\mu)}{r} - \gamma^2(\mu+\beta)\frac{\partial \beta}{\partial r}\right] I\right\} - \frac{\partial}{\partial \nu}\left\{\gamma\left[\frac{\beta(1-\mu^2)}{r} + \gamma^2\mu(\mu+\beta)\frac{\partial \beta}{\partial r}\right]\nu I\right\} + \gamma\left\{\frac{2\mu+\beta(3-\mu^2)}{r} + \gamma^2(1+\mu^2+2\beta\mu)\frac{\partial \beta}{\partial r}\right\} I = \eta - \chi I$$

- Spherical geometry
- $\beta(r) = v(r)/c$
- $\gamma = 1/(1-\beta^2)^{1/2}$
- μ is direction-cosine
- r is the radius
- I is the intensity
- η is the emissivity
- χ is the opacity

Rate Equations

$$\sum_{j<i} b_j \left(\frac{n_j}{n_i}\right)^* (R_{ji} + C_{ji}) - b_i \sum_{j<i} (R_{ij} + C_{ij}) + b_i \sum_{j>i}^{\kappa} (R_{ij} + C_{ij}) = -\sum_{j>i}^{\kappa} b_j \left(\frac{n_j}{n_i}\right)^* (R_{ji} + C_{ji})$$

- Non-LTE level populations; radiative and collisional rates

Radiative Equilibrium Temperature Corrections

$$\int_0^\infty (\eta_\lambda - \chi_\lambda J_\lambda)\, d\lambda = 0$$

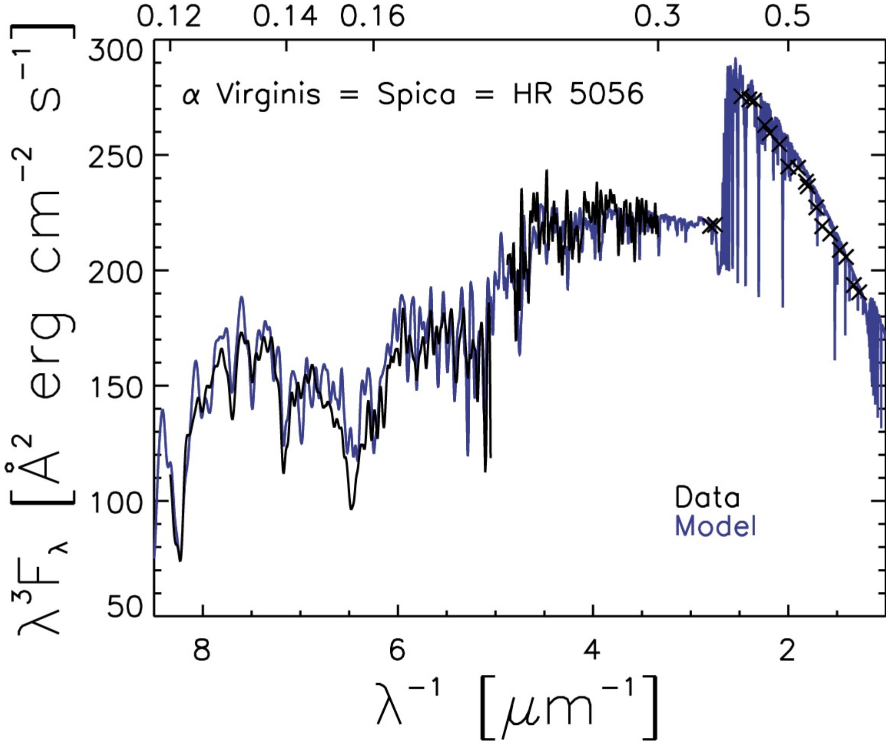
- Temperature structure adjusted to conserve energy; in cool stars convective equilibrium can be established in the inner structure

For Interferometry
Full radiation field: intensities @ every angle

Synthetic Spectrum

Spectroscopic Information

Energy Distribution; Color Temperature



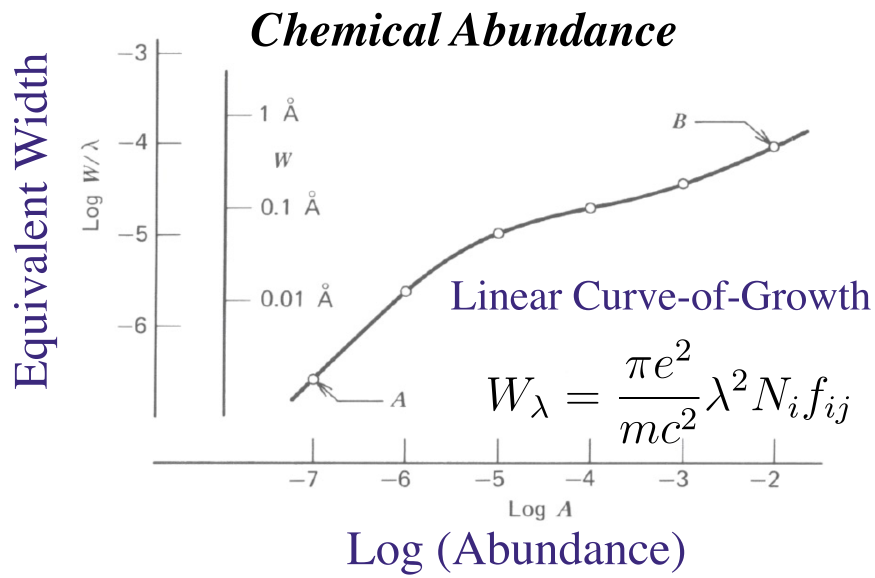
surface flux; flux per unit area at the photosphere

$$\mathcal{F} = \int_0^\infty F_\lambda \, d\lambda = \int_0^\infty B(T_{\rm eff})_\lambda \, d\lambda = \sigma T_{\rm eff}^4$$

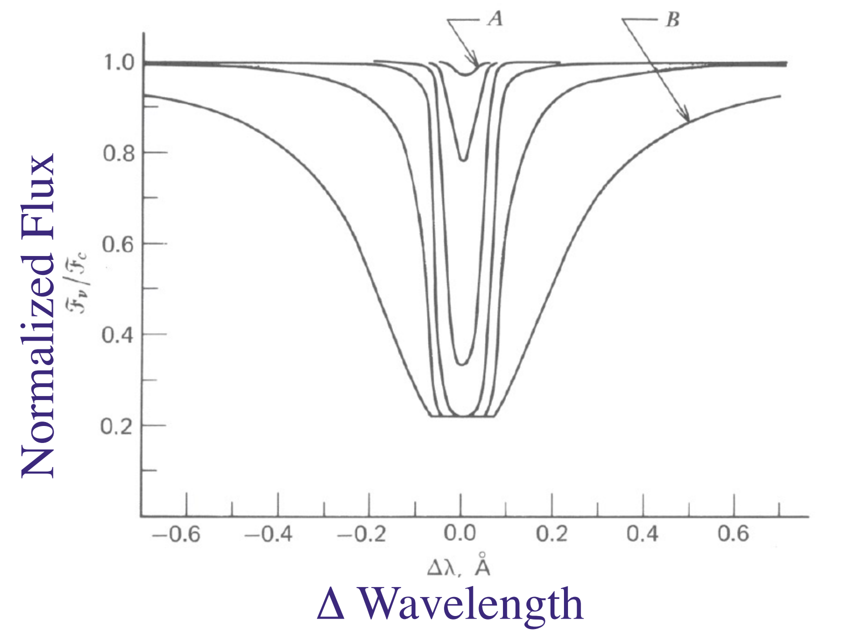
$$T_{\rm eff} = \left[\frac{4\mathcal{F}_\oplus}{\sigma\theta^2}\right]^{\frac{1}{4}}$$

connected to the flux at earth via the angular diameter

Chemical Abundance

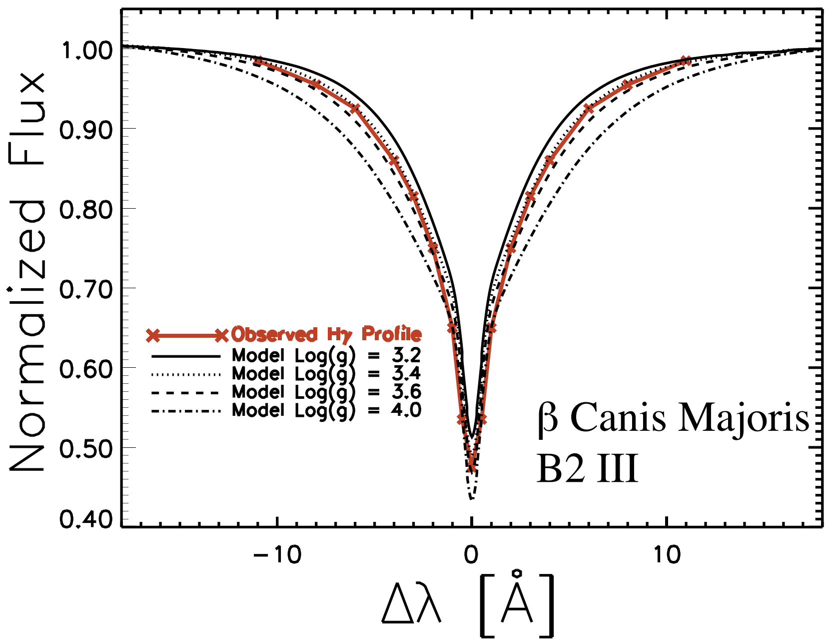


$$W_\lambda = \frac{\pi e^2}{mc^2}\lambda^2 N_i f_{ij}$$

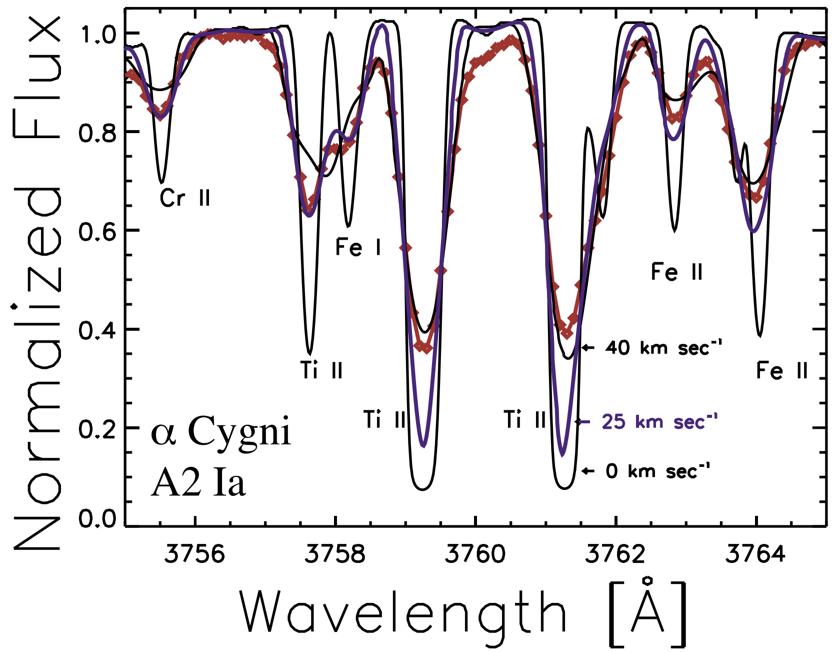


Spectroscopic Information Continued...

