Laboratory nano-flares generated from multiple braided current loops

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Main Points

We present a first-principles mechanism demonstrating how a weakly ionized accretion disk sheds angular momentum and produces the electric power for driving bidirectional astrophysical jets.

1. The basic conserved quantity in an axisymmetric system with a magnetic field is the canonical angular momentum $mrv\theta+q\psi/2\pi$ rather than the ordinary angular momentum $mrv\theta$, This comes from fundamental Hamiltonian/Lagrangian mechanics. ψ is the poloidal magnetic flux.

2. When the Kepler angular velocity and the magnetic field have opposite polarity, collisions between neutrals and charged particles cause:

(i) ions to move radially inwards,

(ii) electrons to move radially outwards,

(iii) both ions and electrons to gain canonical angular momentum,

(iv) neutrals to lose ordinary angular momentum and move radially inwards (accrete)

(v) the total canonical angular momentum of the system (electrons, ions, and neutrals) to be conserved.

3. The accumulation of ions at small radius and accumulation of electrons at large radius produces a radially outward electric field.

4. In 3D, this radial electric field would drive an out-of-plane poloidal current that produces the magnetic forces that drive bidirectional astrophysical jets.

Mathematical Explanation

Motion of charged particles colliding with Kepler-motion neutrals

Equation of motion for charged particles

$$\frac{d\mathbf{u}_\sigma}{dt}=-\frac{GM_*}{r^2}\hat{r}+\omega_{c\sigma}\mathbf{u}_\sigma\times\hat{z}-\nu_{\sigma n}\left(\mathbf{u}_\sigma-\mathbf{u}_n\right)$$
$$\approx\omega_{c\sigma}\mathbf{u}_\sigma\times\hat{z}-\nu_{\sigma n}\left(\mathbf{u}_\sigma-\mathbf{u}_n\right)$$

The solution is

$$\mathbf{u}_\sigma=\left(\mathbf{u}_L\cos\omega_{c\sigma}t+\mathbf{u}_L\times\hat{z}\sin\omega_{c\sigma}t\right)e^{-\upsilon_{\sigma n}t}+\frac{\frac{\upsilon_{\sigma n}}{\omega_{c\sigma}}\mathbf{u}_n\times\hat{B}+\frac{\upsilon_{\sigma n}^2}{\omega_{c\sigma}^2}\mathbf{u}_n}{1+\frac{\upsilon_{\sigma n}^2}{\omega_{c\sigma}^2}}$$

$$\mathbf{u}_\sigma=\frac{\frac{\upsilon_{\sigma n}}{\omega_{c\sigma}}\mathbf{u}_n\times\hat{B}+\frac{\upsilon_{\sigma n}^2}{\omega_{c\sigma}^2}\mathbf{u}_n}{1+\frac{\upsilon_{\sigma n}^2}{\omega_{c\sigma}^2}}$$

