

Quantifying The Information Content In High Contrast Imaging Data

UCSan Diego
SCHOOL OF PHYSICAL SCIENCES

Department of Astronomy and Astrophysics

Jayke Nguyen (UCSD), Quinn Konopacky (UCSD)

The Fisher Information is a way to measure the amount of information that an observable carries about a given model parameter. In turn, that can be used to quantify the sensitivity and uncertainty of model parameters.

We use the Fisher information to investigate how the tunable parameters in post-processing techniques in high-contrast imaging affect the precision of our flux and astrometry.

Model Parameters

$$ec{ heta}=[F,x,y,B]$$
 $\mu_i=F*\Phi(x,y)+B^{5}$

From our model, we can build the Fisher Information Matrix using partial derivatives with respect to our model parameters.

Matrix Terms

$$\mathcal{I}_{j,k} = \sum_{i} \frac{1}{\sigma_i^2} \left(\frac{\partial \mu_i}{\partial \theta_j} \right) \left(\frac{\partial \mu_i}{\partial \theta_k} \right)$$

The matrix terms can be interpreted as the Jacobian weighted by the noise model. The noise model can contain information about the temporal and spatial correlation of speckles!

 $\frac{\mathcal{L}_i}{i} \ \sigma_i^2 \ \ \partial heta_j \ \ \partial heta_i$ Noise

(The noise model can be obtained by taking the frame-wise standard deviation of the de-rotated PSF stack)

Model

$$I_{FF} = \sum_{i} rac{\Phi_{i}^{2}}{\sigma_{i}^{2}}$$

This first term is just same as the convolution using forward model matched filter! [1]

Fisher
Information
Matrix

$$\mathcal{I}(ec{ heta}) = egin{bmatrix} I_{FF} & I_{Fx} & I_{Fy} & I_{FB} \ I_{xF} & I_{xx} & I_{xy} & I_{xB} \ I_{yF} & I_{yx} & I_{yy} & I_{yB} \ I_{BF} & I_{Bx} & I_{By} & I_{BB} \end{bmatrix}$$

The Cramer-Rao lower bound (CRLB) [2] gives us a minimum uncertainty in our model parameters from the Fisher Information Matrix

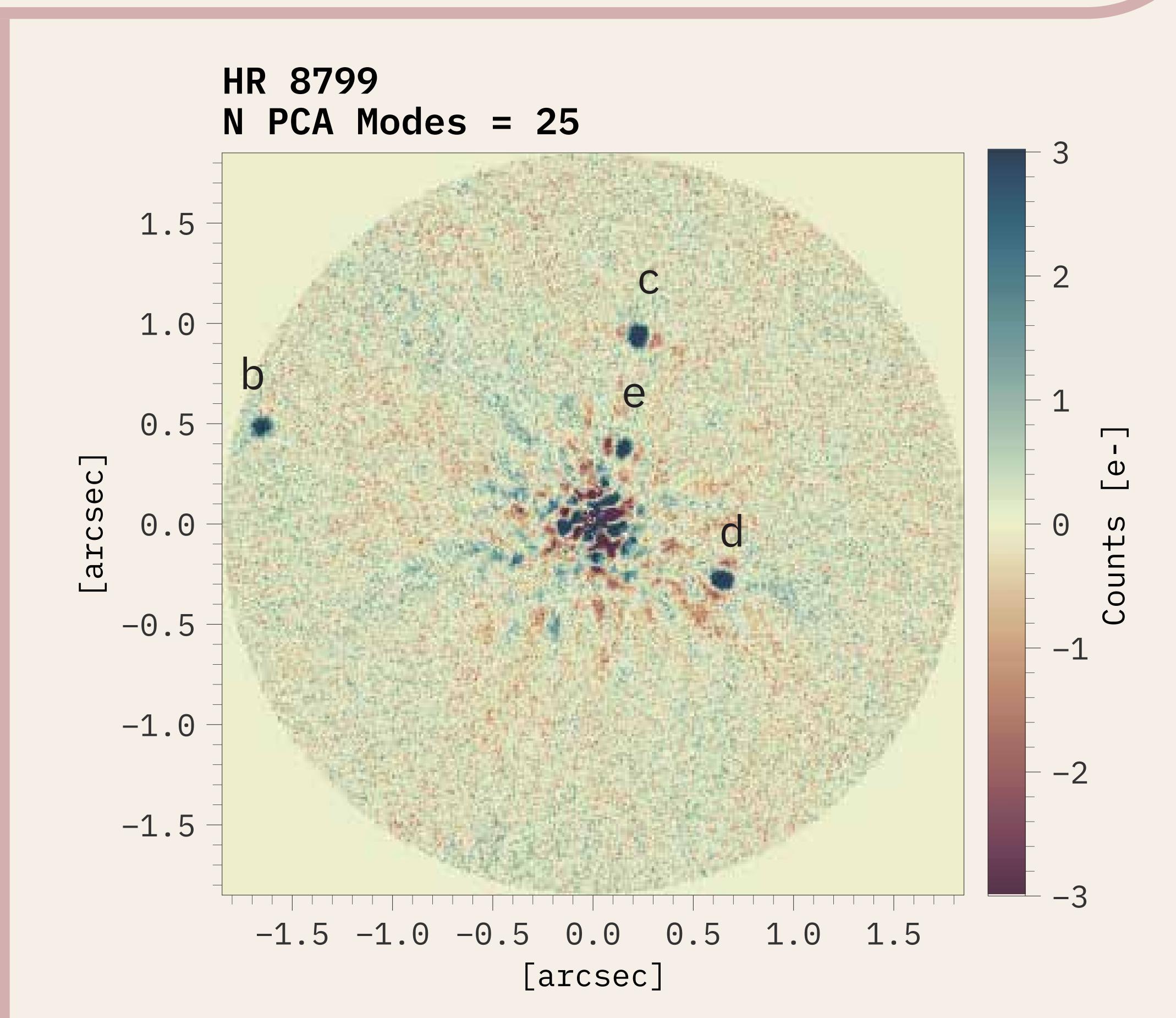
The CRLB is a information-theoretic guarantee on the precision of your model parameters and is typically a conservative estimate, depending on your

data and noise models.