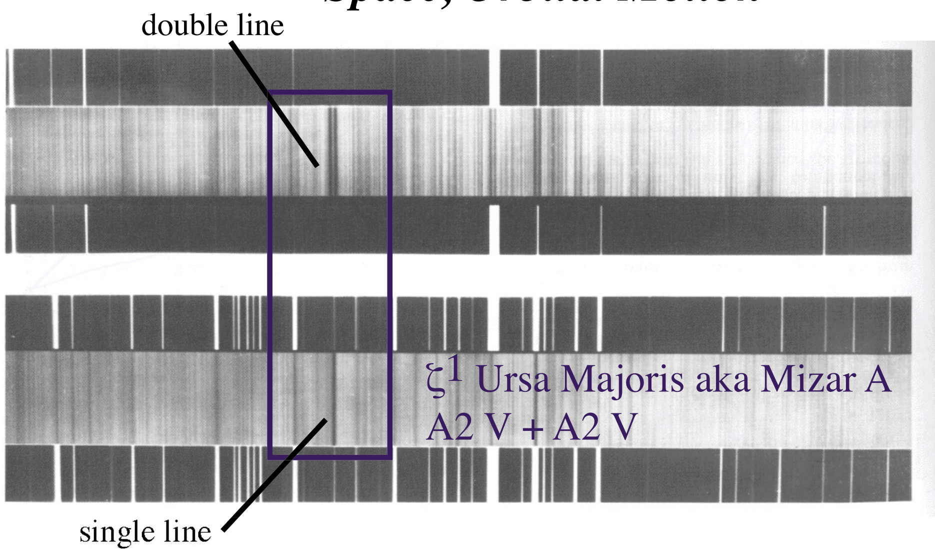
Surface Gravity, Pressure Structure



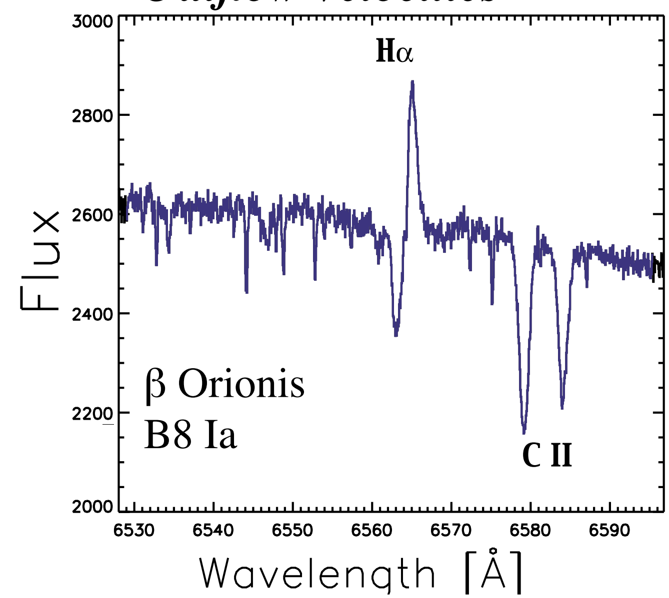
Rotational Velocity



Space, Orbital Motion

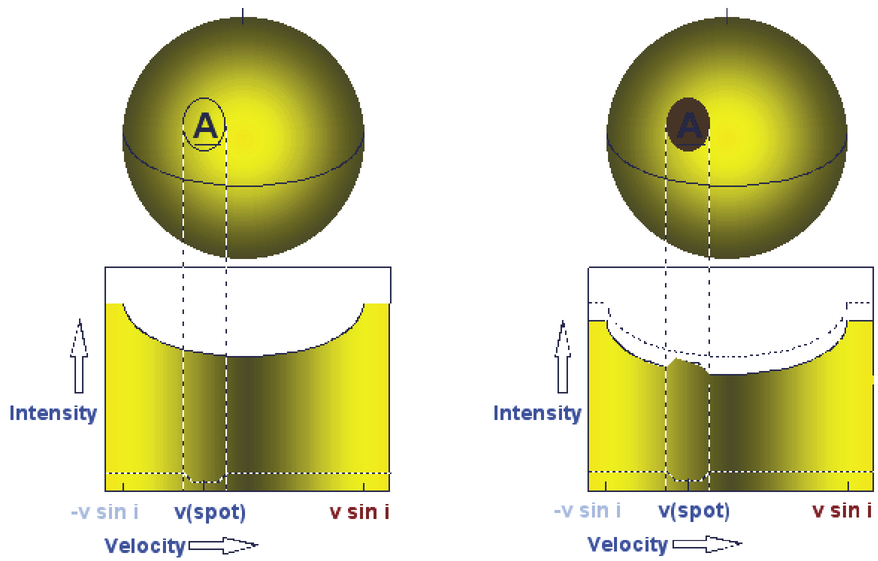


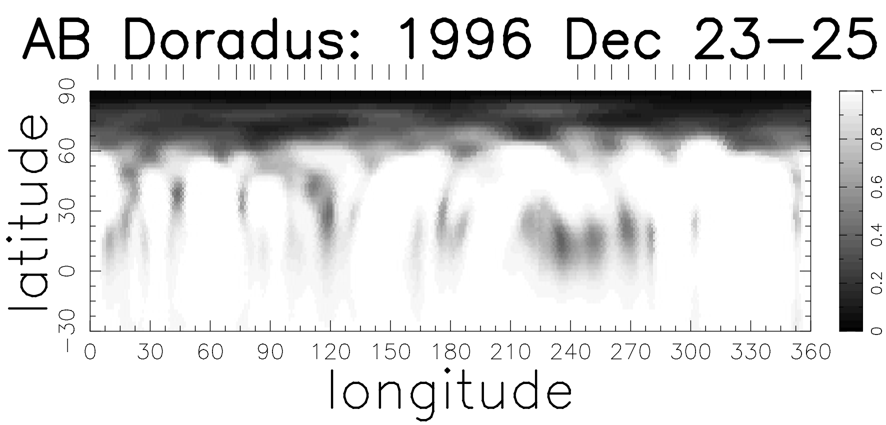
Outflow Velocities



Spectroscopic Information Continued...

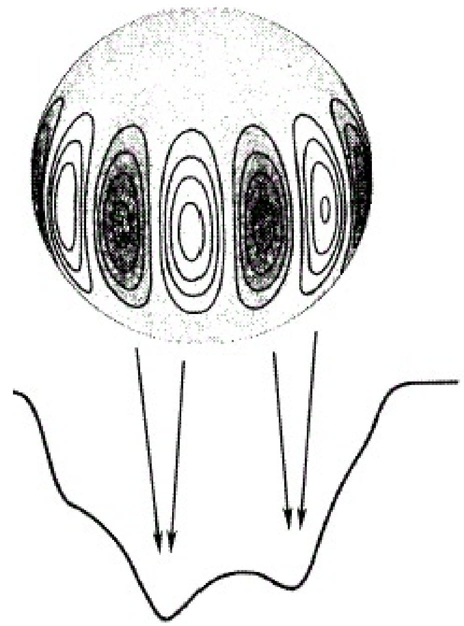
Surface Structure via Doppler Tomography



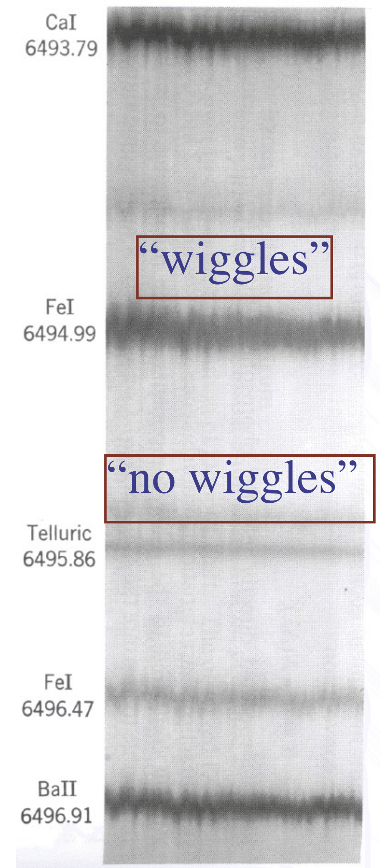


Donati et al. (1999) MNRAS 302, 437

Radial and non-Radial Pulsation; Convection



Vogt & Penrod (1983) ApJ 275, 661



Solar Spectrum

Gray: OASP

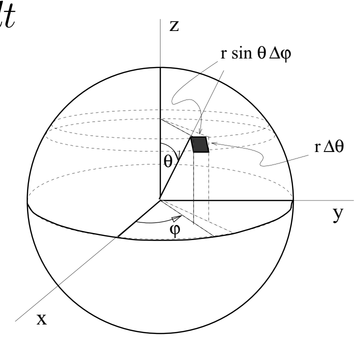
Intensity and the Radiative Transport

Intensity: The basic macroscopic quantity of Radiative Transfer.