$$\boxed{u_{\sigma r}=\frac{\xi_\sigma}{1+\xi_\sigma^2}u_{n\theta}}$$

$$P_\theta=mrv_\theta+\frac{qBr^2}{2}$$

$u_{n\theta}>0$
$B<0$
$\xi_i<0$
$\xi_e>0$

Simulation case

Ions move inward with increasing CAM

$$\boxed{u_{ir}<0\qquad P_{\theta i}\nearrow}$$

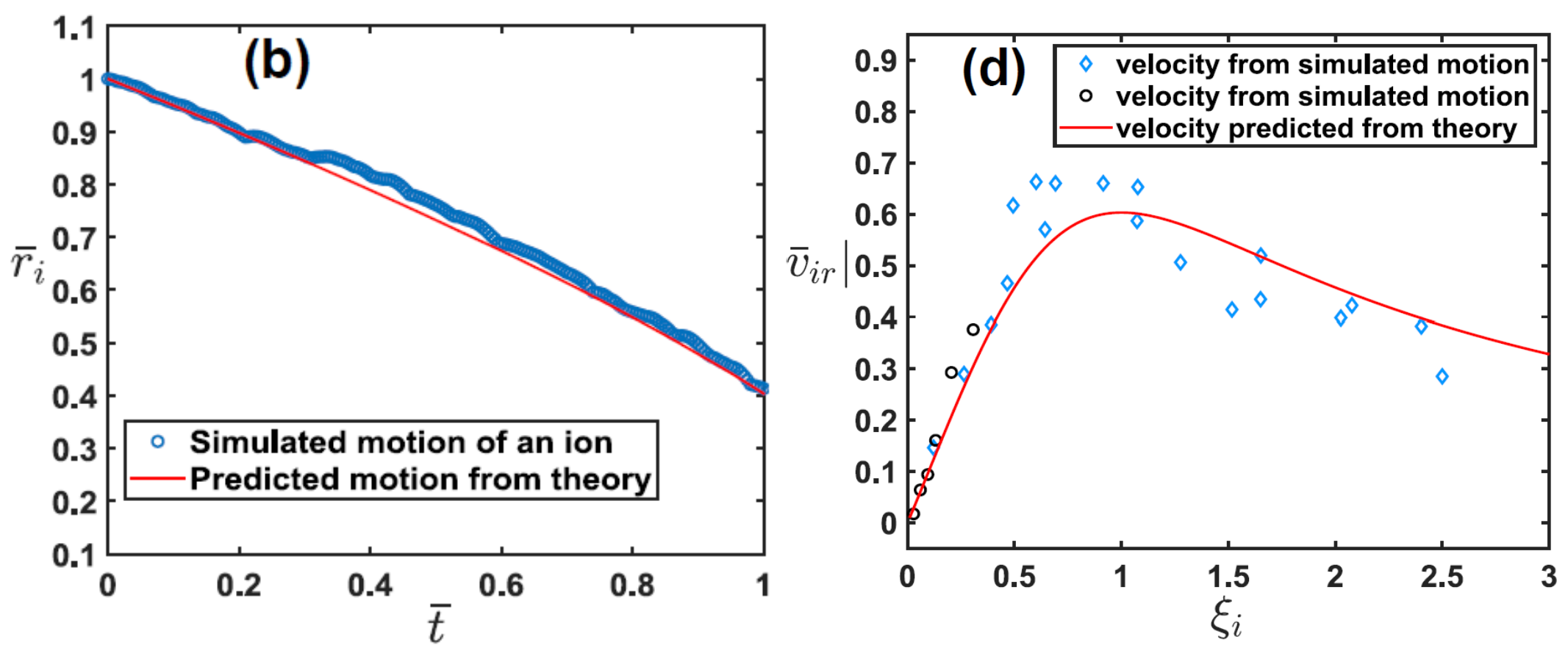
Electrons move outward with increasing CAM

$$\boxed{u_{er}>0\qquad P_{\theta e}\nearrow}$$

Neutrals accrete with decreasing CAM

$$\boxed{u_{nr}<0\qquad P_{\theta n}\searrow}$$

Insensitive to the magnetic field polarity!



Neutral's radial velocity

At a specific radius r, the canonical angular momentum density at this radius is

$$P_\theta=n_nm_nr_nu_{n\theta}+n_im_ir_iu_{i\theta}+n_em_er_eu_{e\theta}+\frac{1}{2}n_iq_iBr_i^2+\frac{1}{2}n_eq_eBr_e^2$$

During a small time Δt, the change of CAM density is

$$\Delta P_\theta=n_nm_n\left(\Delta r_nu_{n\theta}+r_n\Delta u_{n\theta}\right)+n_im_i\left(\Delta r_iu_{i\theta}+r_i\Delta u_{i\theta}\right)+n_em_e\left(\Delta r_eu_{e\theta}+r_e\Delta u_{e\theta}\right)$$
$$+n_iq_iBr_i\Delta r_i+n_eq_eBr_e\Delta r_e=0$$