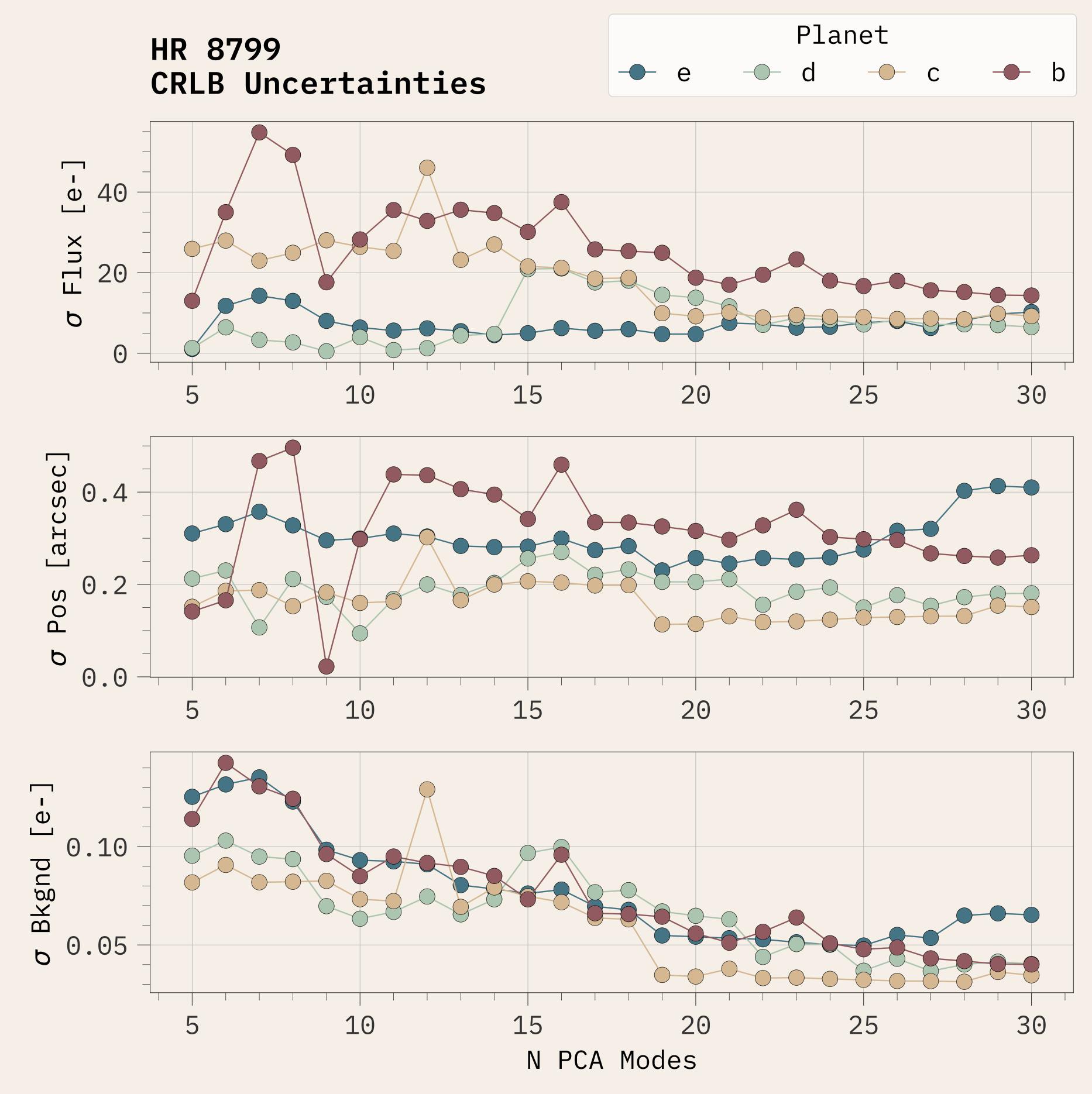
$$var(\vec{\theta}) > \mathcal{I}^{-1}(\vec{\theta})$$

Covariance matrix

The diagonal of this matrix gives the variance, or the square of the uncertainties in each of the parameters.



Using the Fisher Information, we can quantify how the number of PCA modes we subtract affects our precision *before* doing any kind of fitting to the data!



- Flux precision improves until about 25 modes
- Position uncertainty is relatively insensitive to the number of modes
- Background uncertainty drop linearly with the number of modes

None of these results are surprising, but now we have the tools to quantify the information of other tunable post-processing parameters. Future analyses will look at parameters like the number of frames in our reduction or the amount of sky rotation and how those impact our final sensitivity.

References

- 1. Ruffio, J.B., et al. (2017). Improving and Assessing Planet Sensitivity of the GPI Exoplanet Survey with a Forward Model Matched Filter. The Astrophysical Journal, 842 (1):14, Jun 2017. doi: 10.3847/1538-4357/aa72dd. URL https://doi.org/10.3847/1538-4357/aa72dd.
- 2. Rao, C.R. (1992). Information and the Accuracy Attainable in the Estimation of Statistical Parameters. In: Kotz, S., Johnson, N.L. (eds) Breakthroughs in Statistics. Springer Series in Statistics. Springer, New York, NY. https://doi.org/10.1007/978-1-4612-0919-5_16