$$dE_\lambda = I_\lambda \, d\omega \, d\sigma \, d\lambda \, dt$$

Energy per time per wavelength per area per solid angle.

$$I_\lambda = \frac{dE_\lambda}{d\omega \, d\sigma \, d\lambda \, dt}$$



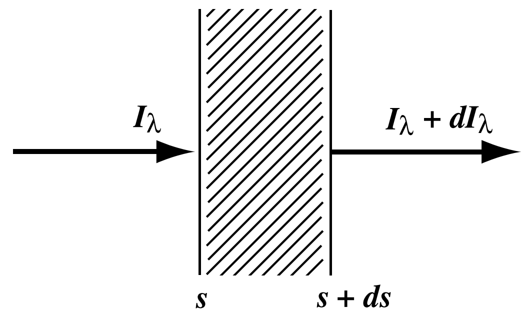
Intensity depends on:

1. location in space
2. direction
3. wavelength

Note: intensity is *independent* of the distance from the source. The flux is not independent of distance.

$$\pi \cdot F_\lambda = \int I_\lambda \cos\theta \, d\omega$$

Transport of intensity along a ray:



absorption coefficient

emission coefficient

$$\frac{dI_\lambda}{ds} = -\kappa_\lambda I_\lambda + \epsilon_\lambda$$

path length

$$\frac{dI_\lambda}{ds} > 0 \qquad \epsilon_\lambda > \kappa_\lambda I_\lambda$$ beam enhanced

$$\frac{dI_\lambda}{ds} < 0 \qquad \epsilon_\lambda < \kappa_\lambda I_\lambda$$ beam extinguished

$$\boxed{S_\lambda = \epsilon_\lambda / \kappa_\lambda}$$ Source Function

$$\frac{dI_\lambda}{\kappa \, ds} = -I_\lambda + \frac{\epsilon_\lambda}{\kappa_\lambda} = -I_\lambda + S_\lambda$$

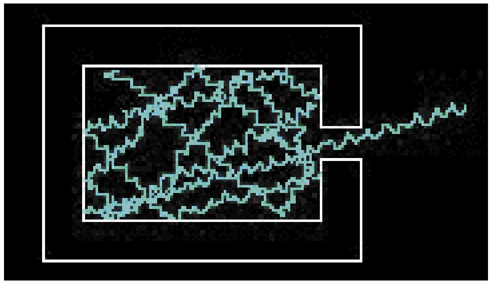
Radiative Transfer Equation

$$\boxed{\frac{dI_\lambda}{d\tau} = -I_\lambda + S_\lambda}$$

Optical Depth

$$\boxed{d\tau_\lambda = \kappa_\lambda \, ds}$$

Perfect Black Bodies; Kirchoff's (Radiation) Law



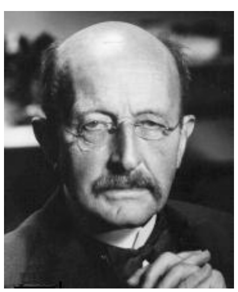
For a perfect ***blackbody***, the intensity is *isotropic and homogeneous*.
(A good approximation *deep* in a stellar atmosphere.)

$$\frac{dI_\lambda}{ds} = -\kappa_\lambda I_\lambda + \epsilon_\lambda \qquad S_\lambda = \epsilon_\lambda / \kappa_\lambda$$

$$\frac{dI_\lambda}{ds} \longrightarrow 0$$

isotropic and homogeneous

$$S_\lambda \equiv I_\lambda \equiv B_\lambda$$



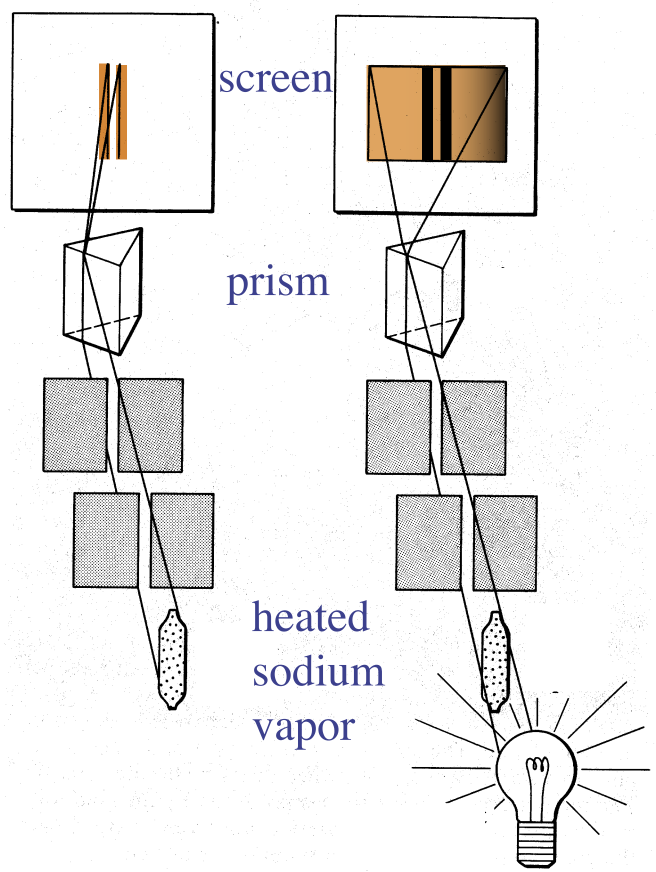
$$B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Planck Function

$$S_\lambda = B_\lambda = \epsilon_\lambda / \kappa_\lambda$$

$$\epsilon_\lambda = \kappa_\lambda \cdot B_\lambda$$

"A good emitter is a good absorber"



This must be true in general not to violate energy conservation.

From *detailed balance*, this must be true at each wavelength λ in order that the 2nd law of thermodynamics is violated!

Basic Spectral Line Formation: Isothermal Slabs

a. Source function *not* a function of optical depth — b. Optical depth is much less than one — c. LTE ($S(\tau) = B(\tau)$)

$$\frac{dI_\lambda}{d\tau} = -I_\lambda + S_\lambda. \quad \Rightarrow a \Rightarrow \quad I_\lambda(\tau_\lambda) = I_\lambda(0)\, e^{-\tau_\lambda} + S_\lambda(1 - e^{-\tau_\lambda}) \quad \Rightarrow b \Rightarrow \quad I_\lambda(\tau) = I_\lambda(0)(1-\tau_\lambda) + S_\lambda(\tau_\lambda).$$

$\Rightarrow$ c $\Rightarrow$

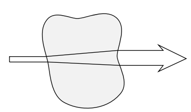
$$I(\tau) = B_{\text{deep layer}} + \tau(B_{\text{outer layer}} - B_{\text{deep layer}})$$

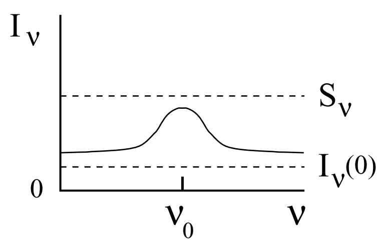
1. Emission

$$B_\lambda(T)_{\text{deep layer}} < B_\lambda(T)_{\text{outer layer}}.$$

$\tau_\nu(D) < 1$

$I_\nu(0) < S_\nu$



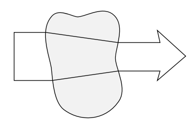


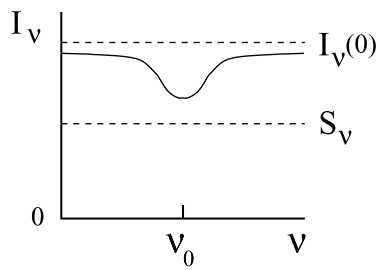
2. Absorption

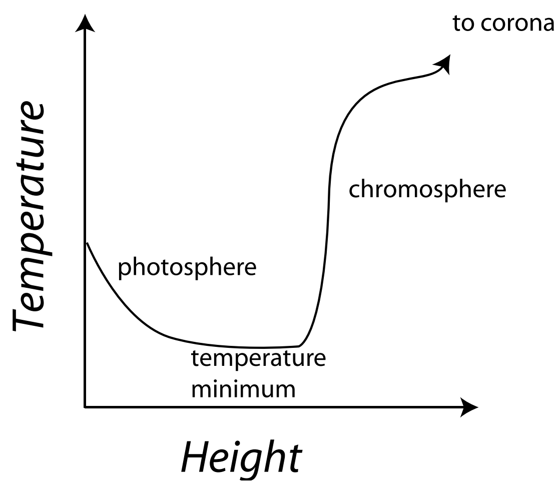
$$B_\lambda(T)_{\text{deep layer}} > B_\lambda(T)_{\text{outer layer}}.$$

$\tau_\nu(D) < 1$

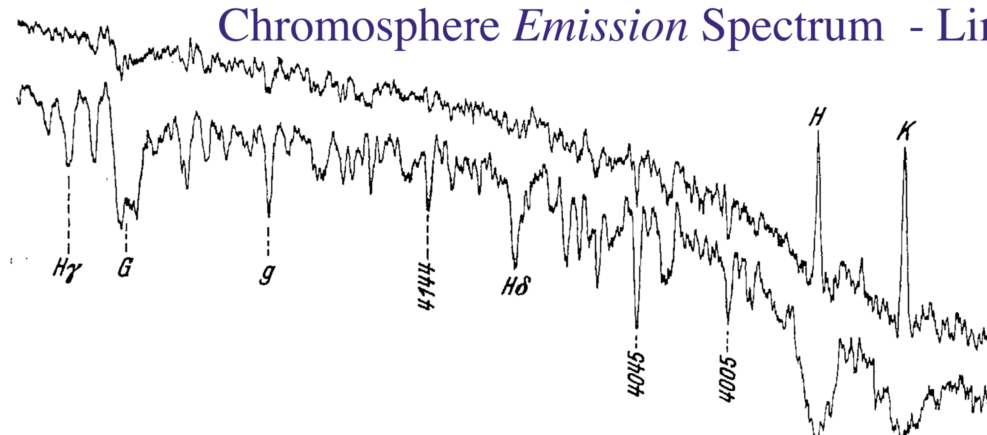
$I_\nu(0) > S_\nu$







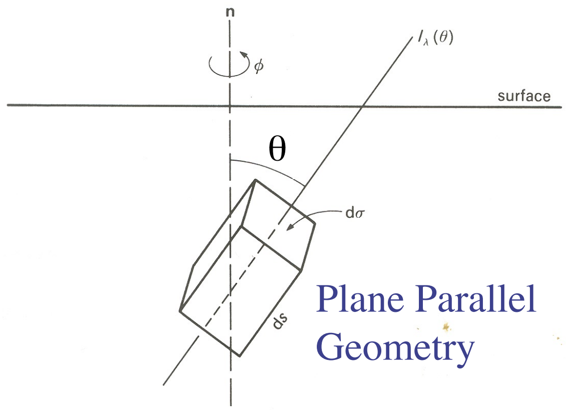
Chromosphere *Emission* Spectrum - Limb



Photosphere *Absorption* Spectrum - Disk Center

More Radiative Transfer: Now with Angles!