During collisions, $n_nm_nr_n\Delta u_{n\theta}+n_im_ir_i\Delta u_{i\theta}+n_em_er_e\Delta u_{e\theta}=0$

$$n_im_i\Delta r_iu_{i\theta}+n_em_e\Delta r_eu_{e\theta}\ll n_iq_iBr_i\Delta r_i+n_eq_eBr_e\Delta r_e$$

$$\Delta P_\theta\approx n_nm_n\Delta r_nu_{n\theta}+n_iq_iBr_i\Delta r_i+n_eq_eBr_e\Delta r_e=0$$

$$\boxed{u_{nr}=-\frac{\omega_{ci}}{\omega_K}\frac{m_i}{m_n}\left(\chi_iu_{ir}-\chi_eu_{er}\right)}$$

Mass accretion rate

$$\dot{M}=2\pi ru_{nr}\Sigma\qquad\chi_\sigma=\frac{n_\sigma}{n_n}\qquad\xi_\sigma=\frac{\nu_{\sigma n}}{\omega_{c\sigma}}$$

$$\boxed{\dot{M}=2\pi\chi n_nr^2h\left|qB\right|\left(\frac{\left|\xi_i\right|}{1+\xi_i^2}+\frac{\left|\xi_e\right|}{1+\xi_e^2}\right)}$$

From our model

$$\boxed{\dot{M}=3\times10^{-8}M_\odot\cdot\text{year}^{-1}}$$

From observation

$$\dot{M}=10^{-9}-10^{-7}M_\odot\cdot\text{yr}^{-1}$$

Simulation Results

- A central star with mass M_* at the origin
- Uniform magnetic field $\mathbf{B}=B\hat{z}$
- Particles restricted to $z=0$ plane
- N-body simulation

Between collisions

$$\frac{d\bar{\mathbf{v}}_\sigma}{d\bar{t}}=-\frac{1}{\bar{r}^2}\hat{r}+\bar{\omega}_{c\sigma}\bar{\mathbf{v}}_\sigma\times\hat{z}$$

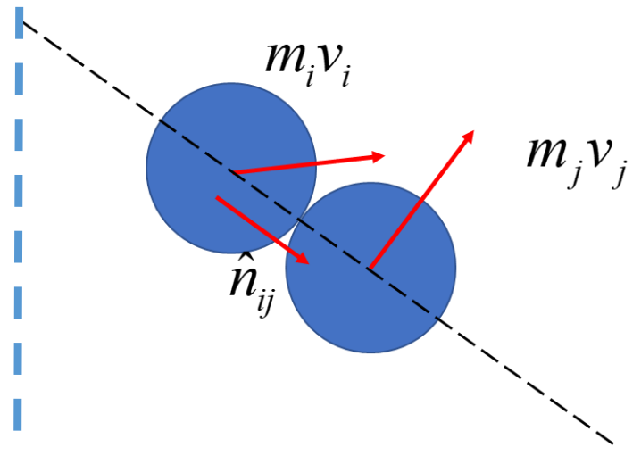
Gravity **Magnetic force**

$r_0=1\text{AU}$

$v_{K0}=\sqrt{GM_*/r_0}$

$\omega_{K0}=\sqrt{GM_*/r_0^3}$

Collision happens when $\left|\mathbf{r}_i-\mathbf{r}_j\right|<2a$



$\mathbf{v}_{ij}=\mathbf{v}_i-\mathbf{v}_j$

$$\delta\mathbf{v}_i=-\frac{2m_j}{m_i+m_j}\left(\mathbf{v}_{ij}\cdot\hat{n}_{ij}\right)\hat{n}_{ij}$$