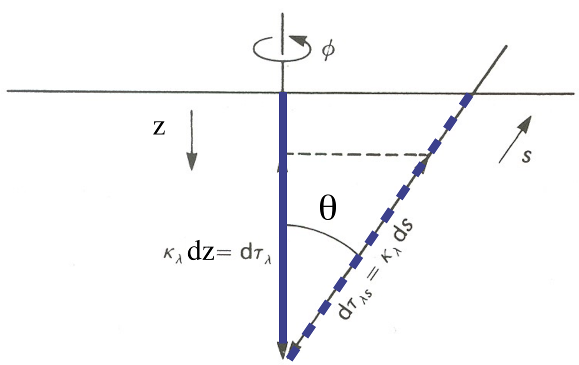
1. Setup Equation



Plane Parallel Geometry

Radiative Transfer Equation

$$\frac{dI(\theta)}{ds} = -\kappa I(\theta) + \epsilon = -\kappa I(\theta) + \kappa S$$



$$d\tau = d\tau_z = d\tau_s \sec\theta$$

$$\frac{dI(\theta)}{\sec\theta d\tau} = I(\theta) - S$$

I, *S* and τ are all functions of wavelength

2. Solve for the Intensity at the Surface!

(use integrating factor : $e^{-\tau\sec\theta}$)

$$\frac{dI(\theta)}{\sec\theta d\tau} = I(\theta) - S$$

$$I(0,\theta) = \int_0^\infty S e^{-\tau\sec\theta}\, d(\tau\sec\theta)$$

This is the ***formal solution*** to the equation of transfer.

$I(0,\theta)$ is the surface (τ=0) viewed from angle θ.

Intensity varies with angle because the source function varies with depth.

3. Trial Source Function

plug in: $S = a + b\tau$

$$I(0,\theta) = \int_0^\infty a\ e^{-\tau\sec\theta}\, d(\tau\sec\theta) + \int_0^\infty b\ \tau e^{-\tau\sec\theta}\, d(\tau\sec\theta)$$

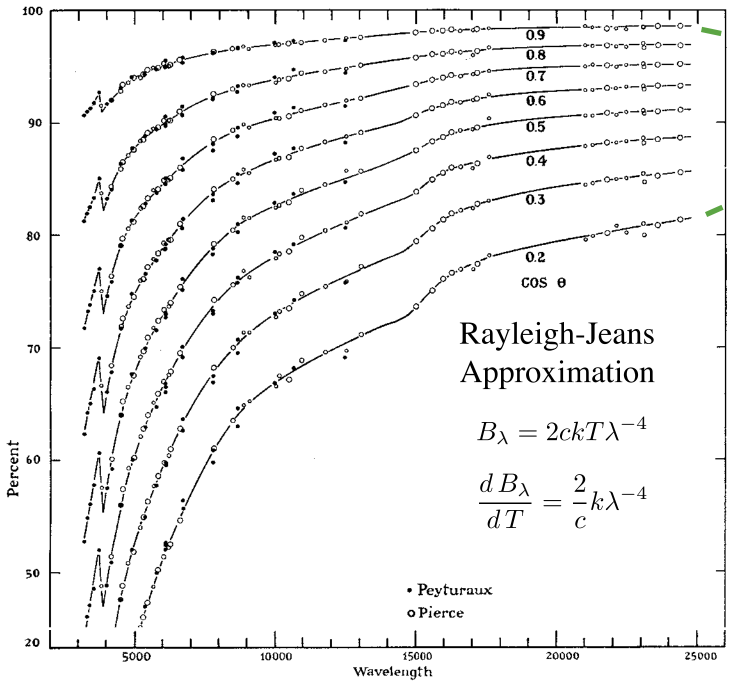
$$I(0,\theta) = a + b\ \cos\theta$$

$$S_\lambda(\tau_\lambda) = \sum_i a_{\lambda i}\tau_\lambda^i \longrightarrow I_\lambda(0,\theta) = \sum_i A_{\lambda i}\cos^i\theta$$

Jason P. Aufdenberg, 10 July 2003, Michelson Interferometry Summer School

Determining the Temperature Structure of the Sun: Limb-Darkening

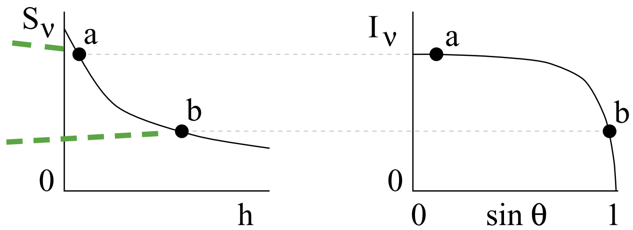
Relative Intensity

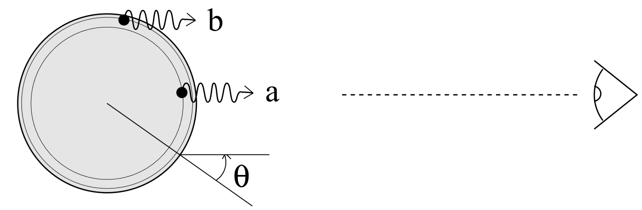


Wavelength

Disk Center

Near Limb





Solve Inverse Problem:

1. Measure intensities

$$I_\lambda(0,\theta) = \sum_i A_{\lambda i} \cos^i \theta \qquad A_{\lambda i} = a_{\lambda i} \cdot i!$$

$$S_\lambda(\tau_\lambda) = \sum a_{\lambda i} \tau_\lambda^i$$

2. Determine fit coefficients, A_0, A_1, A_2

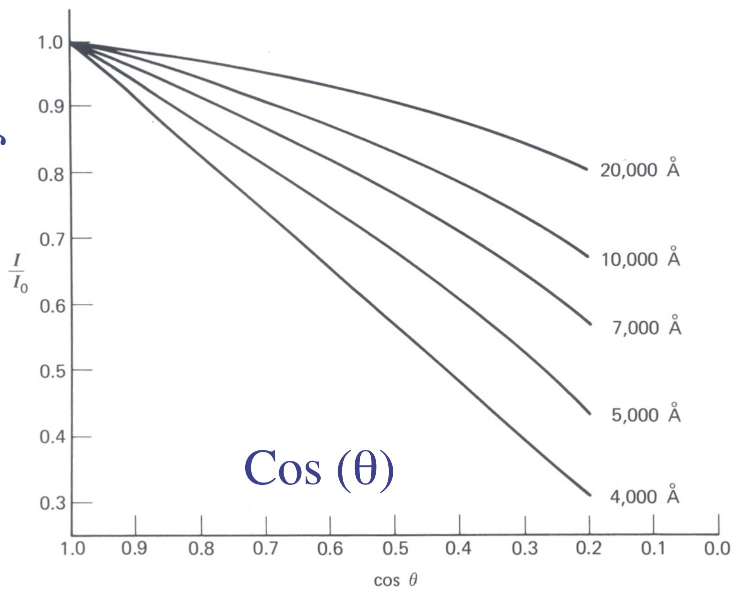
$$I_\lambda(0,\theta) = \left[A_o(\lambda) + A_1(\lambda)\cos\theta + A_2(\lambda)\cos^2\theta\right] I_\lambda(0,0)$$

3. Find source function

$$S_\lambda(\tau_\lambda) = \left[\frac{A_o}{0!}(\lambda) + \frac{A_1}{1!}(\lambda)\tau_\lambda + \frac{A_2}{2!}(\lambda)\tau_\lambda^2\right] I_\lambda(0,0)$$

Central disk intensity in absolute units

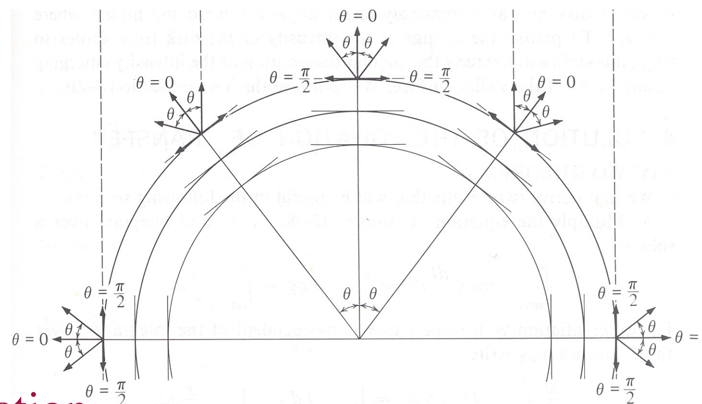
Relative Intensity



Cos (θ)

Determining the Temperature Structure of the Sun: Limb-Darkening Continued...

$$I_\lambda(0,\theta) = \left[A_o(\lambda) + A_1(\lambda)\cos\theta + A_2(\lambda)\cos^2\theta\right] I_\lambda(0,0) \longrightarrow S_\lambda(\tau_\lambda) = \left[\frac{A_o}{0!}(\lambda) + \frac{A_1}{1!}(\lambda)\tau_\lambda + \frac{A_2}{2!}(\lambda)\tau_\lambda^2\right] I_\lambda(0,0)$$



Leads to the solar temperature structure assuming:

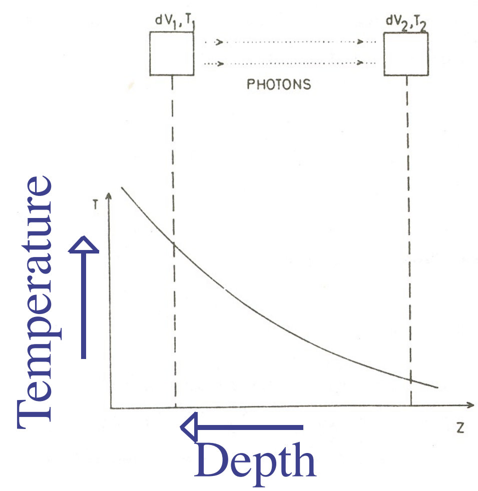
1. Plane-parallel geometry is valid.

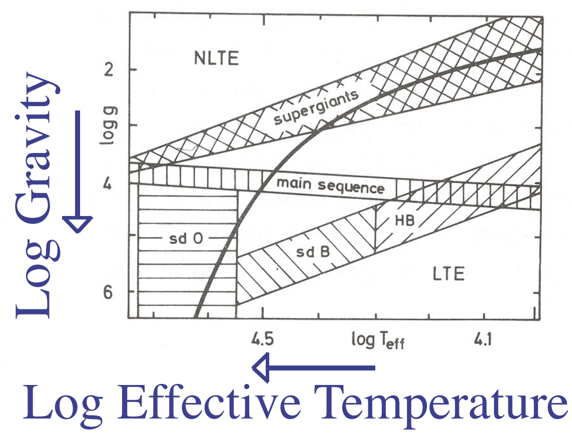
The sun's photosphere is roughly 1000 km thick versus a solar radius of 7 x 10^5 km, or an extenstion of 0.1%.

2. Local thermodynamic equilibrium is a good approximation.

The Planck function connects S(τ) to T(τ), the temperature structure.

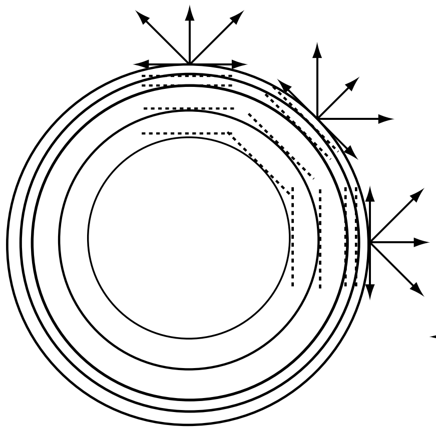
$$S_\lambda \equiv B_\lambda, \qquad B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT} - 1}$$

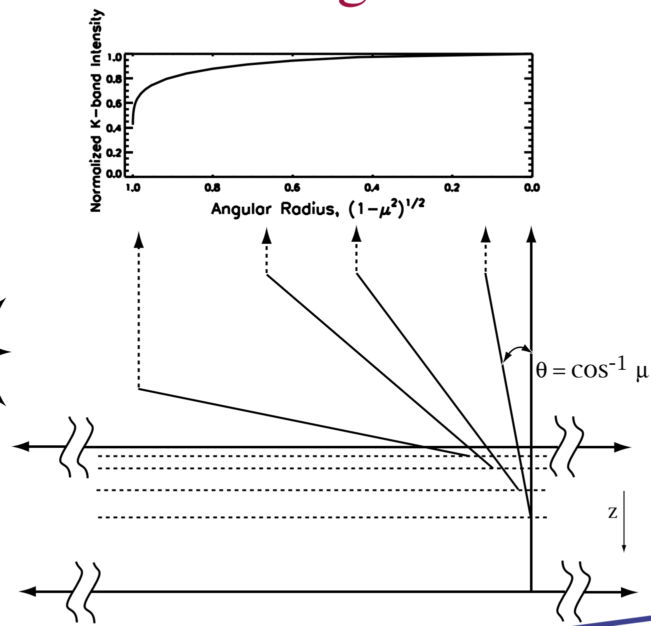




Models for Limb-Darkening: Plane-Parallel vs. Spherical Geometry

a) plane-parallel case

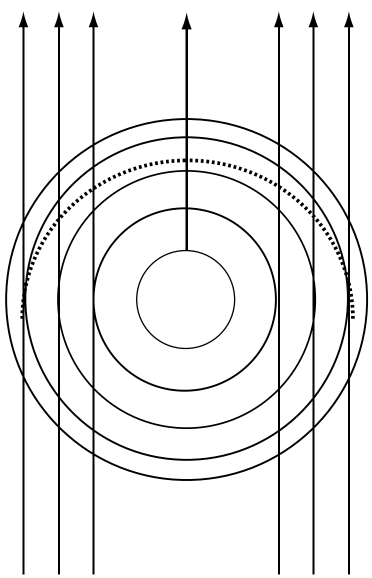


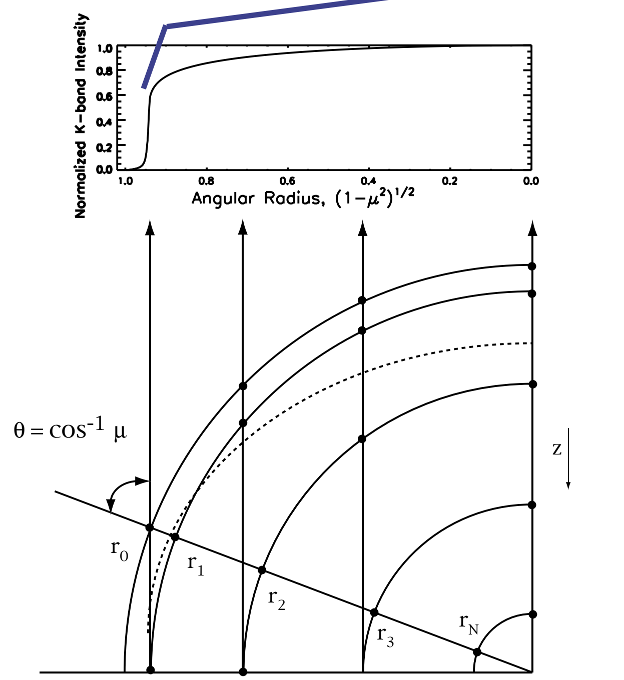


The semi-infinite nature of plane-parallel models means that the atmosphere is optically thick at all angles.

Drop off characteristic of spherical models.

b) spherical case





The rays of a spherical model impact nested shells, of which the outer most are optically thin.

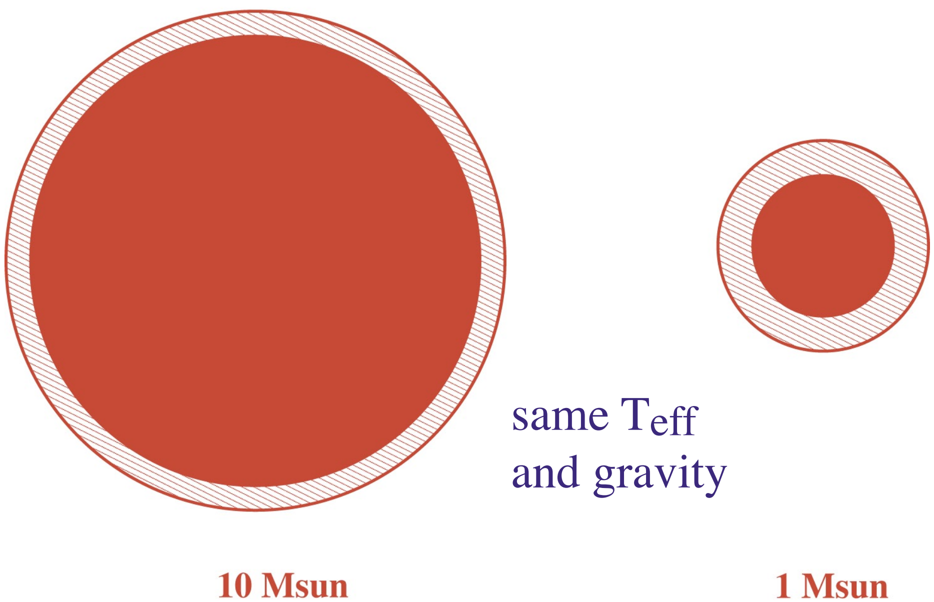
Spherical Models and the 3rd Parameter: Mass (or Radius)
Gravity, Mass, Radius: Choose Two!

The common use of plane-parallel model atmospheres (e.g. Kurucz's ATLAS programs) gets us accustomed to thinking that for a given chemical composition, a model *hydrostatic* atmosphere is primarily characterized by 1) the ***effective temperature*** and 2) the ***surface gravity***. Spherical models require a 3rd parameter, ***mass or radius***, to established the luminosity of the star (remember, flux is not conserved in the spherical case).

$$g(r) = \frac{GM}{r^2}$$

In spherical models the gravity is a function of depth. The gravity parameter must refer to a reference radius consistent with the stellar mass.

Atmospheric Extension is a function of Mass

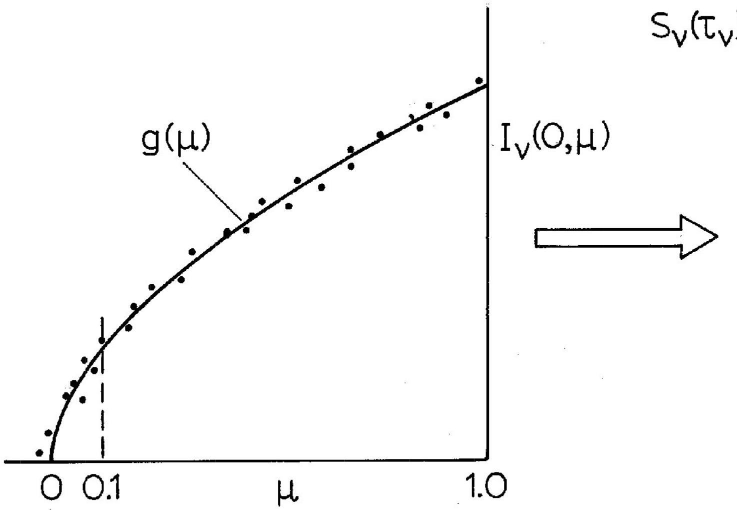


The ratios of angular diameter measurements at several wavelengths, some of which probe strong molecular bands, will depend of the extension of the atmosphere and therefore the mass.