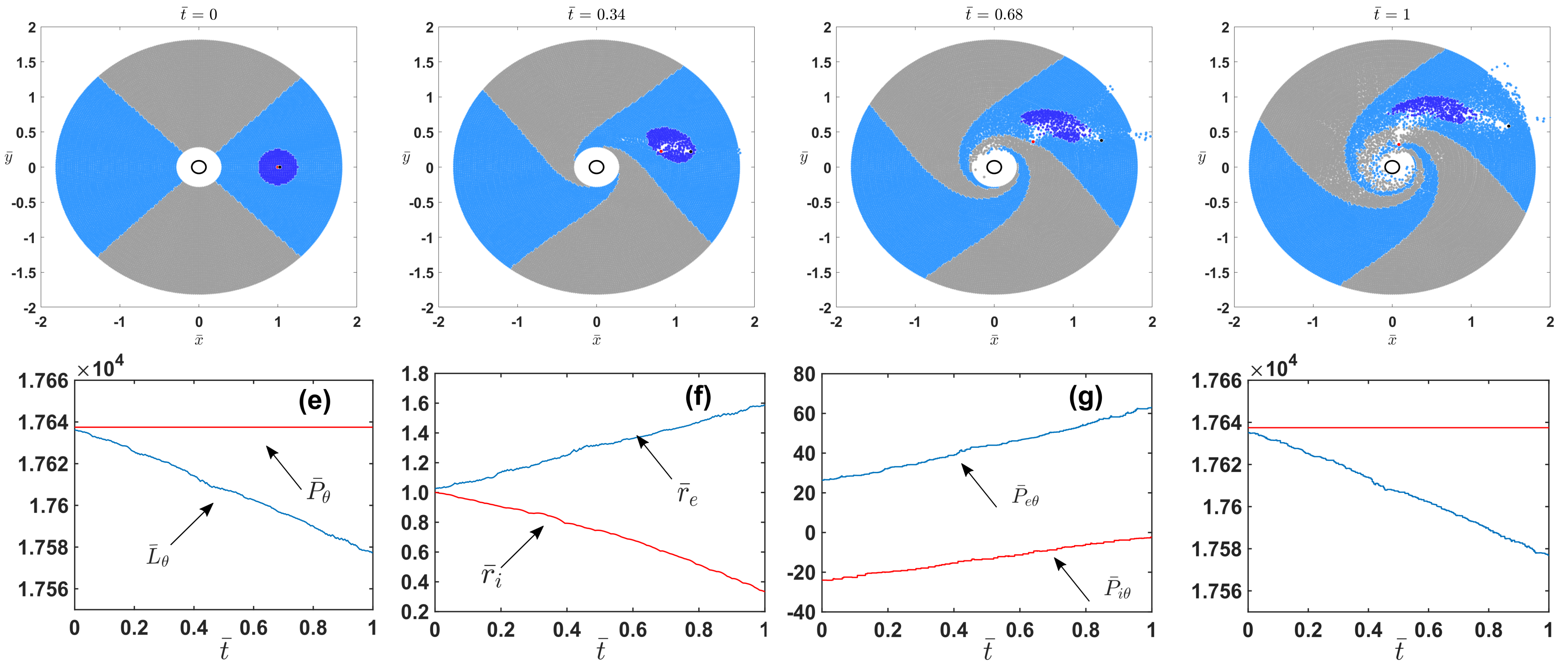


Figure 1. Simulation results when there is an electron–ion pair in the system. (a)–(d) The particle trajectories of the whole system. **Neutrals** are **blue** and **gray**. The **ion** is **red**, and the **electron** is **black**. The neutrals surrounding the electron–ion pair at $\bar{t}=0$ are dark blue. (e) The total canonical angular momentum of the system and the total ordinary angular momentum of the system. (f) The radial positions of the ion and electron. (g) The canonical angular momentum of the ion and electron. (h) The ordinary angular momentum of the neutrals and the total canonical angular momentum of the system