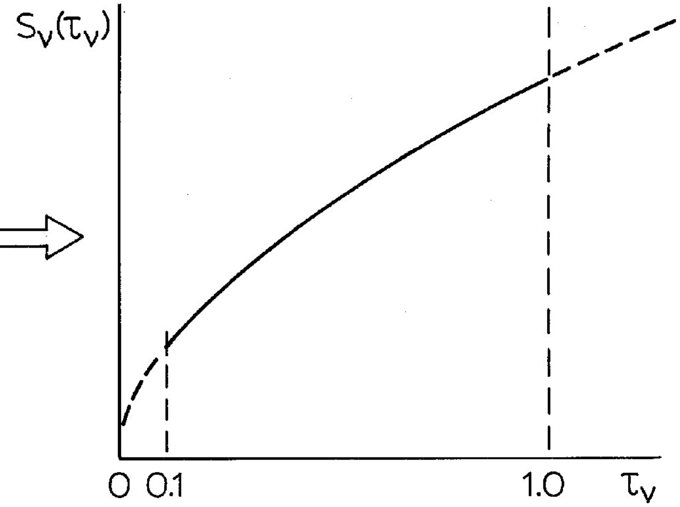
Mass is an additional free parameter for fitting visibility functions, however for near by stars the mass may be well constrained and provide a strong test for the models.

Temperature and Opacity of the Solar Atmosphere: Limb-Darkening Continued...

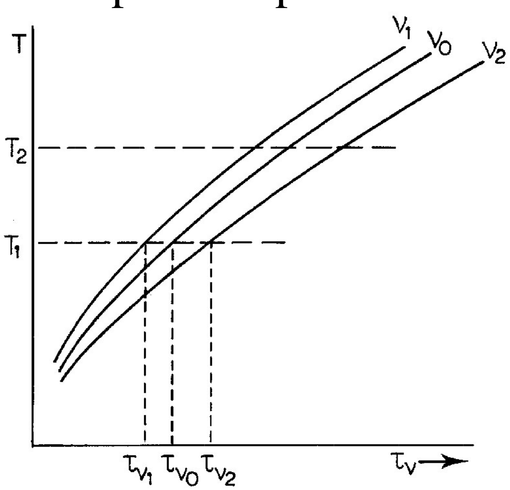
1. Limb darkening measurements



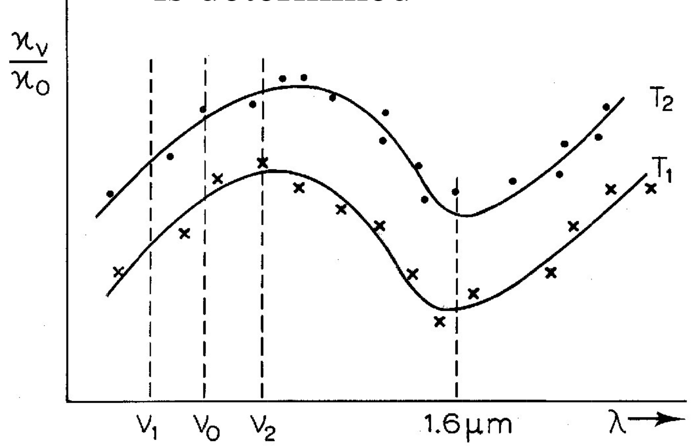
2. Source function vs. optical depth

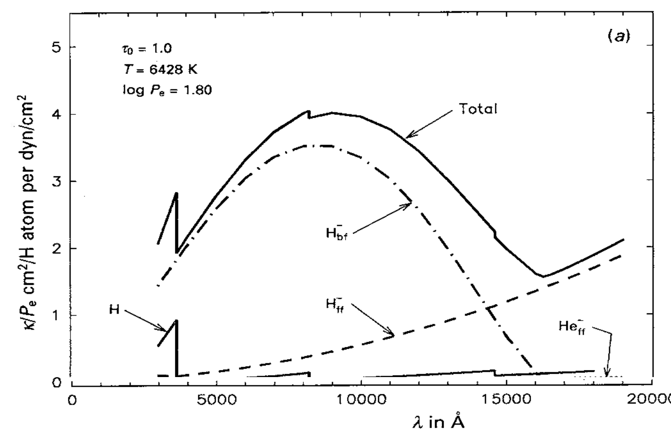


3. Assuming LTE, temperature vs. optical depth



4. For a one-to-one temperature-depth relationship, the optical vs. wavelength is determined





Determining the Temperature Structure of the Sun:
Disk-Center Absolute Intensities

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STRUCTURE OF THE SOLAR CHROMOSPHERE. II. THE UNDERLYING PHOTOSPHERE AND TEMPERATURE-MINIMUM REGION

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1. Compute the ***brightness temperature*** from the measured central intensities, *I*.

$$B(\lambda, T) = \frac{2hc/\lambda^5}{e^{hc/\lambda kT} - 1},$$

Assume *I*=*B* and solve for *T* at each wavelength:

$$T_b^{\text{center}} = \frac{14388}{\lambda \ln\left[(11909/\lambda^5 I_\lambda) + 1\right]},$$

2. Construct a model atmosphere which reproduces the brightness temperature as a function of wavelength.

Models are fine tuned to match the intensity as a function of wavelength.

The best fit model yields a ***temperature-depth relationship***.

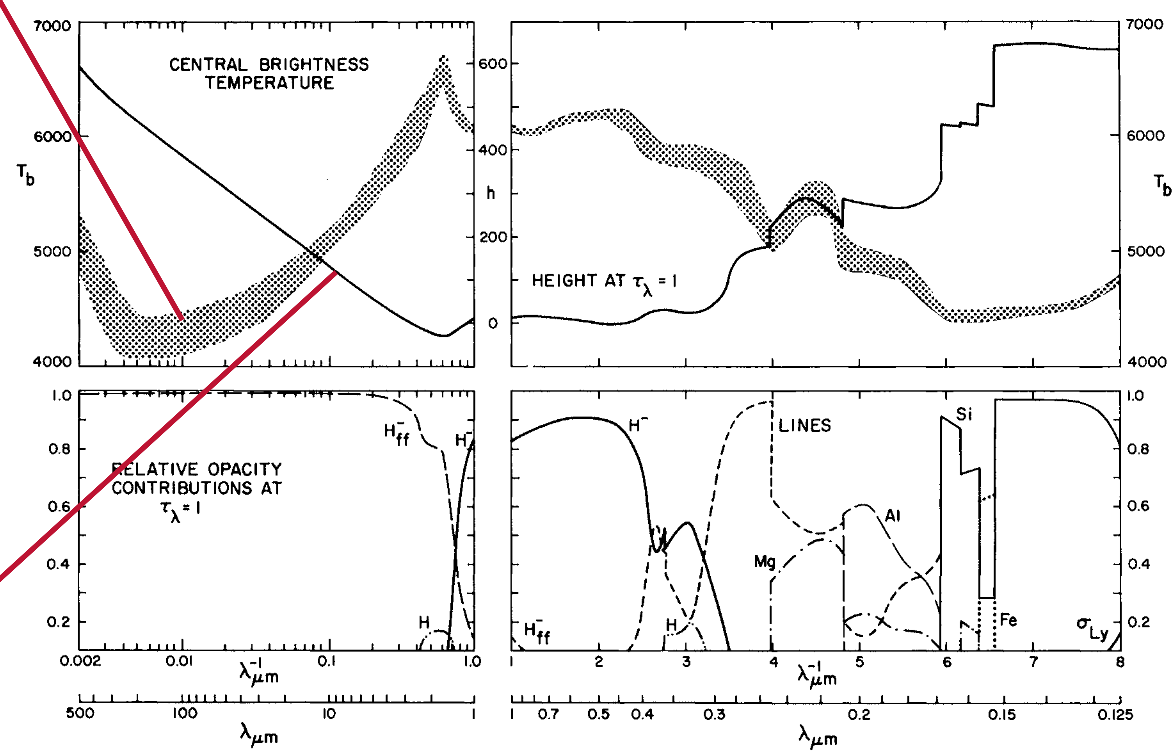


FIG. 1.—The two upper panels show the observed continuum brightness temperature at the disk center (*shaded band*) and the approximate height in the solar atmosphere where the radiation is formed (*solid line*). The lower panels illustrate the relative contributions to the opacity at this height. These quantities are plotted as functions of wavelength in the range 0.125–500 μ. The opacity due to "lines" is explained in § II*i*.

Radiative Equilibrium

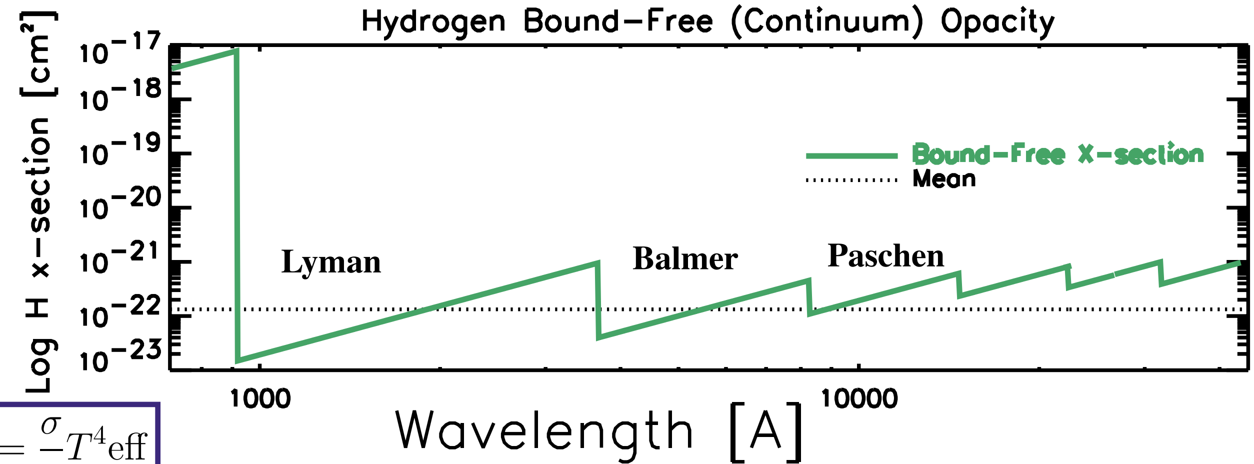
Radiative Equilibrium applies in the tenuous outer layers of stars where radiation is the dominant energy transport mechanism.

R.E. is a special case of thermal equilibrium which says that the temperature structure is not changing with time so the radiative flux must be constant.

This means the total energy absorption must equal the total energy emitted in each layer; there are no sources or sinks of radiative energy.

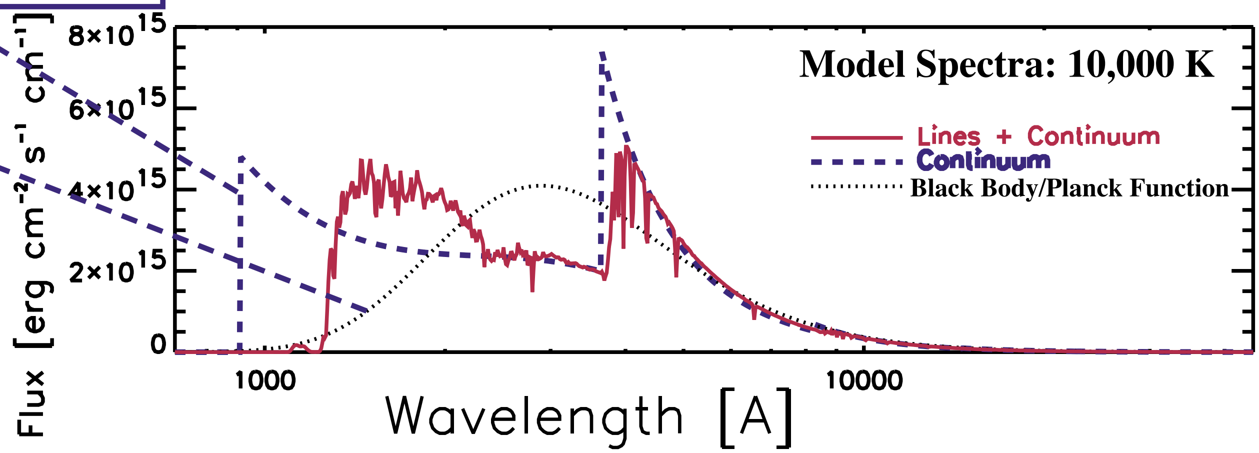
$$\int_0^\infty \kappa_\lambda J_\lambda \, d\lambda = \int_0^\infty \kappa_\lambda S_\lambda \, d\lambda$$

$$\boxed{\text{Absorbed Radiation} = \text{Emitted Radiation}}$$



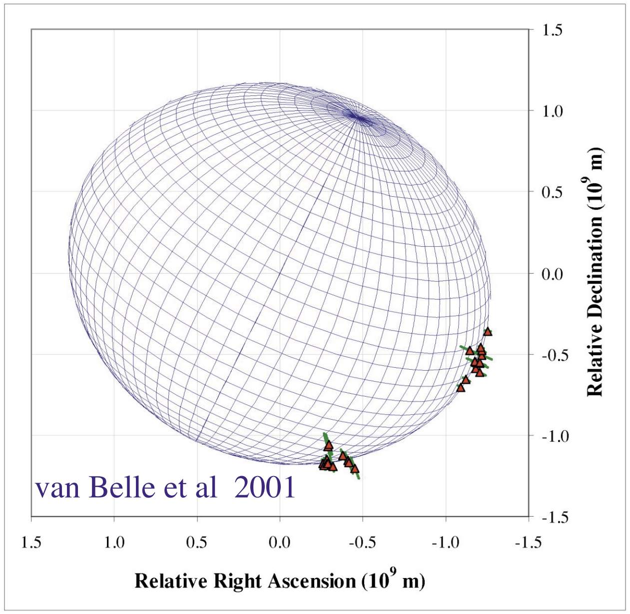
$$(F_\lambda)_{\text{blackbody}} \neq (F_\lambda)_{\text{pure hydrogen}}$$

$$\boxed{\int (F_\lambda)_{\text{blackbody}} \, d\lambda = \int (F_\lambda)_{\text{pure hydrogen}} \, d\lambda = \frac{\sigma}{\pi} T^4 \text{eff}}$$



Extended Atmospheres
For non-plane parallel atmospheres the flux is not constant, but falls off like $1/r^2$. In this case, only the luminosity is constant with depth.

Rapid Rotation: Structural Distortion & von Zeipel Gravity Darkening



Stars must satisfy both *mechanical* and *thermal* requirements for stability.

Mechanical: rotation distorts a star's figure as it adjusts its structure to maintain hydrostatic equilibrium.

Interferometric measurement of Altair (A7 V)!

Ratio of equatorial and polar radii

$$R_e(u)/R_p = \frac{3}{u}\cos\left[\frac{\pi + \arccos(u)}{3}\right]$$

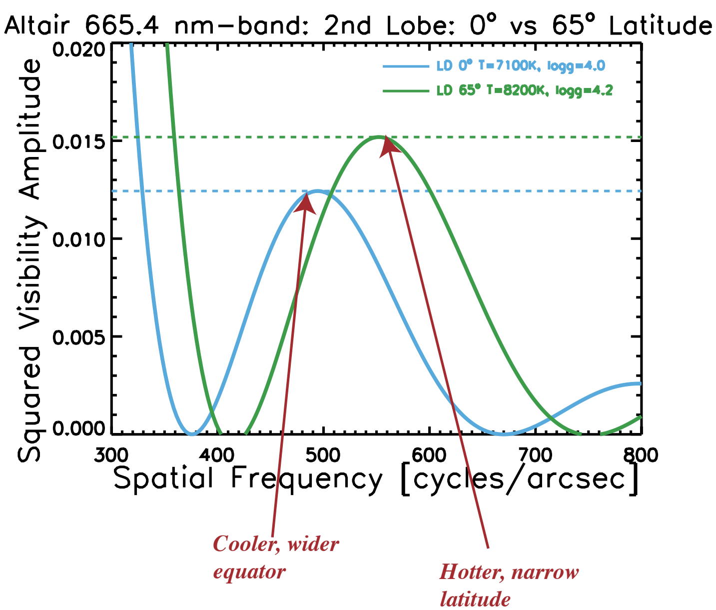
Critical angular rotation velocity

$$\omega_{\rm crit} = \sqrt{\frac{8}{27}\frac{GM}{R_p^3(\omega)}}$$

Equatorial velocity

$$V_{\rm eq} = u\,\omega_{\rm crit}\,R_e(u)$$

$$\omega = u\,\omega_{\rm crit}$$



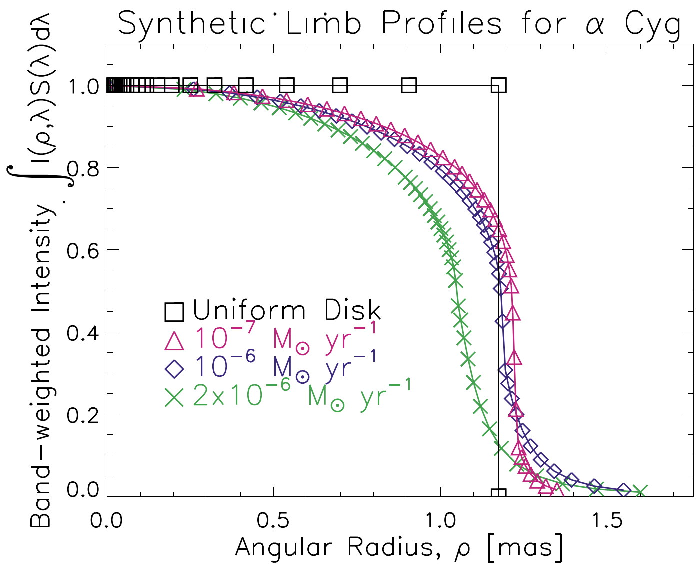
Thermal: At the equator, lower gravity reduces both the pressure and temperature gradients.
Local flux ~ local gravity
Local effective temperature ~ (local gravity)$^{1/4}$
Effective Temperature varies with stellar latitude: cooler equator, hotter pole.

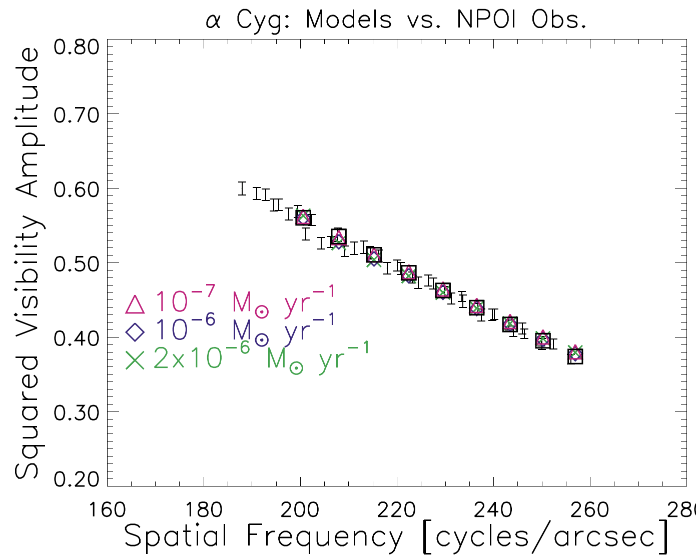
Effect: Limb-darkening varies as a function of stellar latitude

Stellar Winds, An Example
Limb-darkening Effects & Mass Loss Rates for Deneb (A2 Ia)