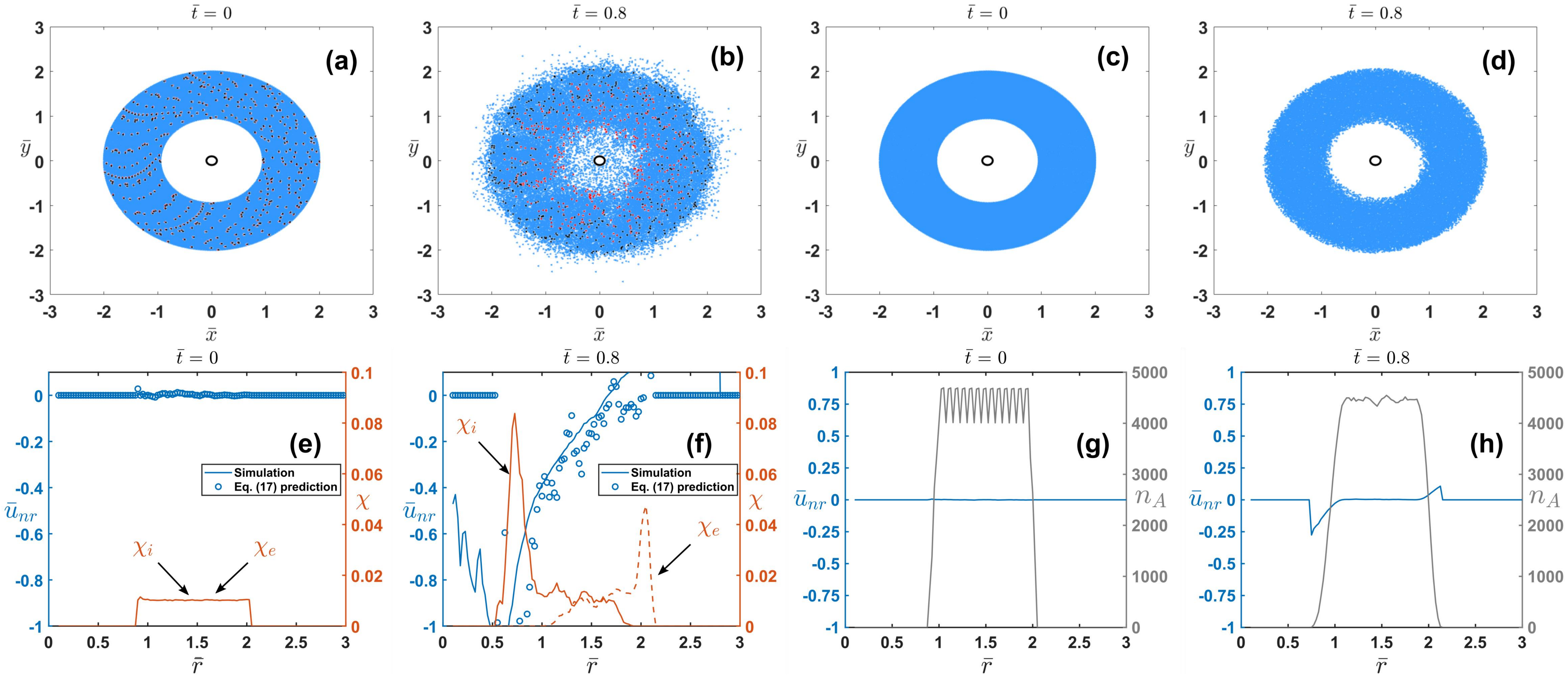
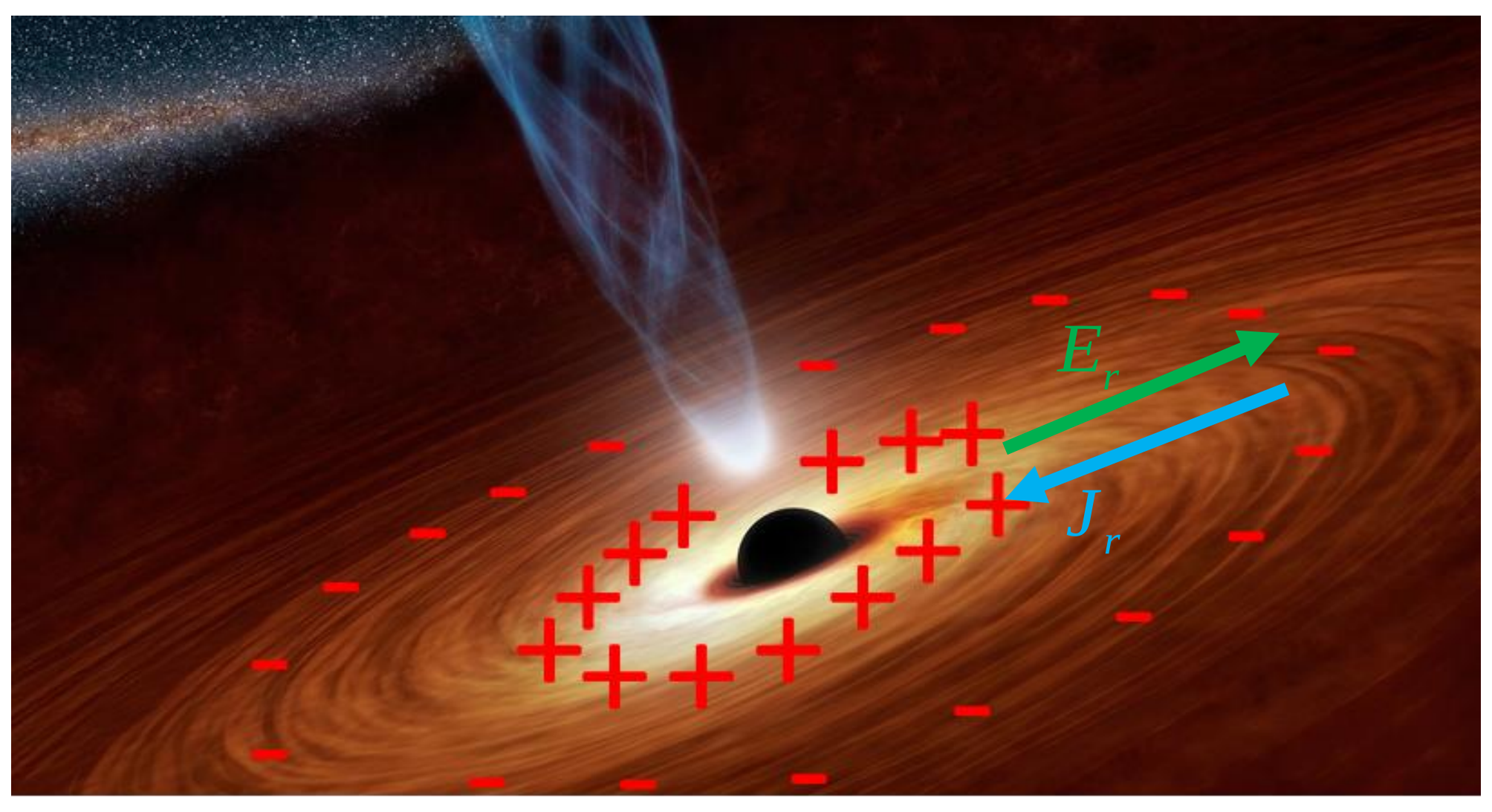
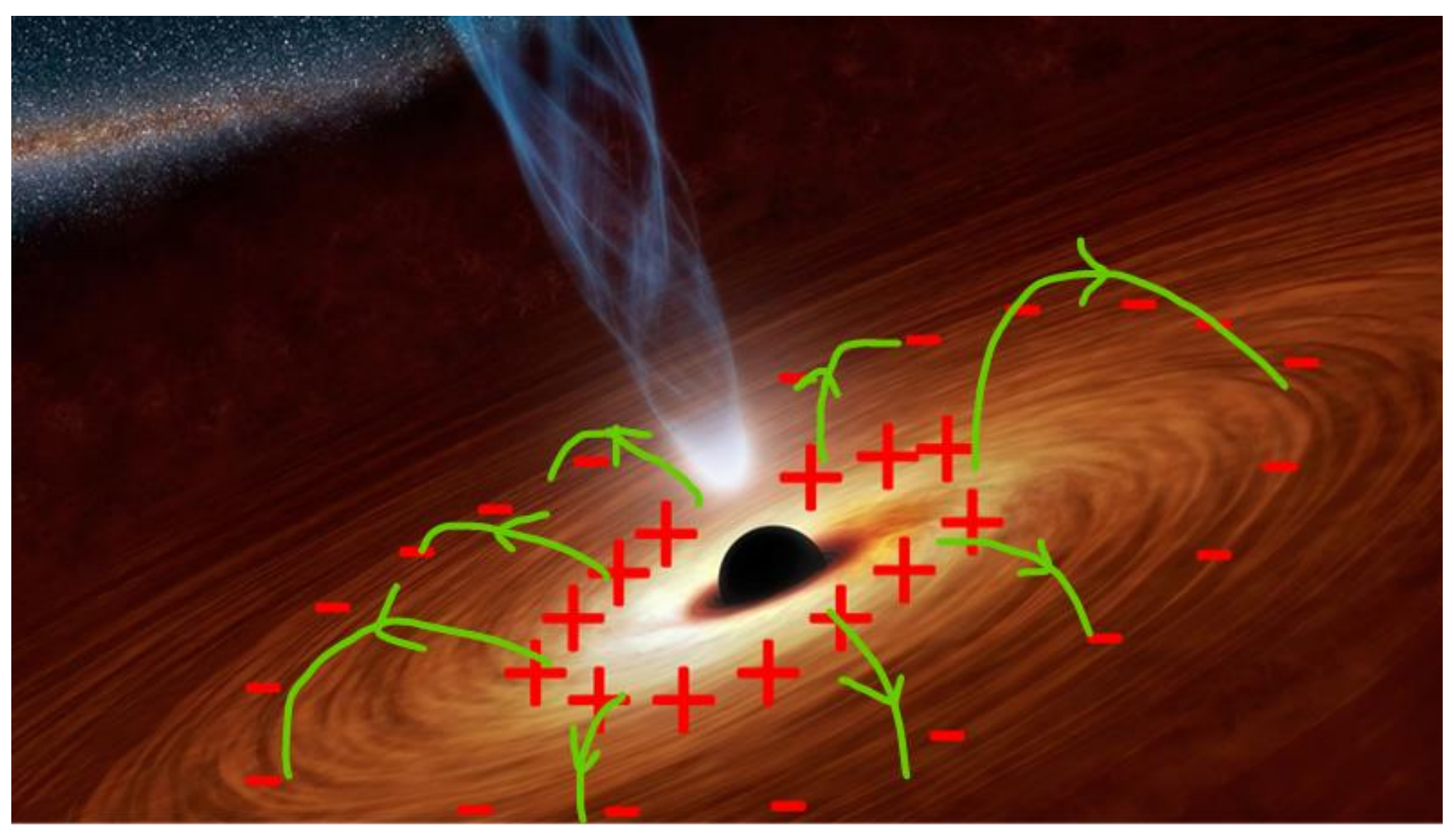


Figure 2. (a), (b) The particle trajectories of a system with ions and electrons. The initial velocity is a Kepler velocity plus a random thermal velocity having 10% of the Kepler velocity magnitude. (c), (d) The particle trajectories of a reference system having neutrals only with the same initial condition for neutrals as in (a). (e), (f) The neutral radial drift velocity profile and the density fraction of ions and electrons of the system in (a), (b). The blue line is the radial drift velocity profile of neutrals obtained from the simulation. The blue circles are the radial velocity of neutrals calculated as a function of the ion radial velocity and electron radial velocity as predicted by Equation (17). The red solid/dashed line shows the ion/electron density fraction vs. radial position. (g), (h) The neutral drift velocity profile and neutral surface density n_A vs. radial position of the system of (c), (d).

Astrophysical Jets Generation



$E_rJ_r<0$



Poloidal Current

Caltech Astrophysical Jets Experiment