Atmospheric Structure

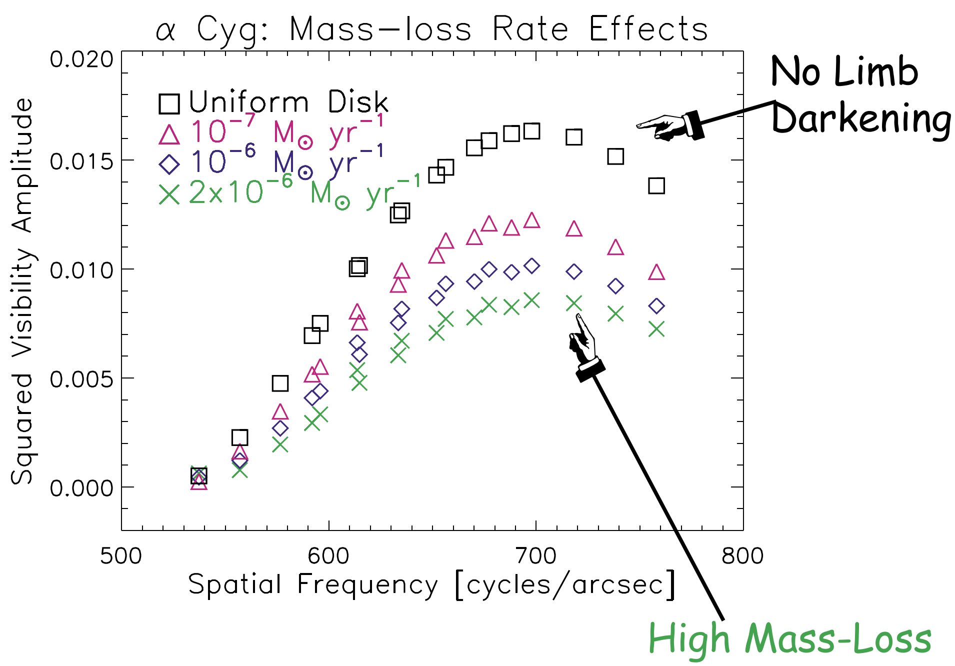
Hydrodynamic	Hydrostatic
$v = v_\infty \left(1 - \frac{R_\star}{r}\right)^{\beta_{\rm wind}}$	$\frac{dP}{d\tau} = \frac{g}{\kappa}$
$\rho = \frac{\dot{M}}{4\pi r^2 v}$	





The limb brightness is sensitive to the density structure, which is a function of the mass-loss rate.

Data in the 1st lobe *do not* constrain the mass-loss rate.

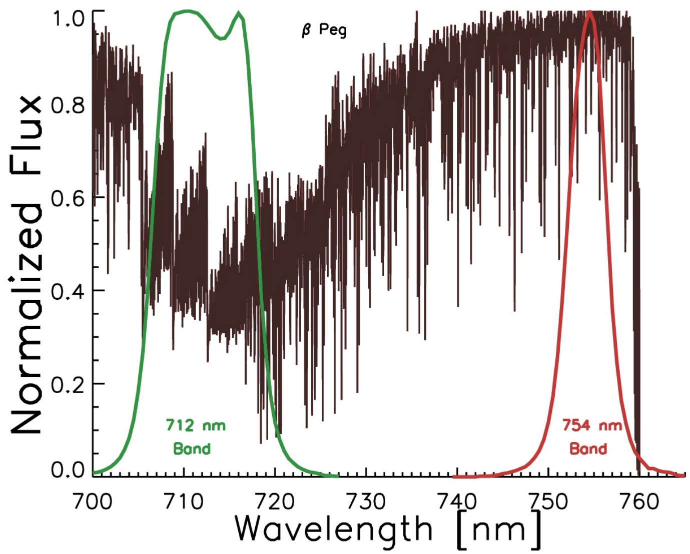


High spatial frequency data is needed to break the angular diameter/mass-loss rate degeneracy.

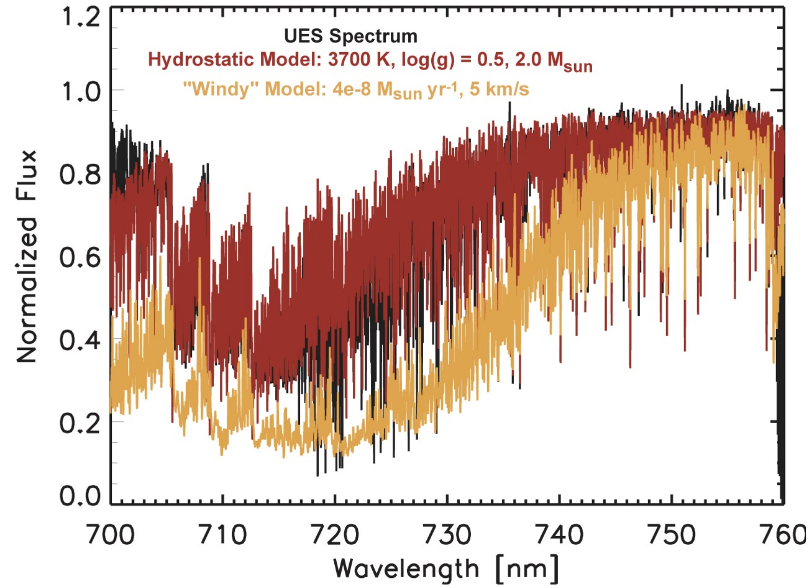
Simulations indicate that measurements of the 2nd lobe could provide a mass-loss diagnostic for hot supergiant, like Deneb and Rigel (B8 Ia).

Extended Cool Star Atmospheres, An Example: β Pegasi (M 2.5 II)

Titanium Oxide Band Head; Narrow Band Filters

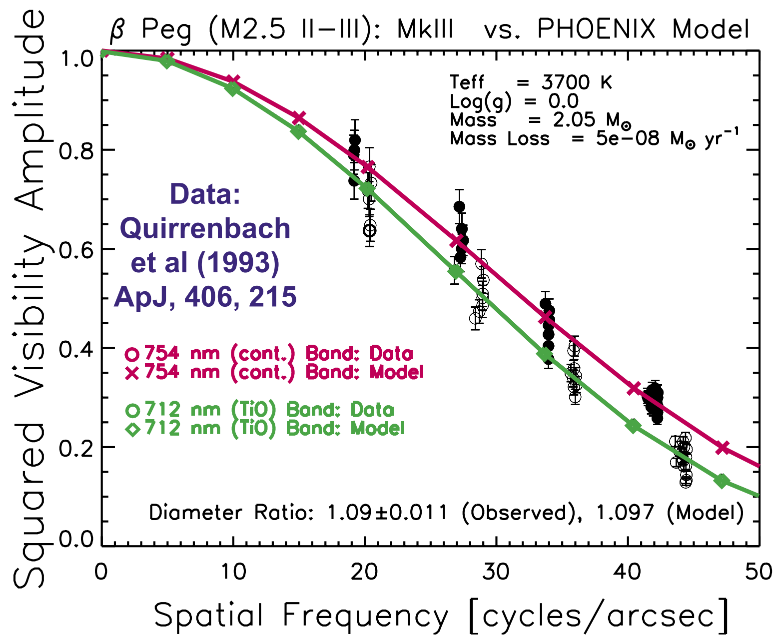


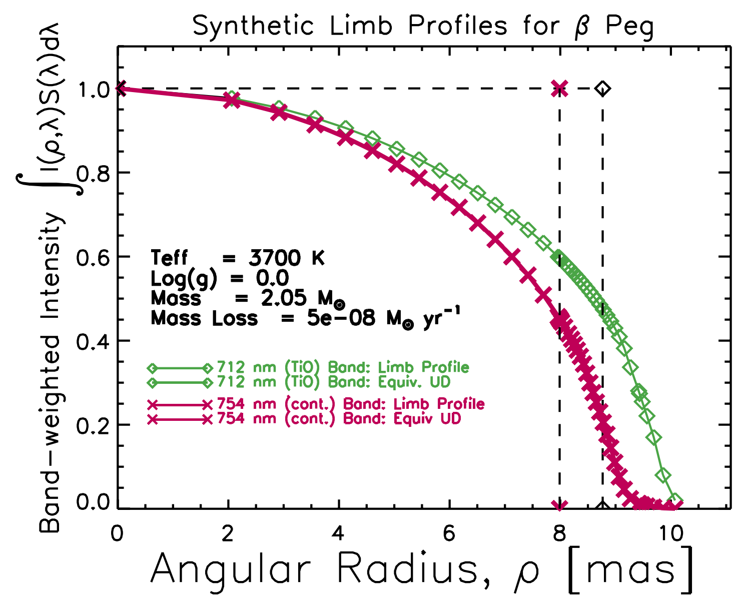
Model Spectra versus Observed Spectra



Spherical wind model fits interferometry, but not spectrum.

Spherical hydrostatic model fits spectrum, but not interferometry.





 Jason P. Aufdenberg, 10 July 2003, Michelson Interferometry Summer School

Closing Thoughts, Summary

Spectroscopy probes stellar *fluxes*, however *interferometry* probes stellar *intensities*, the basic quantity of radiative transfer in stellar atmospheres. That's very cool!

Spectroscopy and interferometry are complementary. How well does that best fit stellar atmosphere model fit both the visibility data *and* the stellar spectrum.

Stellar atmospheres are not black bodies. Published spectrophotometry exists for thousands of bright stars.

Most bright stars are variable. Contemporaneous spectrophotometry/spectroscopy and interferometry should be the goal.

You always see to an optical depth of unity. In spherical atmospheres the limb is very "fuzzy" and optically thin.

Spherical models are parameterized by T_{eff}, log(g) *and* Mass.

Outer boundaries of real stellar atmospheres are complicated by winds, shells, chromospheres, convection, magnetic fields, pulsation, etc. Realistic physical models are beyond challenging. Interferometry will help to further constrain these fascinating problems.

Synthetic Visibilities

Some References for Stellar Atmospheres

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* ***Introduction to Stellar Atmospheres and Interiors***
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* ***Kinetic Theory of Particles and Photons: Theoretical Foundations of Non-LTE Plasma Spectroscopy*** -- J. Oxenius (Springer)

* ***The Analysis of Star Light: One hundred and fifty years of astronomical spectroscopy*** -- J. B. Hearnshaw

* ***Mapping the Spectrum - Techniques of visual representation in research and teaching*** -- K. Hentschel (Oxford)